STA404

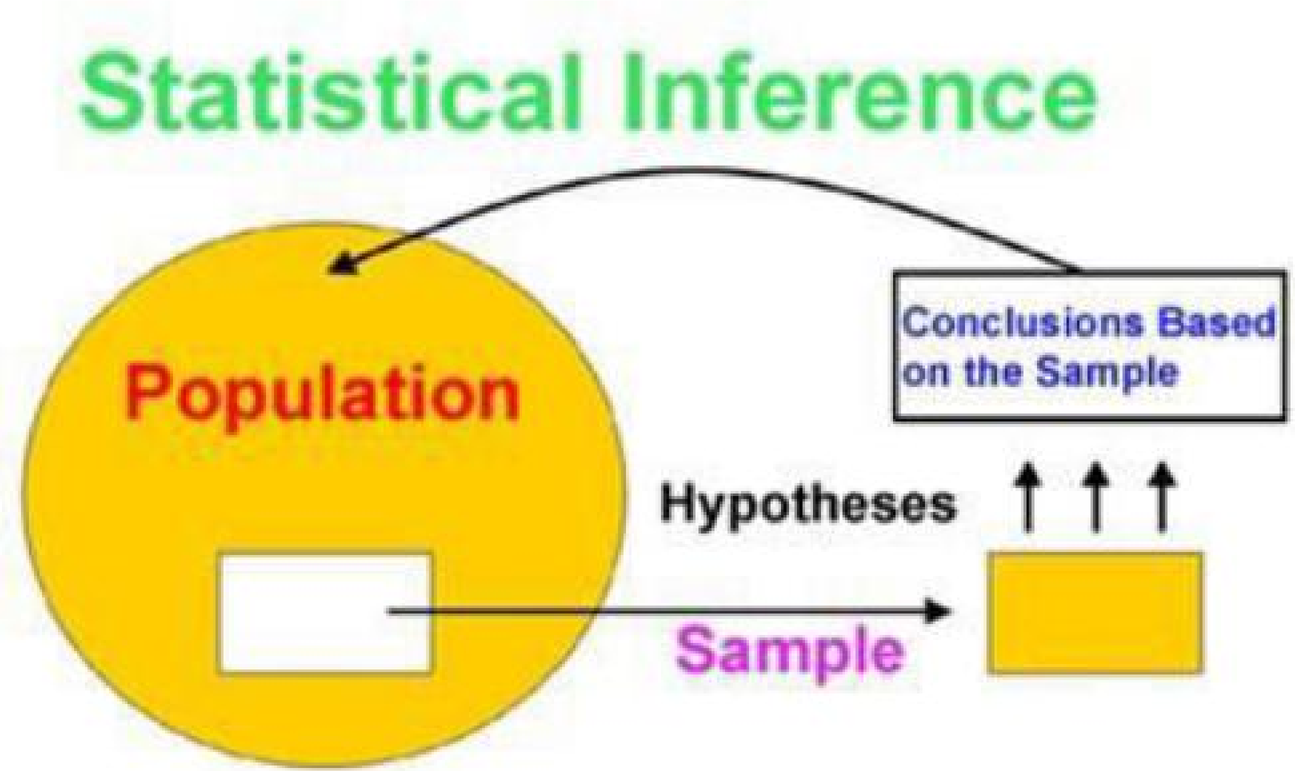
Regression & Correlation Analysis

Lesson 8-1

Statistical Inference about Regression

Explanation of the hypothesis testing procedure for simple linear regression coefficient §

What is Statistical inference?



MCQs

Difference between Population & Sample Regression

- Population Regression

$$y = \alpha + \beta x + \varepsilon$$

- Sample Regression

$$\hat{y} = a + bx + e$$

Hypothesis Testing – General Procedure

Procedure Table

Solving Hypothesis-Testing Problems (Traditional Method)

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s) from the appropriate table

Step 3 Compute the test value.

Step 4 Make the decision to reject or not reject the null hypothesis.

Step 5 Summarize the results.

Testing Hypothesis – Procedure about Regression Coefficient β

Step 1

Stat the hypothesis.

$H_0 : \beta = 0$ and $H_1 : \beta \neq 0$

$H_0 : \beta \leq 0$ and $H_1 : \beta > 0$

$H_0 : \beta \geq 0$ and $H_1 : \beta < 0$

Step 2

Decide the level of significance.

Here, we set the value of $\alpha = 5\% = 0.05$

General formula format

Many hypotheses are tested using a statistical test based on the following general formula:

$$Test\ Value = \frac{(Observed\ value) - (Expected\ value)}{Standard\ error}$$

The observed value is the "statistic" (such as the mean) that is computed from the **sample data.**

The expected value is the "parameter" (such as the mean) that you would expect to obtain if the null hypothesis were true. In other words, the hypothesized value.

The denominator is the standard error of the statistic being tested (in this case, the standard error of the mean).

$$t = \frac{b - \beta_0}{S.E(b)}$$

It is t-test. It depends on *Degree of Freedom* = df = n-2

Required calculations

Step 4

Know and do the required calculations and find the test value.

$$t = \frac{b - \beta_0}{S.E(b)}$$

$\beta_0 \Rightarrow$ Hypothesis / given

$$b = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum X^2 - (\sum X)^2}$$

$$a = \bar{Y} - b\bar{X}$$

$$s_{YX} = \sqrt{\frac{\sum(Y - \hat{Y})^2}{n-2}} \text{ or } = \sqrt{\frac{\sum Y^2 - a\sum Y - b\sum XY}{n-2}}$$

$$S.E(b) = s_b = \frac{s_{YX}}{\sqrt{\sum(X - \bar{X})^2}}$$

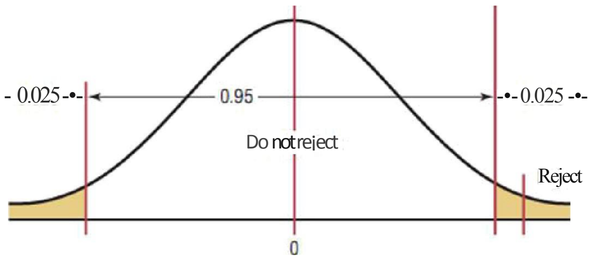
Short Q and MCQ

Critical Value, Comparison and Conclusion

Step 5

- Use t-table to find the critical values.
- Plot these values on bell shaped curve.
- Compare calculated value with table value
- Conclude and summarize the result

d.f.	intervals	80%	90%	95%	98%	99°4
	One tail, a	0.10	Q.Q5	0.025	0.01	0.0IH
	Two tails, α	0.20	0. IO	0.Q5	Q.02	OQ1
I		30 i8	6.3 I4	12.706	3I.R2J	63.637
3		T.ñB6	2.920	4.303	6.965	9.923
4		T.63B	2.3S3	3.IB2	4.541	5.B4I
5		1533	2.132	2.776	3./4	4.fiT)4
6		1 476	2.0 15	2.57I	3.365	4.032
7		I.440	1.943	2.447	3.143	3.70/
8		I.397	1.B95	2.365	2.99R	3.499
			1.B6P	2.306	2.B96	3.,55
				2 262	2.R2 I	3C50
10		I.3•2	I.B12	2.225	2. i64	3.169
11		T.k3	1.796	2J01	2.i18	3.10fi
12		J.356	1.7B2	2,179	2.681	3.055
13		I %0	1.771	2.160	2.6S0	3.012
14		I.345	1.761	2145	2.624	2.97
15		1 34 I	1.?S3	2.131	2.602	2.947
16		I.M	1.746	2.120	2.5B3	2.921
17		J33S	1.740	2.110	2.567	2.B9B
IB		I J30	I. 34	2.101	2.552	2.R7B
19		I,328	L72R	2.093	2.539	2.B61
		1.]2S	I.723	2.086	2.528	2.R45
22		I. 323	I.72 I	2.0Sfl	2.518	2.831
		I.M1	I.717	2.074	2.508	2.f119
23		1.319	I.?I4	2.059	2.500	2.807



Testing Hypothesis – Example

- For the Given data:
- Estimate the regression line from the following data of height (X) and Weight (Y)
- Test the hypothesis that the height and weight are independent.

Height (X)	Weight(Y))
60	110
60	135
60	120
62	120
62	140
62	130
62	135
64	150
64	145
70	170
70	185
70	160

Testing Hypothesis – Example

Step 1

Stat the hypothesis.

H_0 : $\beta = 0$ (There is NO slop. There is NO linear relationship.)

H_1 : $\beta \neq 0$ (There exist linear relationship)

Step 2

We set the level of significance (alpha) $\alpha = 0.05$

Step 3

The test-statistic (formula) to be used is:

$$t = \frac{b - \beta_0}{S.E(b)}$$

It follows t-distribution with degree of freedom ***df = n-2***

Computation

X	Y	XX	YY	XY
60	110	3600	12100	6600
60	135	3600	18225	8100
60	120	3600	14400	7200
62	120	3844	14400	7440
62	140	3844	19600	8680
62	130	3844	16900	8060
62	135	3844	18225	8370
64	150	4096	22500	9600
64	145	4096	21025	9280
70	170	4900	28900	11900
70	185	4900	34225	12950
70	160	4900	25600	11200
766	1700	49068	246100	109380

What we need to calculate?

Step 4

Do necessary computations from data and solve the formula using these values.

$$t = \frac{b - \beta_0}{s_b}$$

- $$s_b = \frac{s_{YX}}{\sqrt{\sum (X - \bar{X})^2}}$$

$$b = \frac{n \sum XY - (\sum X)(\sum Y)}{n \sum X^2 - (\sum X)^2}$$

$$s_{YX} = \sqrt{\frac{\sum (Y - \hat{Y})^2}{n-2}} = \sqrt{\frac{\sum Y^2 - a \sum Y - b \sum XY}{n-2}}$$

$$a = \bar{Y} - b\bar{X}$$

$$\sum (X - \bar{X})^2 = \sum X^2 - \frac{(\sum X)^2}{n}$$

Let's do calculations

Step 4

$$b = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum X^2 - (\sum X)^2} = \frac{(12)(109380) - (766)(1700)}{(12)(49068) - (766)^2} = \frac{10360}{2060} = 5.03$$

$$a = \bar{Y} - b\bar{X} = \left(\frac{1700}{12}\right) - (5.03)\left(\frac{766}{12}\right) = -179.41$$

$$\Sigma(Y-\hat{Y})^2 = \Sigma Y^2 - a\Sigma Y - b\Sigma XY = 246100 - (-179.41)(1700) - (5.03)(109380)$$
$$= 915.60$$

$$\Sigma(X-\bar{X})^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n} = 49068 - \frac{(766)^2}{12} = 171.67$$

Let's do calculations

Step 4

$$s_{YX} = \sqrt{\frac{\sum(Y-\hat{Y})^2}{n-2}} = \sqrt{\frac{915.60}{12-2}} = 9.5687$$

- $$s_b = \frac{s_{YX}}{\sqrt{\sum(X-\bar{X})^2}} = \frac{9.5687}{\sqrt{171.67}} = \frac{9.5687}{13.10} = 0.73$$

- $$t = \frac{b-\beta_0}{s_b} = \frac{5.03-0}{0.73} = 6.89$$

Calculated value = 6.89

Critical Value from t-table

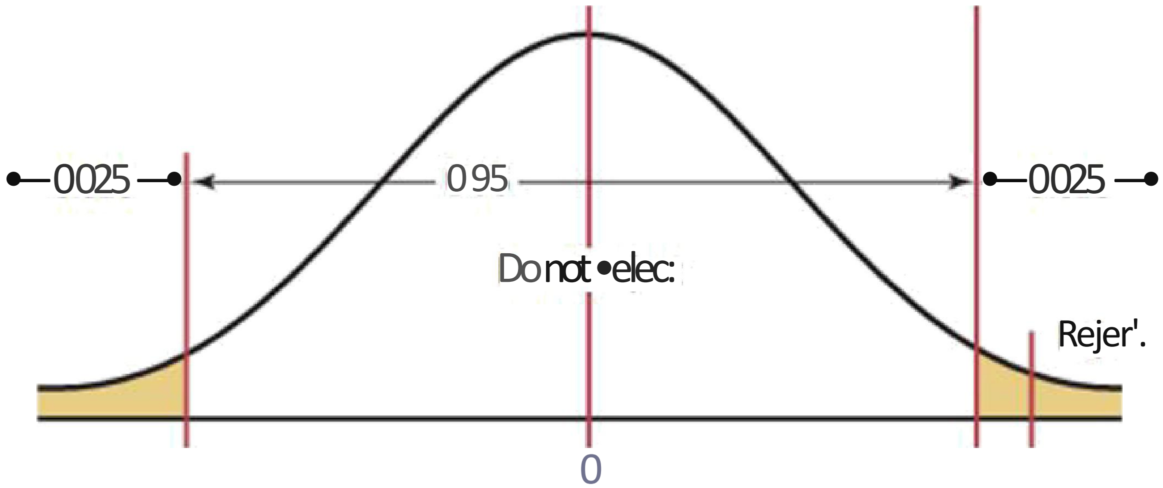
Step 5 : Find the 0.05 column in the top row and 16 in the left-hand column. Where the row and column meet, the appropriate critical value is found.

Table F The *t* Distribution

d.f.	Confidence intervals	80%	90%	95%	98%	99%
	One tail, α	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921
17		1.333	1.740	2.110	2.567	2.898
18		1.330	1.734	2.101	2.552	2.878
19		1.328	1.729	2.093	2.539	2.861
20		1.325	1.725	2.086	2.528	2.845
21		1.323	1.721	2.080	2.518	2.831
22		1.321	1.717	2.074	2.508	2.819
23		1.319	1.714	2.069	2.500	2.807

Decision Rule

Step 5



Conclusion and Summarizing:

Since the computed value of t=6.89 falls in the critical region, so we reject the null hypothesis and may conclude that there is sufficient reason to say with 95% confidence that height and weight are related.

An other example – Do yourself

In linear regression problem, the following sums were computed from a random sample of size 10.

$$\Sigma X = 320,\ \Sigma Y = 250,\ \Sigma X^2 = 12400,\ \Sigma XY = 9415\ and\ \Sigma Y^2 = 7230$$

Using 5% level of significance, test the hypothesis that the population regression coefficient, β is greater than 0.5.

Hint:

$H_1: \beta > 0.5$

$H_0: \beta \leq 0.5$

Hint

$H_0: \beta \quad 0$

$H_1: \beta > 0$ (Given claim)

T

$est\ formula = t = \frac{b - \beta_0}{s_b}$

- $s_b = \frac{s_{YX}}{\sqrt{\sum(X-\bar{X})^2}}$

$$b = \frac{n \sum XY - (\sum X)(\sum Y)}{n \sum X^2 - (\sum X)^2}$$

$$s_{YX} = \sqrt{\frac{\sum(Y - \hat{Y})^2}{n-2}} = \sqrt{\frac{\sum Y^2 - a\sum Y - b\sum XY}{n-2}} \qquad a = \bar{Y} - b\bar{X}$$

$$\Sigma(X - \bar{X})^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n}$$

Summary

- **Remember the all hypothesis-testing situations using the traditional method should include the**

$$Test\ Value = \frac{(Observed\ \text{rolue}) - (Expected\ value)}{Standard\ error}$$

1. **State the null and alternative hypotheses and identify the claim.**
2. **State an alpha level and find the critical value(s).**
3. **Compute the test value.**
4. **Make the decision to reject or not reject the null hypothesis.**
5. **Summarize the results.**

MCQ

The END

Get Ready ...

- Check mic position
- Check resolution size
- Check pen
- Say something....

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Regression & Correlation Analysis Lesson 9-1

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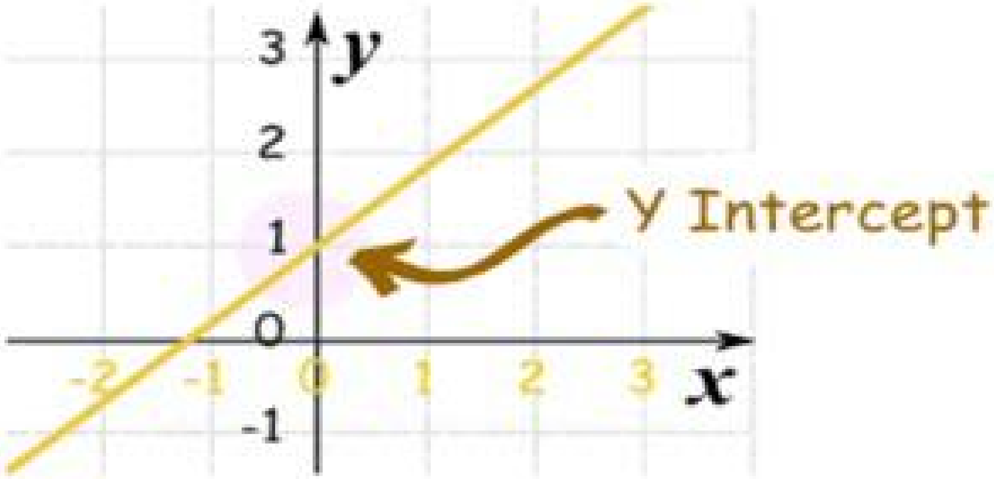
Statistical Inference about Regression

Using the t-test for hypothesis testing of

– the intercept of regression line

What is intercept of regression line?

- The Y intercept of a straight line is simply where the line crosses (cut) the Y axis.

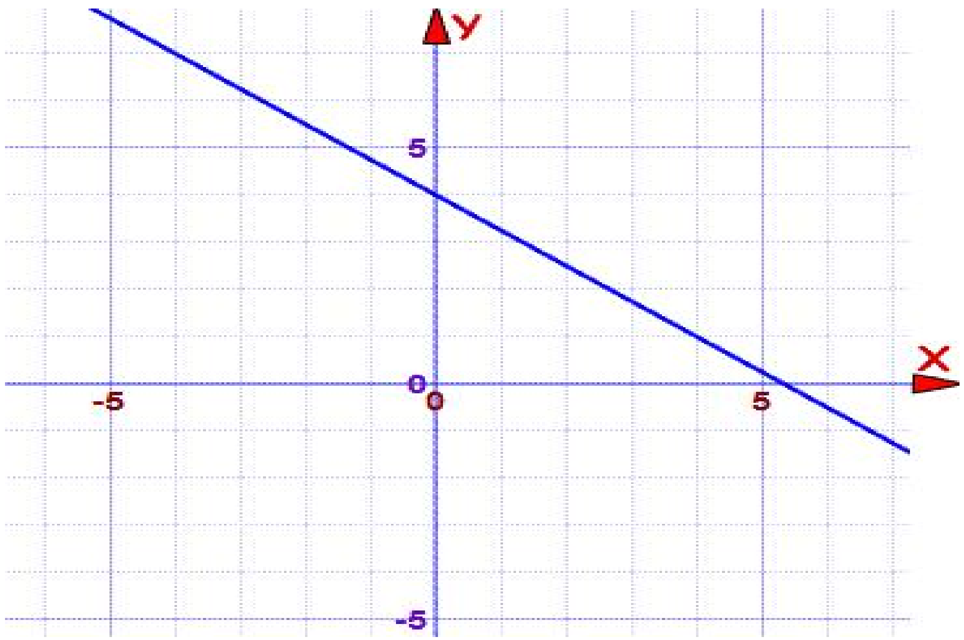


$$Y = a + bX$$

- In the above diagram the line crosses the Y axis at 1.
- So the Y intercept is equal to 1.

Can you tell?

- What is intercept of the given line?



Difference between Population & Sample Regression

- Population Regression

$$Y = \alpha + \beta X + \varepsilon$$

- Sample Regression

$$Y = a + bX + e$$

Hypothesis Testing – General Procedure

Procedure Table

Solving Hypothesis-Testing Problems (Traditional Method)

Step 1	State the hypotheses and identify the claim.
Step 2	Find the critical value(s) from the appropriate table
Step 3	Compute the test value.
Step 4	Make the decision to reject or not reject the null hypothesis.
Step 5	Summarize the results.

Testing Hypothesis – Example

- For the Given data:

- Test the hypothesis that

$H_0 : \alpha = 32$ against $H_1 : \alpha \neq 32$

X	Y
65	85
50	74
55	76
65	90
55	85
70	87
65	94
70	98
55	81
70	91
50	76
55	74

Testing Hypothesis – Example

Step 1

Stat the hypothesis.

$H_0: \alpha = 32$

$H_1: \alpha \neq 32$

Step 2

We set the **level of significance (alpha)** $\boldsymbol{\alpha = 0.01}$ **(1%)**

Step 3

The test-statistic (formula) to be used is: $\text{Test Value} = \dfrac{(\text{Observed value}) - (\text{Expected value})}{\text{Standard error}}$

$$t = \frac{a - \alpha_0}{S.E(a)}$$

It follows t-distribution with degree of freedom ***df = n-2***

What we need to calculate?

Do necessary computations from data and solve the formula using these values.

$$t = \frac{a - \alpha_0}{S.E(a)}$$

$$b = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum X^2 - (\sum X)^2}$$

S.E(a) = $s_a = s_{YX}\sqrt{\frac{1}{n} + \frac{\bar{X}^2}{\sqrt{\sum(X-\bar{X})^2}}}$

$$s_{YX} = \sqrt{\frac{\sum(Y-\hat{Y})^2}{n-2}} = \sqrt{\frac{\sum Y^2 - a\sum Y - b\sum XY}{n-2}}$$

$$a = \bar{Y} - b\bar{X}$$

$$\Sigma(X-\bar{X})^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n}$$

Computation

X	Y	XX	YY	XY
65	85	4225	7225	5525
50	74	2500	5476	3700
55	76	3025	5776	4180
65	90	4225	8100	5850
55	85	3025	7225	4675
70	87	4900	7569	6090
65	94	4225	8836	6110
70	98	4900	9604	6860
55	81	3025	6561	4455
70	91	4900	8281	6370
50	76	2500	5776	3800
55	74	3025	5476	4070
725	1011	44475	85905	61685

Let's do calculations

Step 4

$$b = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum X^2 - (\sum X)^2} = \frac{12(61685) - (725)(1011)}{12(44475) - (725)^2} = \frac{7245}{8075} = 0.897$$

$$a = \bar{Y} - b\bar{X} = \left(\frac{1011}{12}\right) - (0.897)\left(\frac{725}{12}\right) = 30.056$$

$$\Sigma(Y - \hat{Y})^2 = \Sigma Y^2 - a\Sigma Y - b\Sigma XY = 85905 - (30.056)(1011) - (0.897)(61685)$$
$$= 186.939$$

$$\Sigma(X - \bar{X})^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n} = 44475 - \frac{(725)^2}{12} = 672.917$$

Let's do calculations

Step 4

$$s_{YX} = \sqrt{\frac{\sum(Y-\hat{Y})^2}{n-2}} = \sqrt{\frac{186.939}{12-2}} = 4.3236$$

$$\text{S.E(a)} = s_a = s_{YX}\sqrt{\frac{1}{n} + \frac{\bar{X}^2}{\sqrt{\sum(X-\bar{X})^2}}} = (4.3236)\sqrt{\frac{1}{12} + \frac{(68.42)^2}{672.917}} = \cdots = 11.47$$

$$t = \frac{a-\alpha_0}{S.E(a)} = \frac{30.056 - 32}{11.47} = -0.169$$

Critical Value from t-table

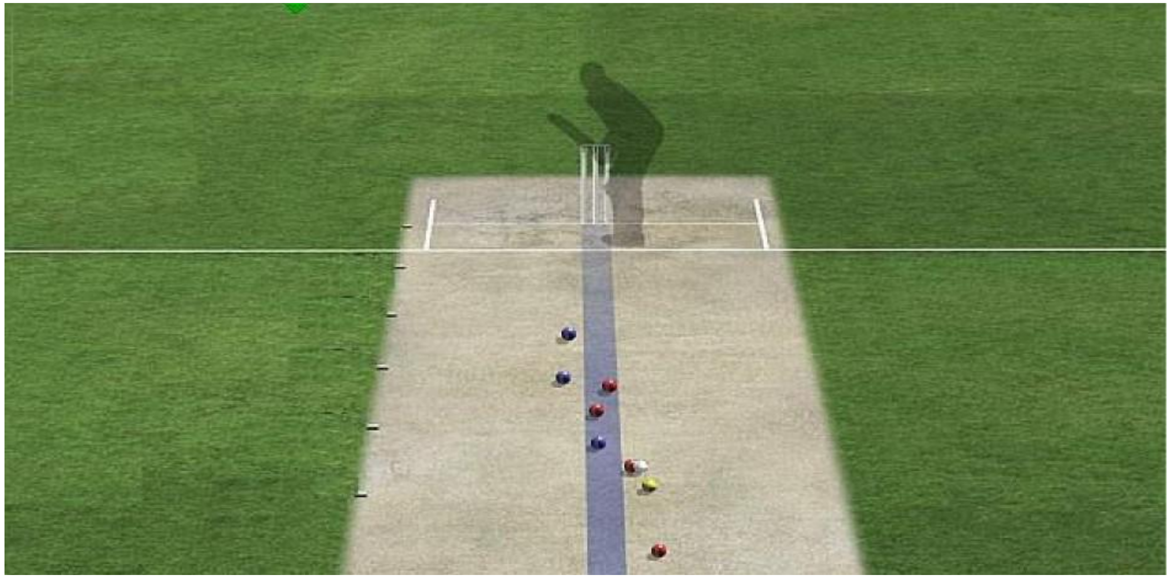
Step 5 : Find the 0.01 column in the top row and 10 in the left-hand column. Where the row and column meet, the appropriate critical value is found.

Table F The *t* Distribution

	Confidence intervals	80%	90%	95%	98%	99%
	One tail, α	0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921
17		1.333	1.740	2.110	2.567	2.898
18		1.330	1.734	2.101	2.552	2.878
19		1.328	1.729	2.093	2.539	2.861
20		1.325	1.725	2.086	2.528	2.845
21		1.323	1.721	2.080	2.518	2.831
22		1.321	1.717	2.074	2.508	2.819
23		1.319	1.714	2.069	2.500	2.807

Decision Rule

Step 5



Conclusion and Summarizing:

Since the computed value of I = -0.169 does NOT fall in the critical region, so we DO NOT reject the null hypothesis.

An other example – Do yourself

In linear regression problem, the following sums were computed from a random sample of size 10.

$$\Sigma X = 320,\ \Sigma Y = 250,\ \Sigma X^2 = 12400,\ \Sigma XY = 9415\ and\ \Sigma Y^2 = 7230$$

$$\hat{Y} = 4.04 + 0.655X$$

$$\Sigma(Y - \hat{Y})^2 = 53.175$$

$$\Sigma(X - \overline{X})^2 = 2160$$

Using **5% level** of significance, test the hypothesis that the intercept of regression line, α is zero.

Hint:

Hint

$H_0: \alpha = 0$ (Given claim)

$H_1: \alpha \neq 0$

T

$Test\ formula = t = \frac{a-\alpha_0}{S.E(a)}$

S.E(a) = $s_a = s_{YX}\sqrt{\frac{1}{n} + \frac{\bar{X}^2}{\sqrt{\sum(X-\bar{X})^2}}}$

$$s_{YX} = \sqrt{\frac{\sum(Y-\hat{Y})^2}{n-2}} = \sqrt{\frac{\sum Y^2 - a\sum Y - b\sum XY}{n-2}}$$

$$\Sigma(X-\bar{X})^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n}$$

$$\hat{Y} = 4.04 + 0.655X$$

$$b = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum X^2 - (\sum X)^2}$$

$$a = \bar{Y} - b\bar{X} = 4.04$$

Summary

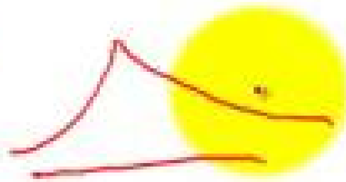
- Remember the all hypothesis-testing situations using the traditional method should include the

$$Test\ Value = \frac{(Observed\ value) - (Expected\ value)}{Standard\ error}$$

$\alpha = 0,\ \alpha \geq 0,\ \alpha \leq 0$

1. State the null and alternative hypotheses and identify the claim.
2. State an alpha level and find the critical value(s).
3. Compute the test value.
4. Make the decision to reject or not reject the null hypothesis.
5. Summarize the results.

critical



The END

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Regression & Correlation Analysis Lesson 10-1

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Statistical Inference about Simple Correlation

Testing hypothesis that the population correlation coefficient ρ is equal to some specified value other than zero

Hypothesis Testing – General Procedure

Procedure Table

Solving Hypothesis-Testing Problems (Traditional Method)

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s) from the appropriate table

Step 3 Compute the test value.

Step 4 Make the decision to reject or not reject the null hypothesis.

Step 5 Summarize the results.

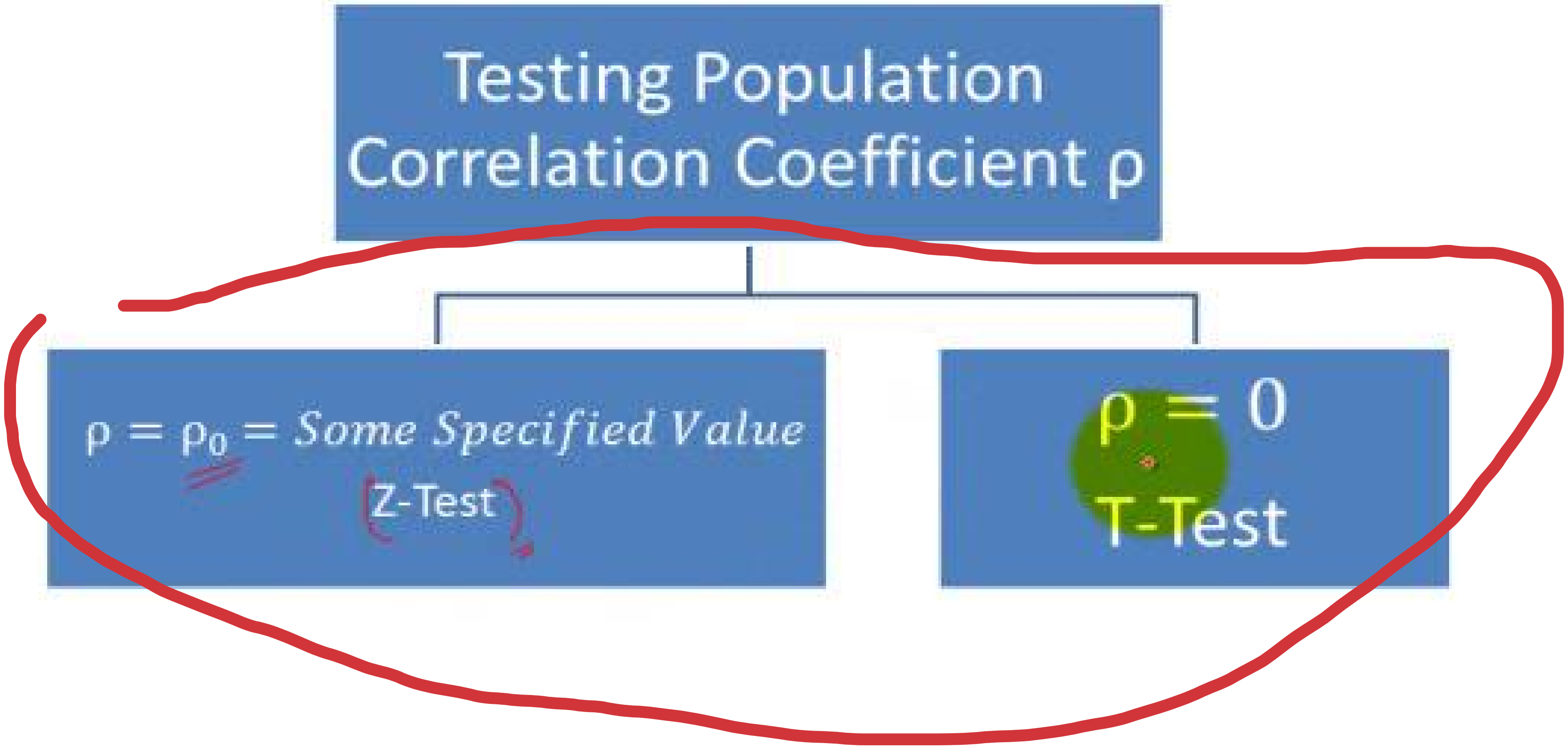
Notations:

- Sample Correlation Coefficient
- English letter "r"

- Population Correlation Coefficient
- Greek small letter "p" (rho)

MCQ

Two Cases


Testing Population Correlation Coefficient ρ
ρ = ρ0 = Some Specified Value
Z-Test
ρ = 0
T-Test

Some Main Points

- **Problem:** Sampling distribution of "r" is
  - Neither normal
  - Nor becomes approximately normal for large sample size
- **Solution:** Do Transformation:

Old variable → new variable

"r" --------------- "Z_f"

$$"r" \ldots\ldots\ldots\ldots\ldots Z_f = 1.1513 \log \frac{1+r}{1-r}$$

Testing formula

$$\text{Test Value} = \frac{(\text{Observed value}) - (\text{Expected value})}{\text{Standard error}}$$

$$Z = \frac{Z_f - \mu_z}{1/\sqrt{n-3}}$$

Where,

$$Z_f = 1.1513 \log \frac{1+r}{1-r}$$

$$\mu_z = 1.1513 \log \frac{1+\rho}{1-\rho}$$

sample pop.

MLQ

Testing Hypothesis – Example

- A random sample of 28 pairs from a bivariate normal population showed a correlation coefficient of 0.7.
- Is this value consistent with the assumption that the correlation coefficient in the population is 0.5?

Testing Hypothesis – Example

Step 1

Stat the hypothesis.

$H_0: \rho = 0.5$ (Given claim)

$H_1: \rho \neq 0.5$

Step 2

We set the **level of significance (alpha)** $\alpha = 0.05$ **(5%)**

Step 3

The test-statistic (formula) to be used is: $Test\ Value = \frac{(Observed\ value) - (Expected\ value)}{Standard\ error}$

$$Z = \frac{Z_f - \mu_z}{1/\sqrt{n-3}}$$

– *It is z-test.*

Calculations

$$Z = \frac{Z_f - \mu_z}{1/\sqrt{n-3}}$$

Given : $r = 0.7,\ \rho = 0.5$ and $n = 28$

$$Z_f = 1.1513\ \log\frac{1+r}{1-r} = 1.1513\ \log\frac{1+0.7}{1-0.7} = 1.1513\ \log\frac{1.7}{0.3}$$

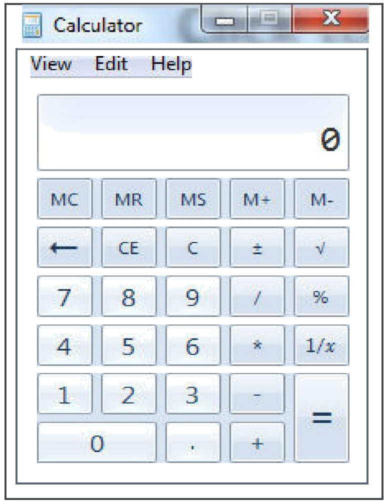
$$Z_f = 1.1513\ \log(5.66) = 1.1513\ (0.7528) = 0.87$$

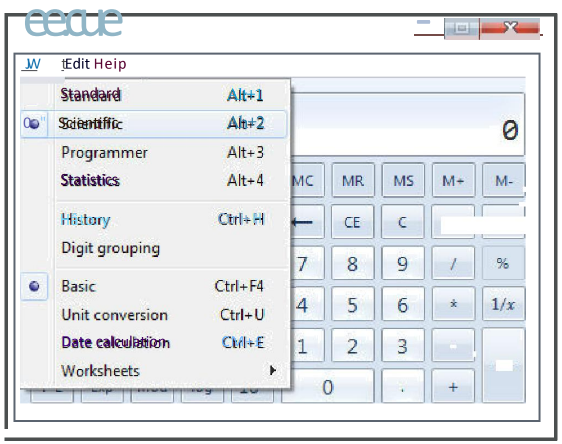
$$\mu_z = 1.1513\ \log\frac{1+\rho}{1-\rho} = 1.1513 \log\frac{1+0.5}{1-0.5} = 1.1513 \log\frac{1.5}{0.5} = 1.1513 \log(3)$$

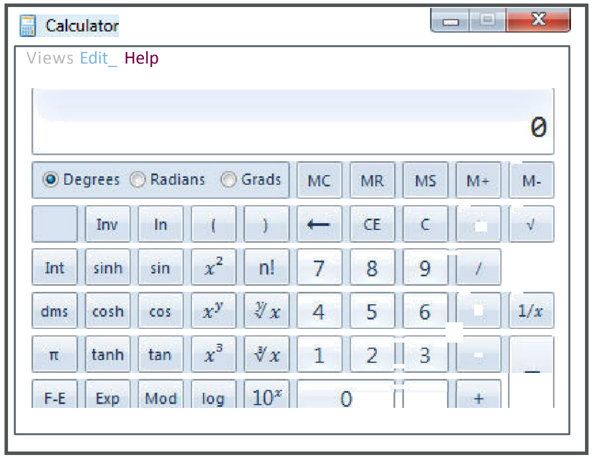
$$= 1.1513\,(0.4771) = 0.55$$

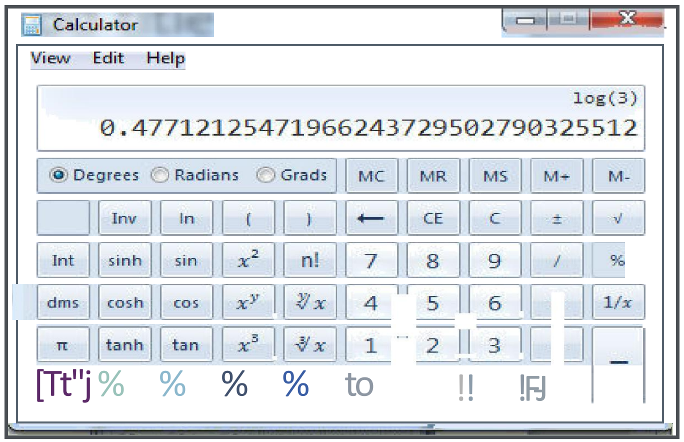
Calculating log Value

Windows calculator









Calculations

Now we have: $r = 0.7,\ n = 28,\ Z_f = 0.87,\ \mu_z = 0.55$

$$Z = \frac{Z_f - \mu_z}{1/\sqrt{n-3}}$$

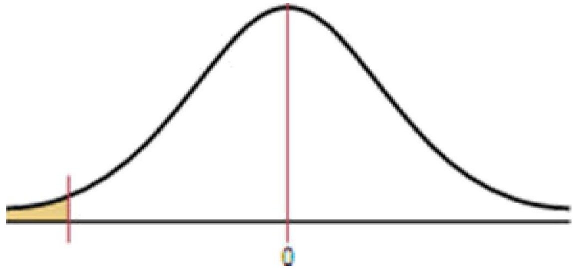
$$Z = \frac{0.87 - 0.55}{1/\sqrt{28-3}} = \frac{(0.32)}{1/\sqrt{25}} = (0.32)\times(5) = 1.60$$

$$\Rightarrow Z_{calculated} = 1.60$$

Frequently used Z Critical Values

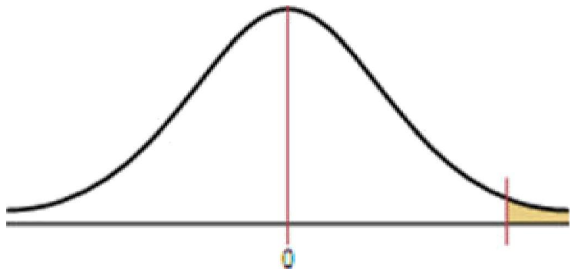
I°lypotksais Testing
wd Cnt"icalValues

$H_0: \mu = k$
$H_1: \mu < k$
$\begin{cases} \alpha = 0.10,\ C.V. = -1.28 \\ \alpha = 0.05,\ C.V. = -1.65 \\ \alpha = 0.01,\ C.V. = -2.33 \end{cases}$



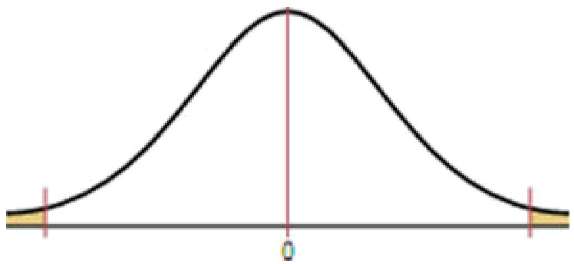
$H_0: \mu = k$
$H_1: \mu > k$
$\begin{cases} \alpha = 0.10,\ C.V. = +1.28 \\ \alpha = 0.05,\ C.V. = +1.65 \\ \alpha = 0.01,\ C.V. = +2.33 \end{cases}$

(6)RIy6t-\slls6



a •0.10.I V. = ±1.65

'"!!(@t"iig?



Decision Rule

Compare Calculated value with Table Value

Step 5

Conclusion and Summarizing:

Since the computed value of z = 1.60 does NOT fall in the critical region, so we do not reject our null hypothesis and may conclude that the given claim is correct.

Summary

- When $\rho \neq 0$, then distribution of "r" is
  - Neither normal
  - Nor becomes normal on large sample
- We transform "r" into a new variable which is distributed normally. Hence we can use to for testing.

$$Z = \frac{Z_f - \mu_z}{1/\sqrt{n-3}}$$

- Its z-test.
- Rest of the testing procedure is same.

The END

Get Ready …

- Check mic position
- Check resolution size
- Check pen
- Say something….

STA404
Regression & Correlation Analysis Lesson 10-2

Department of
Statistics Virtual
University of Pakistan

Statistical Inference about Simple Correlation

Testing hypothesis that the population correlation coefficient ρ is equal to zero

Hypothesis Testing – General Procedure

Procedure Table

Solving Hypothesis-Testing Problems (Traditional Method)

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s) from the appropriate table

Step 3 Compute the test value.

Step 4 Make the decision to reject or not reject the null hypothesis.

Step 5 Summarize the results.

Notations:

- Correlation Coefficient
  - from SAMPLE data
- English letter "r"

Correlation Coefficient from POPULATION data

- Greek small letter "p" (rho)

MLO

Two Cases

Testing Population Correlation Coefficient ρ

- $\rho = \rho_0 = Some\ Specified\ Value$
  Z-Test
- $\rho = 0$
  T-Test

Some Main Points

- Sampling distribution is NOT symmetry BUT it is interesting to note that when p = 0, then its symmetrical.
- Above make it possible to use the t-test.
- T-test can be applied for any sample size
- What does it means No: p = 0
  - There is NO linear correlation between two variables

Testing Hypothesis – Example

- A random sample of 20 pairs of observations gives a coefficient of 0.45.
- Test the hypothesis at the 0.05 level of significance that the correlation coefficient in the population is zero.

Testing Hypothesis – Example

Step 1

Stat the hypothesis.

$H_0: \rho = 0$ (There is NO linear correlation between two variables)

$H_1: \rho \neq 0$

Step 2

We set the **level of significance (alpha)** $\boldsymbol{\alpha = 0.05}$ **(5%)**

Step 3

The test-statistic (formula) to be used is:

$$Test\ Value = \frac{(Observed\ value) - (Expected\ value)}{Standard\ error}$$

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

It follows t-distribution with degree of freedom ***df = n-2***

Calculations

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

$$t = \frac{0.45\sqrt{20-2}}{\sqrt{1-(0.45)^2}}$$

$$t = \frac{1.91}{0.89} = 2.14$$

t = t-Calcuated Value = 2.14

Critical Value from t-table

Step 5 : Find the 0.05 column in the top row and 18 in the left-hand column. Where the row and column meet, the appropriate critical value is found.

Table F The *t* Distribution

	Cnnfidencc intervals	80%	90%	95%	98%	SS•Z
	One tail, o	o.io	0.05	0.025	0.01	0.005
d.f.	Too tails, n	R20	0. IO	0.05	0.02	0.01
I		30 8	6.314	12. 06	31.Pzl	6s.s57
2		i.éaa	2.920	4. 03	s.9s3	9.925
3		1,638	2.353	3. B2	4.541	5.841
4		1.533	2.132	2.76	3. /4	4.fi04
5		1.476	2.0 1S	2. 71	3.365	4.032
6		1.440	1.943	2.47	3.143	2.70
7		1.415	l.a95	2. 65	2.99g	3.499
8		1.397	l.B6n	2. 06	z.ass	3.351
9		1.383	1.833	2.. 62	2.R2\	ZZSO
io		1.3i2	1.BI2	2.. 25	2. /64	3.1s9
11		1,3b3	1.796	2. 01	z.› iB	3.10d
12		i31a	1.7g2	2. 79	2.681	3.055
12		l no	1.771	2. 60	2.dSfl	3.012
14		1.345	1.76 I	2. 45	2.624	2.97
15		1.341	1.?53	2. 31	Zfi02	2.947
ld		iza‹	1.74fi	2. 20	2 583	2.921
17		333	1.74O	2.'0	2.5e7	2.898
18				2.101	2ss2	2.R78
19		t. I	1.729	2.093	2539	2.861
20		1.325	1.72S	z.eas	2.528	2.R46
21		1.323	1.721	2.0an	2.518	2.831
22		1.321	1.717	z.o74	2.5o8	2.s19
23		1319	1.? l4	2.069	2.500	2.807

Decision Rule

Compare Calculated value with Table Value

Step 5

Conclusion and Summarizing:

Since the computed value of I = 2.14 falls in the critical region, so we reject our null hypothesis and may conclude that the correlation coefficient in the population differ from zero.

It is similar to test the ....

- Ho: $\rho = 0$
  - There is NO linear correlation between two variables.
  - The two variables are INDEPENDET
- Ho: $\beta = 0$
  - There is NO linear correlation between two variables.
  - The two variables are INDEPENDET

Summary

- When $H_0 : \rho = 0$, then distribution of "r" is symmetry. Therefor we can use t-test
- The formula is: $$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$
- Testing $H_0 : \rho = 0$ is equivalent to test $H_0 : \beta = 0$
- T-test can be applied to any sample size.

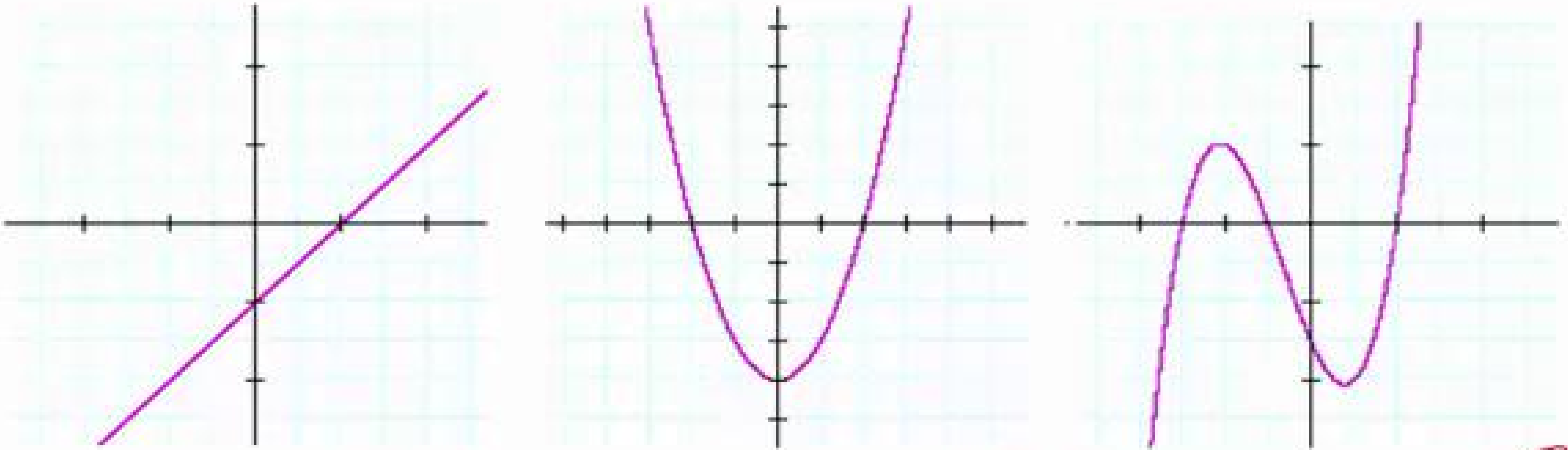
The END

Regression & Correlation Analysis

How to fit a Second Degree Parabola line?

Setting up Normal Equations

Shape of equations/data …



$y = a + bx^{1}$

Straight line

$y = a + bx + cx^{2}$

Second degree Eq.

Parbola

$y = a + bx + cx^{2} + dx^{3}$

Third degree Eq.

Normal Equations?

- Normal equations are equations obtained by setting equal to zero the partial derivatives of the sum of squared errors (least squares)
- This is a technique for computing coefficients for equations
- It finds the regression coefficients *analytically*

How to make Normal Equations …

$$Y = a + b\,X \qquad\qquad Y = a + b\,X + cX^2$$

The normal Equations are …..

$$Y = a + b\,X ----> \Sigma Y = na + b\Sigma X$$

$$XY = aX + b\,X^2 ---> \Sigma XY = a\Sigma X + b\Sigma X^2$$

How to make Normal Equations …

$$Y = a + bX + cX^2$$

$1 \qquad X \qquad X^2$

The normal Equations are …..

$$1\left(Y = a + bX + cX^2\right) \qquad \bullet\ \Sigma Y = na + b\Sigma X + c\Sigma X^2$$

$$X\left(Y = a + bX + cX^2\right) \qquad \bullet\ \Sigma XY = a\Sigma X + b\Sigma X^2 + c\Sigma X^3$$

$$X^2\left(Y = a + bX + cX^2\right) \qquad \bullet\ \Sigma X^2 Y = a\Sigma X^2 + b\Sigma X^3 + c\Sigma X^4$$

Test your Knowledge ...

Can you find normal equation for the following equation ?

$$Y = a + bX + cX^2 + dX^3$$

- $\Sigma Y =$ $na + b\Sigma X + c\Sigma X^2 + dX^3$
- $\Sigma XY =$ $a\Sigma X + b\Sigma X^2 + c\Sigma X^3 + d\Sigma X^4$
- $\Sigma X^2 Y =$ $a\Sigma X^2 + b\Sigma X^3 + c\Sigma X^4 + d\Sigma X^5$
- $\Sigma X^3 Y =$ $a\Sigma X^3 + b\Sigma X^4 + c\Sigma X^5 + d\Sigma X^6$

- The
END -

Regression &Correlation Analysis

How to fit a Second Degree Parabola line?

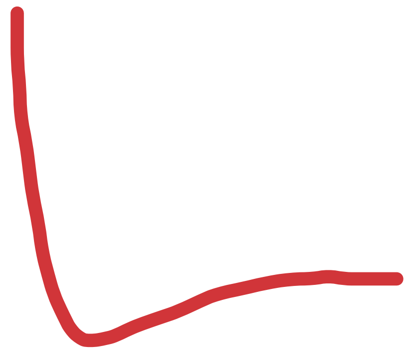
Procedure and step by step calculation with example

How to fit a Second Degree Parabola?

Example:

Fit a second degree parabola to the following data, taking X as independent variable.

[redacted]	
0	1
1	1.8
2	1.3
3	2.5
4	6.3



Equation of Second Degree ...

$$Y = a + bX + cX^2$$

The normal Equations are .....

- $\Sigma Y = na + b\Sigma X + c\Sigma X^2$
- $\Sigma XY = a\Sigma X + b\Sigma X^2 + c\Sigma X^3$
- $\Sigma X^2 Y = a\Sigma X^2 + b\Sigma X^3 + c\Sigma X^4$

What we need...?

ΣY Σx Σx^2

Σxy Σx^3

$\Sigma x^2 y$ Σx^4

n

Required Calculation

X	Y	XY	XX	XXY	XXX	XXXX
0	1.0	0	0	0	0	0
1	1.8	1.8	1	1.8	1	1
2	1.3	2.6	4	5.2	8	16
3	2.5	7.5	9	22.5	27	81
4	6.3	25.2	16	100.8	64	256
Total	12.9	37.1	30	130 130.3	100	354

$\Sigma x = 10$

Substituting the values in the normal Equations.....

$$\Sigma Y = na + b\Sigma X + c\Sigma X^2$$

$$\Sigma XY = a\Sigma X + b\Sigma X^2 + c\Sigma X^3$$

$$\Sigma X^2 Y = a\Sigma X^2 + b\Sigma X^3 + c\Sigma X^4$$

$$12.9 = 5a + 10b + 30c \quad \text{--- (1)}$$

$$37.1 = 10a + 30b + 100c \quad \text{--- (2)}$$

$$130.3 = 30a + 100b + 354c \quad \text{--- (3)}$$

Solve these equation to eliminate "a"

$$12.9 = 5a + 10b + 30c \quad (1)$$

$$37.1 = 10a + 30b + 100c \quad (2)$$

$$130.3 = 30a + 100b + 354c \quad (3)$$

Multiply eq (1) by 2, and subtract it from eq (2)

$$37.1 = 10a + 30b + 100c$$

$$25.8 = 10a + 20b + 60c$$

$$11.3 = 10b + 40c \quad (4)$$

Multiply eq (2) by 3 and subtract it from eq (3)

$$130.3 = 30a + 100b + 354c$$

$$111.3 = 30a + 90b + 300c$$

$$19 = 0 + 10b + 54c$$

Now, Solve these equation to eliminate "b"

$$11.3 = 10b + 40c \quad \text{— (4)}$$

$$19 = 10b + 54c \quad \text{— (5)}$$

$$-7.7 = -14c$$

$$\Rightarrow c = \frac{-7.7}{-14} = 0.55 \qquad \boxed{c = 0.55}$$

Now put the value of c in eq (4)

$$11.3 = 10b + 40(0.55)$$

$$11.3 = 10b + 22$$

$$-10.7 = 10b \Rightarrow b = -1.07$$

Now, put value of "b" and "c" in Eq...1

$$12.9 = 5a + 10b + 30c \quad \text{—} \textcircled{1}$$

put the value of b & c in eq

$$12.9 = 5a + 10(-1.07) + 30(0.55)$$

$$12.9 = 5a - 10.7 + 16.5$$

$$7.1 = 5a$$

$$\Rightarrow \boxed{a = 1.42}$$

So by solving three normal equations simultaneously, we are able to find the values

$$a = 1.42$$

$$b = -1.07$$

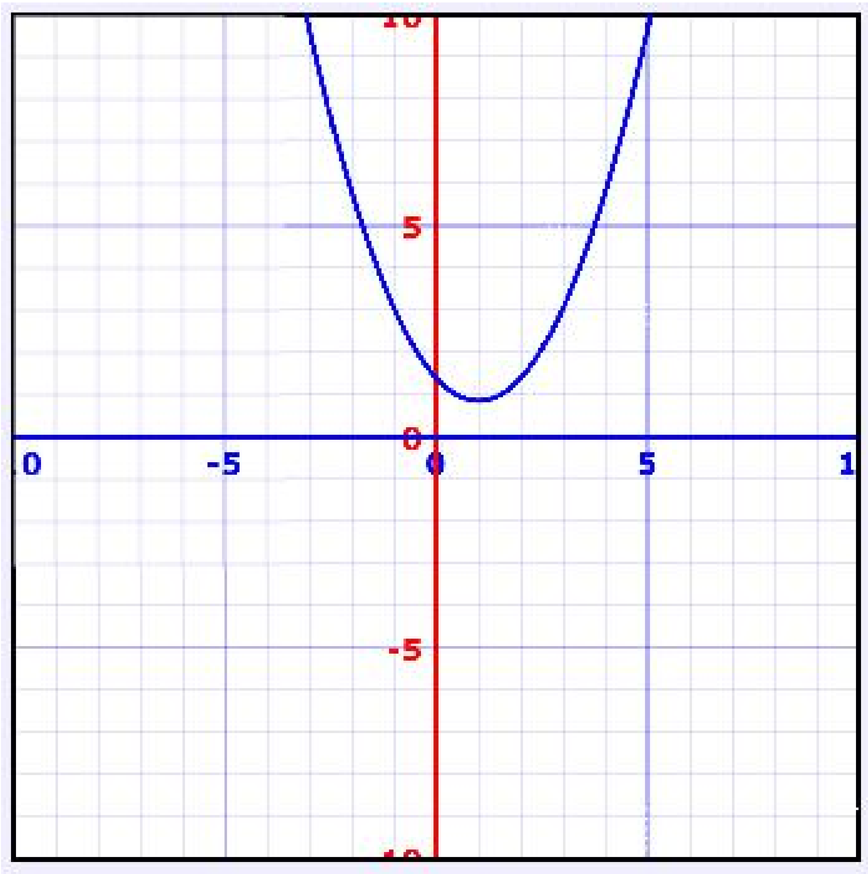
$$c = 0.55$$

$$Y = a + bX + cX^2$$

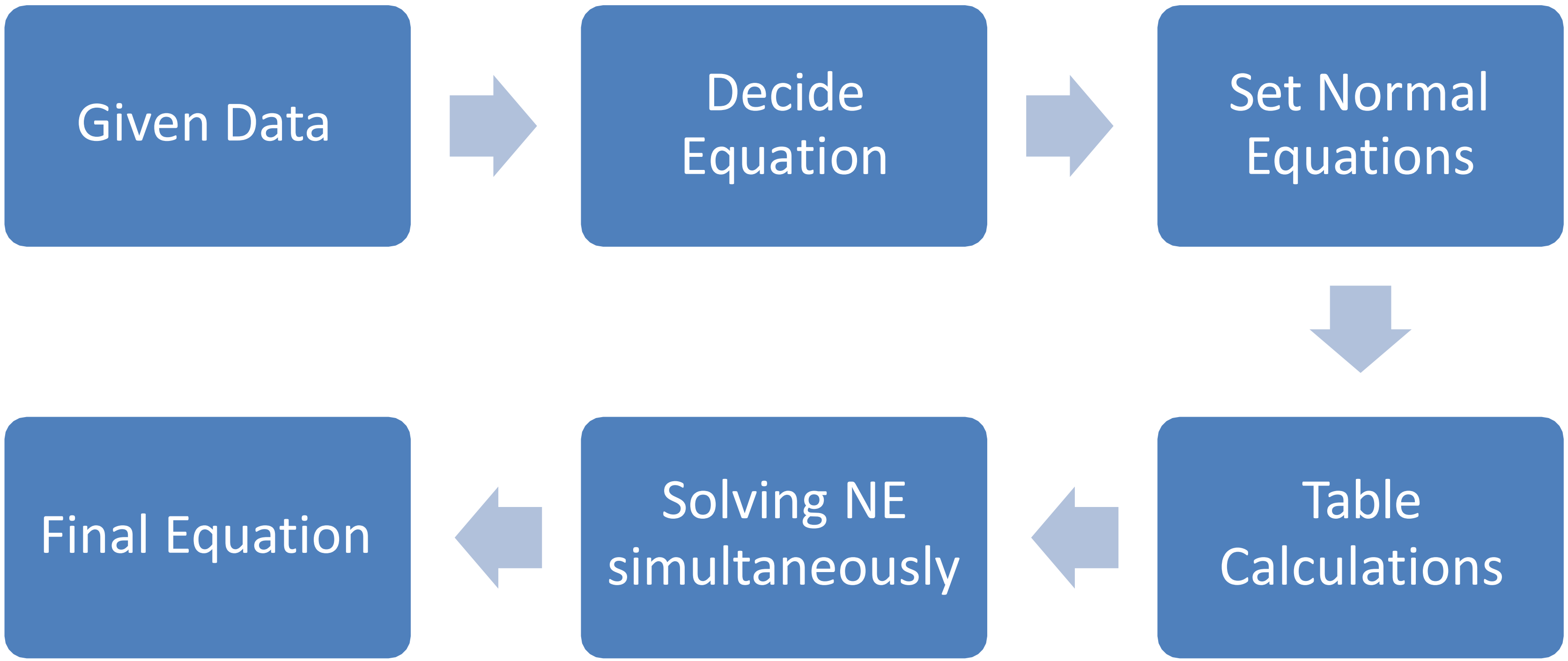
$$\hat{y} = 1.42 - 1.07X + 0.55X^2$$

Find fitted eq. OK

Just for fun … Graph of this equation



Recap


Given Data
Decide Equation
Set Normal Equations
Table Calculations
Solving NE simultaneously
Final Equation

- The
END -

Test 1…2…3

- Mic/Audio test
- Screen Area test
- Pen test

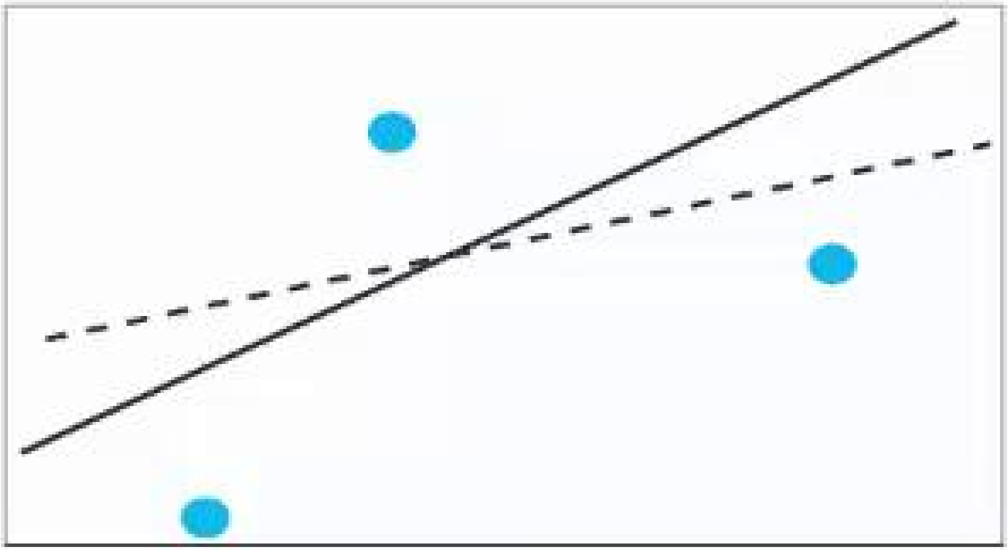
How to Find the Standard Deviation of Regression lecture 12

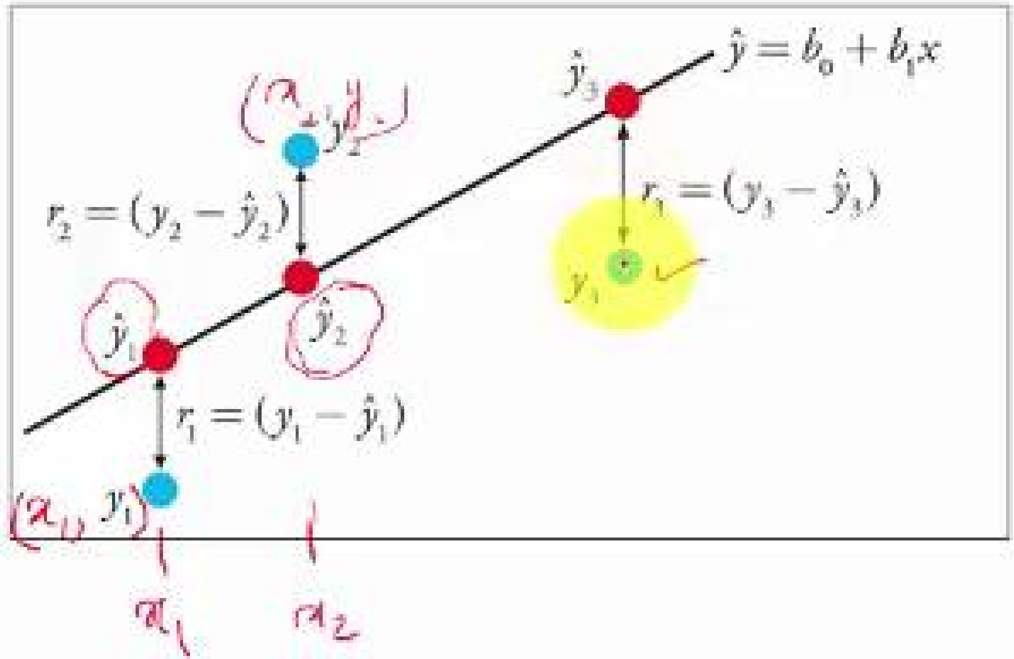
It is also called Standard Error of Estimate

Basic Idea

- The observed values of (X, Y) do not fall on the regression Line but scatter away from it.
- The degree of scatter (dispersion) of the observed values about the regression line is measured by a method/formula.
- It is called standard deviation of regression
- It is also know as “Standard Error of Estimate”

Visual Seen ...




$\hat{y} = b_0 + b_1 x$
$\hat{y}_3$
y_2
$r_2 = (y_2 - \hat{y}_2)$
$r_3 = (y_3 - \hat{y}_3)$
$\hat{y}_2$
$\hat{y}_1$
$r_1 = (y_1 - \hat{y}_1)$
y_1

Formula …

- For population data

$$\sigma_{Y.X} = \sqrt{\frac{\Sigma(Y - \hat{Y})^2}{N}}$$

- For sample data

$$s_{y.x} = \sqrt{\frac{\Sigma(Y - \hat{Y})^2}{n-2}}$$

Example

Example:

- Find the values of $\hat{Y}$.
- Find the residual values "e"
- Show that $\Sigma(Y-\hat{Y})=0$
- Compute the standard error of estimate $s_{y \cdot x}$

X	Y
5	16
6	19
8	23
10	28
12	36
13	41
15	44
16	45
17	50

Fitting regression line

You can do it by yourself ...

$$Y = a + bX$$

b = ✓ → 2.831 ✓ Slop

a = ✓ → a = 1.47

D.Y.S

Finding $\hat{Y}$

X	Y	$\hat{Y} = 1.47 + 2.831X$
5	16	1.47+2.83(5) = 15.63
6	19	1.47+2.83(6) = 18.46
8	23	24.12
10	28	
12	36	
13	41	
15	44	
16	45	
17	50	

Residual values and showing $\Sigma(Y-\hat{Y})=0$

obs. Estimator

X	Y	$\hat{Y}$	$e=Y-\hat{Y}$
5	16	15.63	16 – 15.63 = 0.38 e_1
6	19	18.46	19 – 18.46 = 0.54 e_2
8	23	24.12	23 - 24.12 = -1.12 e_3
10	28	29.78	28 - 29.78 = -1.78
12	36	35.44	0.56
13	41	38.27	2.73
15	44	43.94	0.07
16	45	46.77	-1.77
17	50	49.60	0.40
			$\Sigma(Y_i-\hat{Y}_i)=0$

$\Sigma y = \Sigma \hat{y}$

Showing Standard rror

X	Y	$\hat{Y}$	$e = Y - \hat{Y}$	$(Y - \hat{Y})^2$
5	16	15.63	0.38	0.14
6	19	18.46	0.54	0.30
8	23	24.12	-1.12	1.25
10	28	29.78	-1.78	3.17
12	36	35.44	0.56	0.31
13	41	38.27	2.73	7.44
15	44	43.94	0.07	0.00
16	45	46.77	-1.77	3.12
17	50	49.60	0.40	0.16
			SUM =	
				15.89

Formula ...

$$s_{y.x} = \sqrt{\frac{\Sigma(Y - \hat{Y})^2}{n-2}}$$

$$= \sqrt{\frac{15.89}{9-2}}$$

$$S_{y.x} = 1.51$$

Standard Error!

Recap

Given Data

Find a and b

Find Y Hat

(Y – Y hat)

Square and simplify

Final Value

- The
END -

Test 1…2…3

- Mic/Audio test
- Screen Area test
- Pen test

Regression & Correlation Analysis

How to Find the Standard Deviation of Regression lecture12

Example with formula

Example #2

Example:

- Compute the standard error of estimate $s_{y.x}$

X	Y
20	280
24	360
36	450
32	420
28	400
38	500
32	475
26	320

Formula for Standard Error…

$$s_{y.x} = \sqrt{\frac{\Sigma(Y - \hat{Y})^2}{n-2}} = \sqrt{\frac{\Sigma Y^2 - a\Sigma Y - b\Sigma XY}{n-2}}$$

So

$$s_{y.x} = \sqrt{\frac{\Sigma(Y - \hat{Y})^2}{n-2}}$$

or

$$s_{y.x} = \sqrt{\frac{\Sigma Y^2 - a\Sigma Y - b\Sigma XY}{n-2}}$$

First we fit the Regression line

$$\hat{y} = a + b\,x$$

Formula for a:

$$a = \bar{y} - b(\bar{x})$$

Formula for b:

$$b = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2}$$

$\bar{x} = ?$

$\bar{y} = ?$

$n = ?$

$\Sigma xy = ?$

$\Sigma x = ?$

$\Sigma y = ?$

$\Sigma x^2 = ?$

Required Calculation

Day	X	Y	XY	X^2	Y^2
1	20	280	5600	400	78400
2	24	360	8640	576	129600
3	36	450	16200	1296	202500
4	32	420	13440	1024	176400
5	28	400	11200	784	160000
6	38	500	19000	1444	250000
7	34	475	16150	1156	225625
8	26	320	8320	676	102400
Total	**238**	**3205**	**98550**	**7356**	**1324925**

- From previous table we have:

$$n = 8$$
$$\Sigma x = 238$$
$$\Sigma y = 3205$$
$$\Sigma x^2 = 7356$$
$$\Sigma xy = 98550$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{3205}{8} = 400.62$$
$$\bar{x} = \frac{\Sigma x}{n} = \frac{238}{8} = 29.75$$

- Now putting these values in "b" and "a", and by solving, we get:

$$b = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2} = \frac{8(98550) - (238)(3205)}{8(7356) - (238)^2} = \frac{25610}{2204} = \boxed{11.62}$$

$$a = \bar{y} - b(\bar{x}) = 400.62 - 11.62(29.75) = \boxed{54.92}$$

– Putting the values of "a" and "b" in the equation, we get:

$$\hat{y} = 54.92 + 11.62(x)$$

Using Formula …

$$s_{y.x} = \sqrt{\frac{\Sigma Y^2 - a\Sigma Y - b\Sigma XY}{n-2}}$$

$n = 8$

$a = 54.92$

$b = 11.62$

$\Sigma y = 3205$

$\Sigma y^2 = 1324925$

$\Sigma xy = 98550$

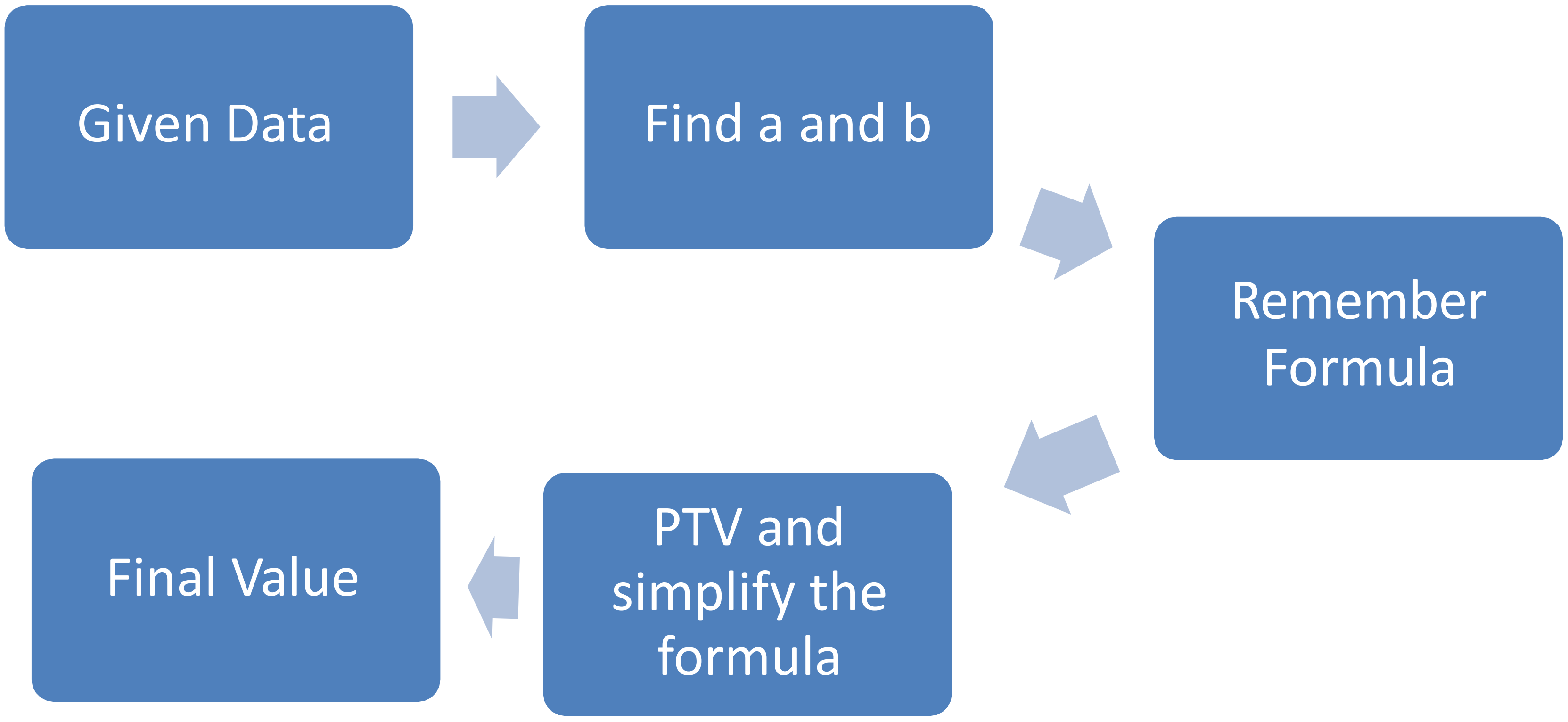
$$S_{yx} = \sqrt{\frac{1324925 - (54.92)(3205) - (11.62)(98550)}{8-2}}$$

$$= \sqrt{\frac{3755.4}{6}}$$

$$= 25.02$$

S. Easlunde.

Recap


Given Data
Find a and b
Remember Formula
PTV and simplify the formula
Final Value

- The

END -

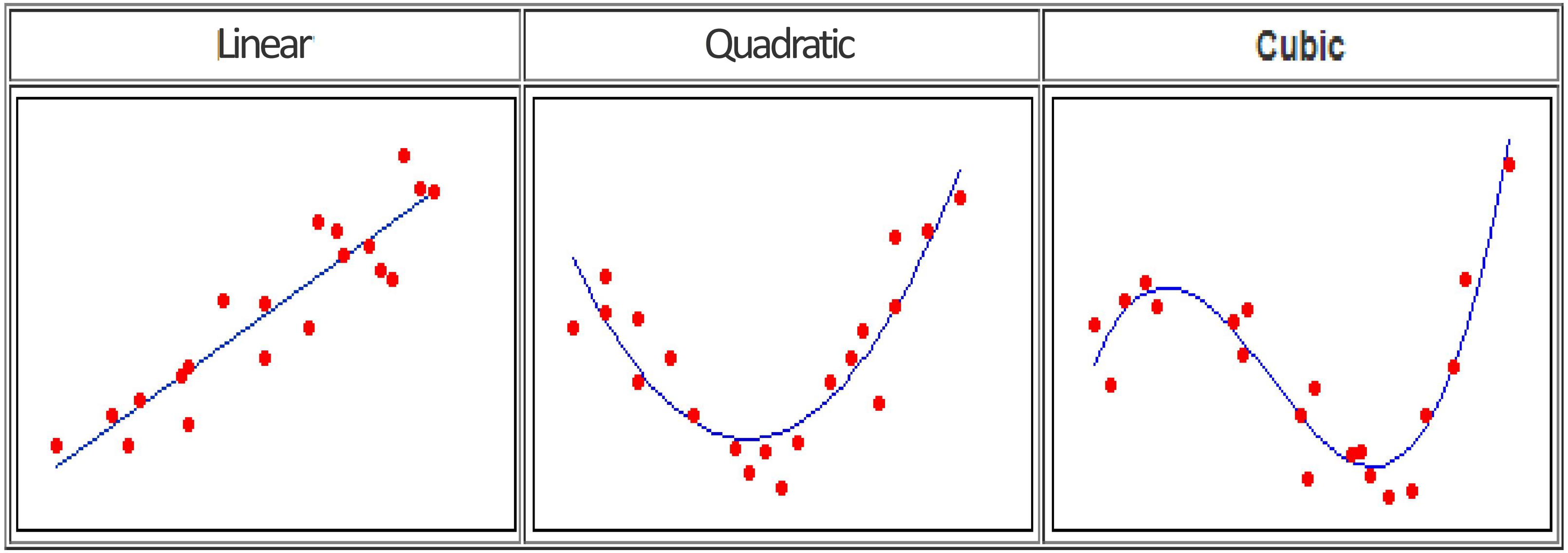
Test 1…2…3

- Mic/Audio test
- Screen Area test
- Pen test

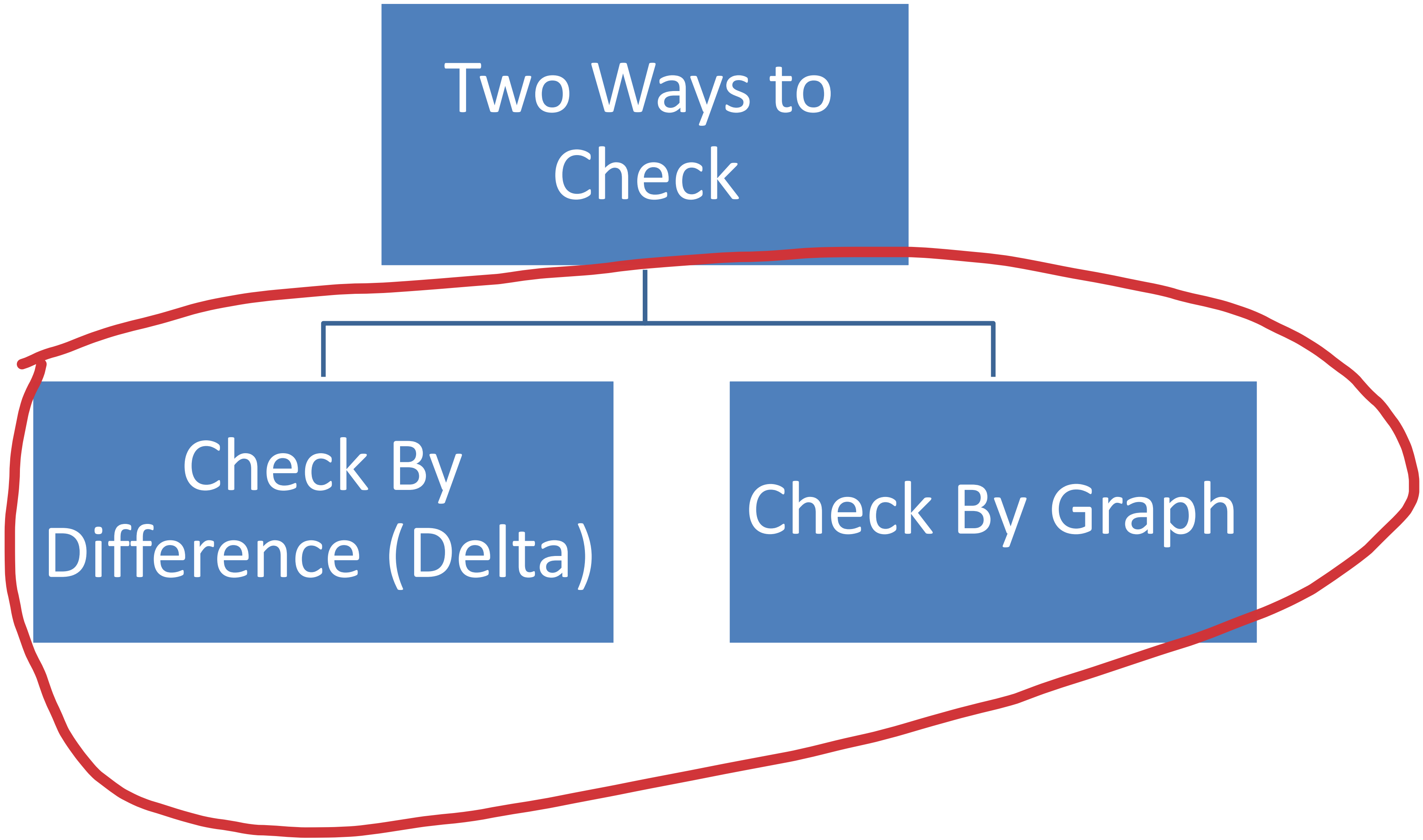
Deciding the Type of Curve to fit

Which curve is suitable in which situation

Lines/Curves come in ifferent shapes ..



Choosing the right line…



Example #1

üuestïon:

Decide the suitable curve for the following data.

[redacted]	
1	12
2	15
3	18
4	21

Lets do it ....

Solution:

Let, first we take its first difference as below and see whether it is constant or not?

X	Y	First Difference
1	12	
2	15	15 – 12 = 3
3	18	18 – 15 = 3
4	21	21 – 18 = 3

Delta Y → ΔY

Criteria for suitable curve

Some more rules:

- If the FIRST DIFFERENCES of Y are *approximately* constant, use a straight line

- If the SECOND DIFFERENCES of Y are *approximately* constant, use a second degree order equation (parabola)

- If the THIRD DIFFERENCES of Y are *approximately* constant, use a third degree order equation

Criteria for suitable curve

Here are few rules:

- If the FIRST DIFFERENCES of <u>log Y</u> are *approximately* constant, use a exponential curve

- If the FIRST DIFFERENCES of <u>log Y</u> and <u>log X</u> are *approximately* constant, use a Geometric curve

- If the FIRST DIFFERENCES of <u>reciprocals of Y</u> are *approximately* constant, use a third degree parabola

Example #2 ....

üuestïon:

Decide the suitable curve for the following data.

[redacted]	
1	10
2	14
3	21
4	31
5	44

Lets do it ….

SOLUTION:

– we take its first difference

– and see whether it is constant or not?

– If not then go for second difference

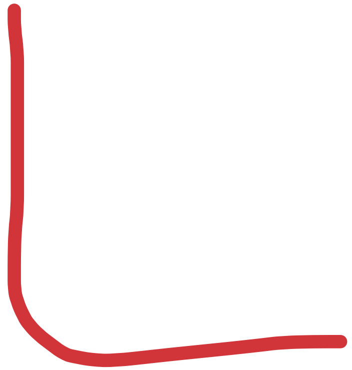
X	Y	First Difference	Second Difference
1	10	–	–
2	14	14 – 10 = 4	–
3	21	21 – 14 = 7	7 – 4 = 3
4	31	31 – 21 = 10	10 – 7 = 3
5	44	44 – 31 = 13	13 – 10 = 3

Example #3 ….

Questłon:

Suggest a suitable curve for the given data?

[redacted]	
0	10
1	17
2	28
3	43
4	62



Remember

- Straight Line:

$$Y = a + bX$$

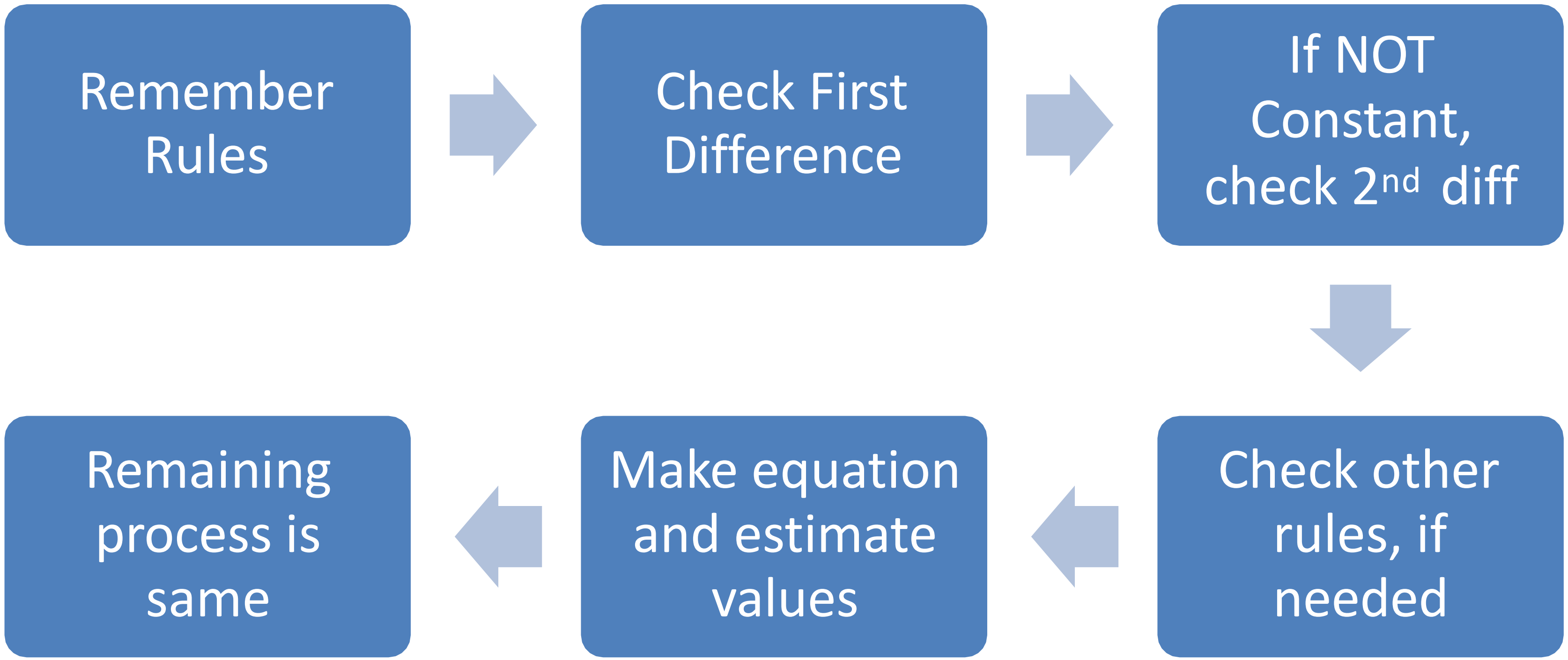
- Second Degree line (Also called Parabola)

$$Y = a + bX + cX^2$$

- Third Degree Line

$$Y = a + bX + cX^2 + dX^3$$

Recap


Remember Rules
Check First Difference
If NOT Constant, check 2nd diff
Check other rules, if needed
Make equation and estimate values
Remaining process is same

- The
END -

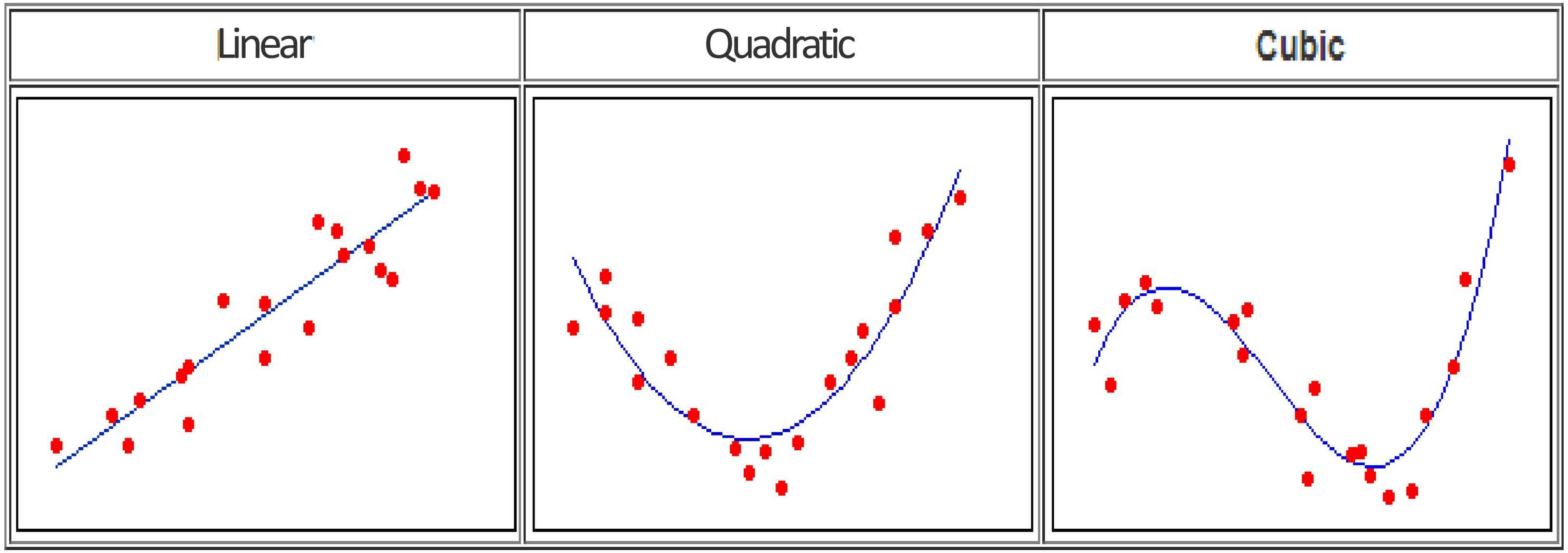
Test 1…2…3

- Mic/Audio test
- Screen Area test
- Pen test

Deciding the Type of Curve to fit

Use graph to get the idea of suitable equation

Lines/Curves come in ifferent shapes ..



Criteria for suitable curve

Here are few guidelines:

- If the graph of the values of X and Y gives a straight line, use a straight line .
- If the graph of the values of **X and Y** gives a curve with only one bend, use a second degree parabola.
- If the graph of the values of **X and Y** gives a curve of the shape of reverse "S", use a third degree parabola.

Criteria for suitable curve

Few more guidelines:

- If the graph of the values of X and log Y gives a straight line, use an exponential curve .
- If the graph of the values of log X and log Y gives a straight line, use a geometric or logarithmic curve .
- If the graph of the values of X and 1/Y gives a straight line, use a hyperbola curve.

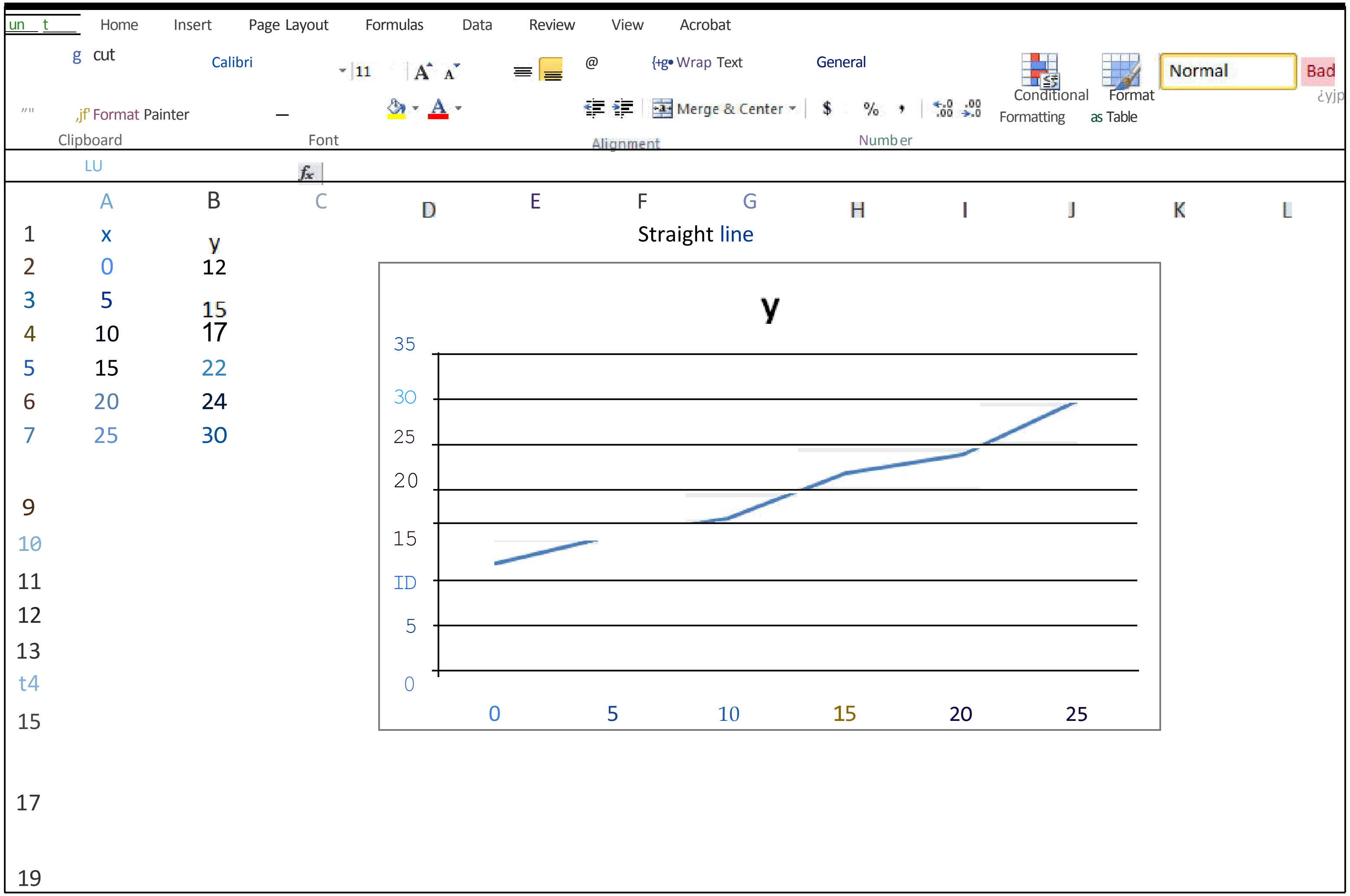
Lets explore few … Example #1

üuestïon:

By plotting, which type of line/curve is appropriate for the data?

x	y
0	12
5	15
10	17
15	22
20	24
25	30

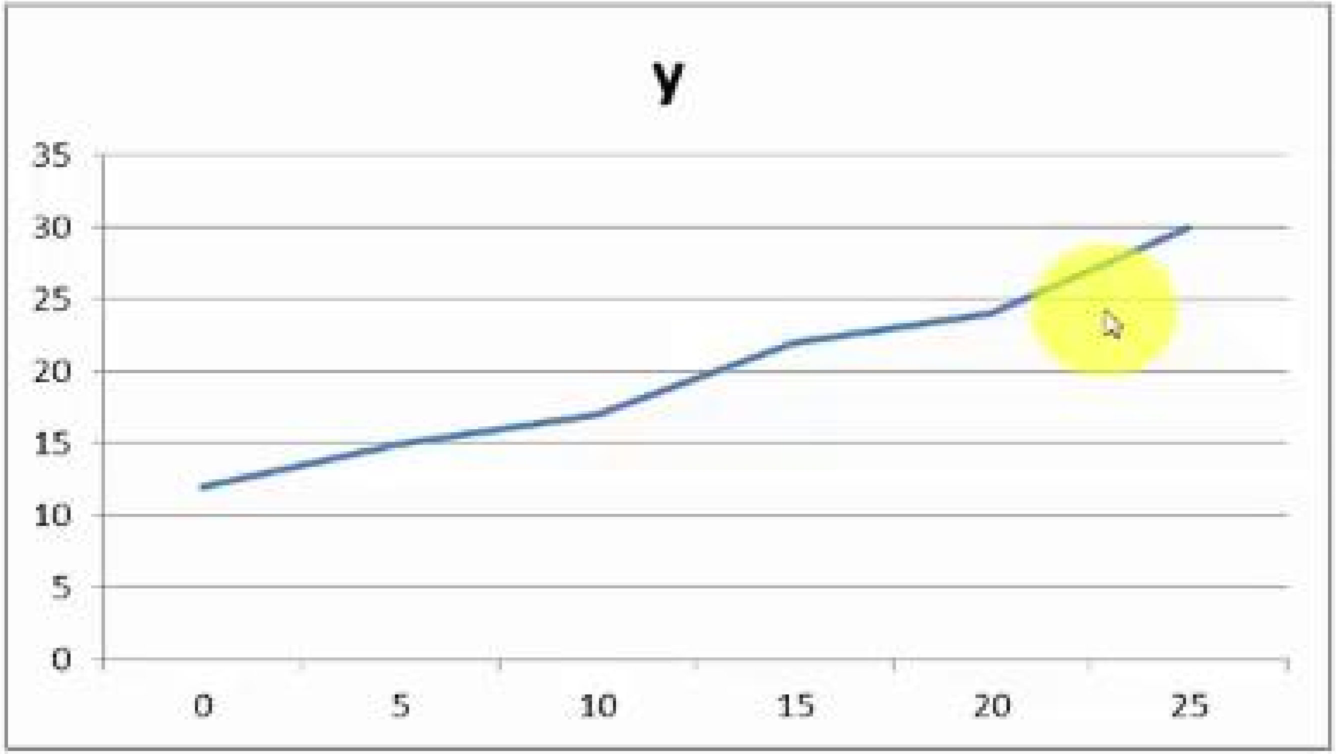
Lets Go To Excel ....



Straight line …

Solution:

- If the graph of the values of X and Y gives a <u>straight line</u>, use a straight line .



Example #2 ....

üuestïon:

Decide the suitable curve for the following data.

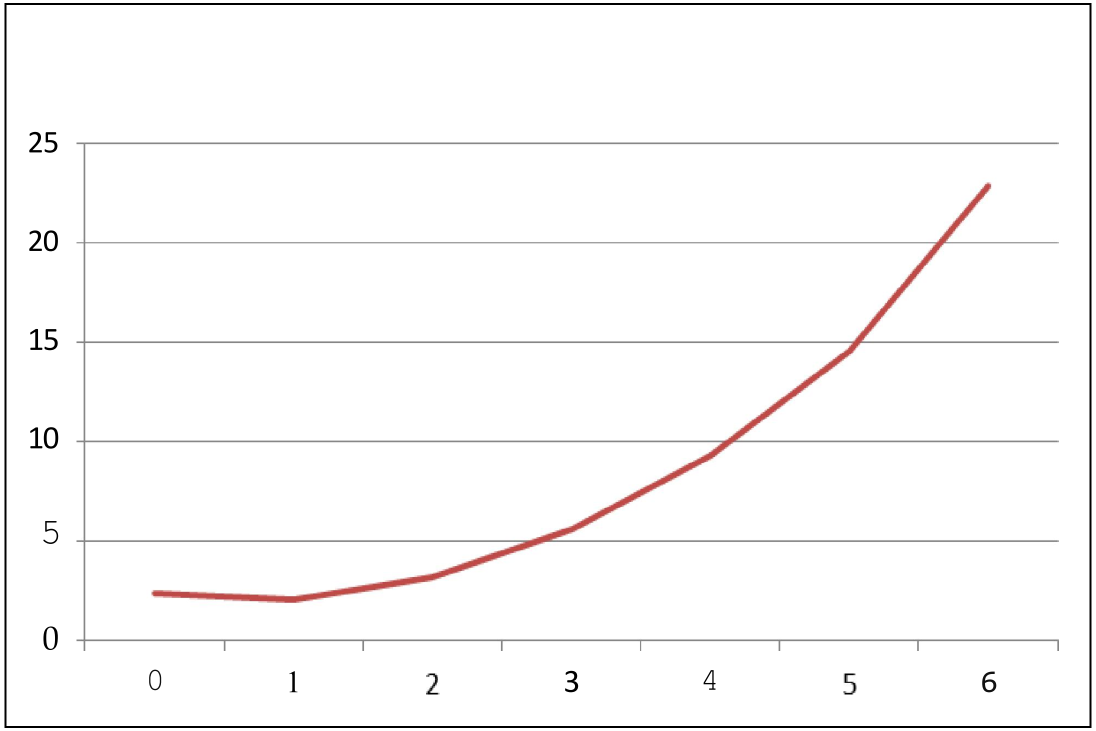
[redacted]

0	2.4
1	2.1
2	3.2
3	5.6
4	9.3
5	14.6
6	22.9

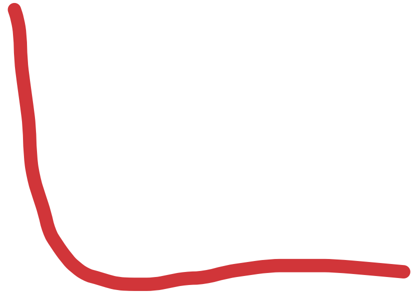
Second Degree Parabola ...

SOLUTION:

- If the graph of the values of gives a <u>curve with</u> only one bend, use a second degree parabola.

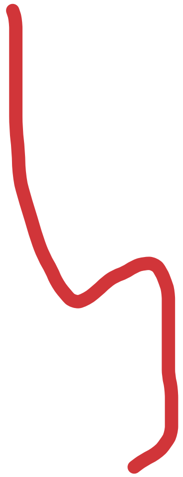


Example #3 ....



Question:

Decide the suitable curve for the following data.

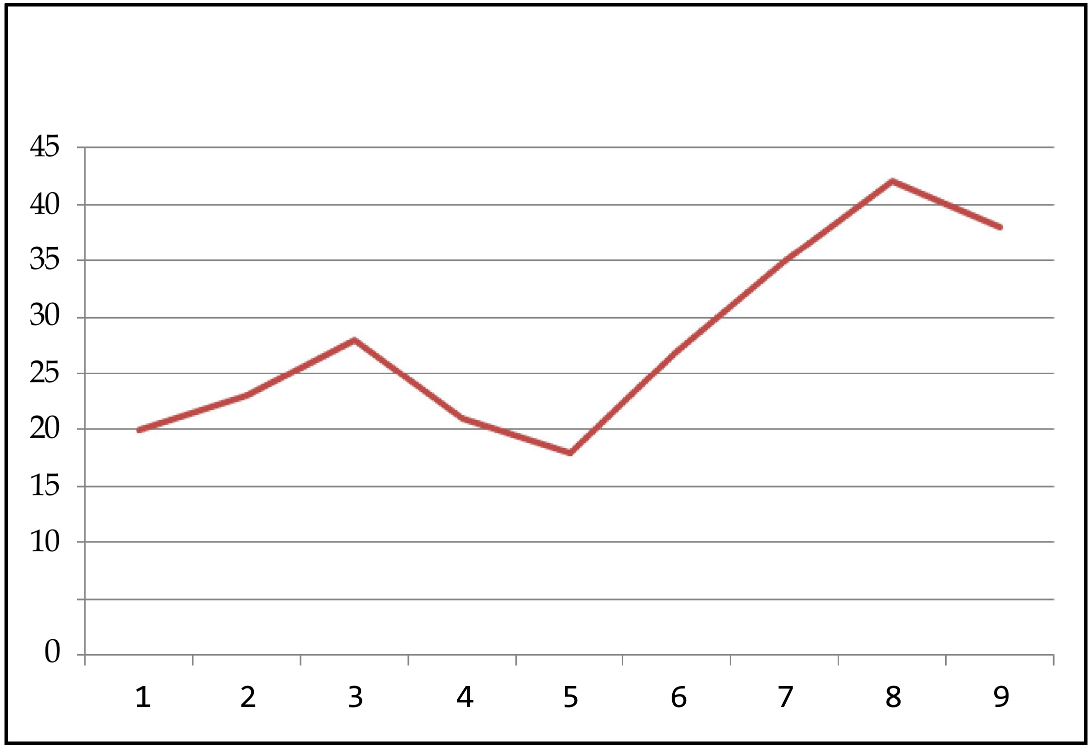


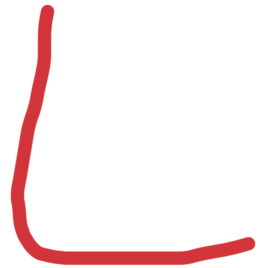
1	20
2	23
3	28
4	21
5	18
6	27
7	35
8	42
9	38

Third Degree Parabola…

SOLUTION:

- If the graph of the values of gives a <u>curve of the</u> shape of reverse "S", use a third degree parabola.





Example #4....

Questïon:

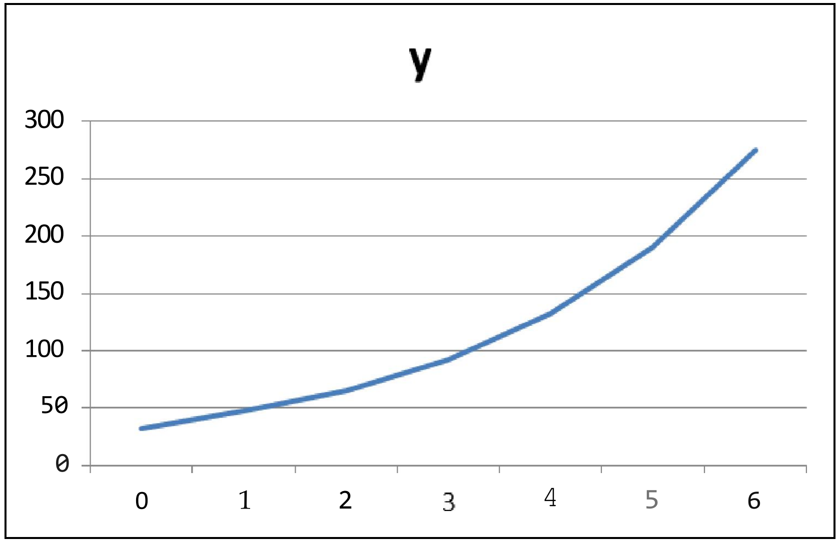
Suggest a suitable curve for the given data?

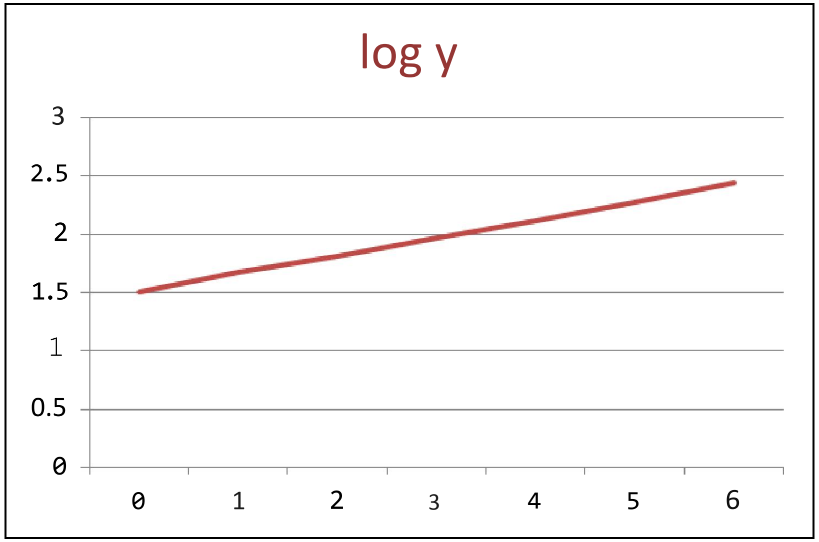
[redacted]	
0	32
1	47
2	65
3	92
4	132
5	190
6	275

Exponential Curve ...

SOLUTION:

- If the graph of the values of X and log Y gives a straight line, use an exponential curve .





Remember

- Straight Line:

$$Y = a + bX$$

- Second Degree line (Also called Parabola)

$$Y = a + bX + cX^2$$

- Third Degree Line

$$Y = a + bX + cX^2 + dX^3$$

Remember

$$Y = ab^X \quad >>> \quad \log Y = \log a + X \log b$$

$$Y' = A + BX$$

$$Y = ae^{bX} \quad >>> \quad \log Y = \log a + (b \log e) X$$

$$Y' = A + Bx$$

$$\frac{1}{Y} = a + bX \quad >>> \quad Y' = a + bX$$

Recap

Remember Rules

Draw plot of data

Observe the plot

Also apply verify from mathematical tools

Graph only gives you rough ideas

If not clear picture, draw some other values

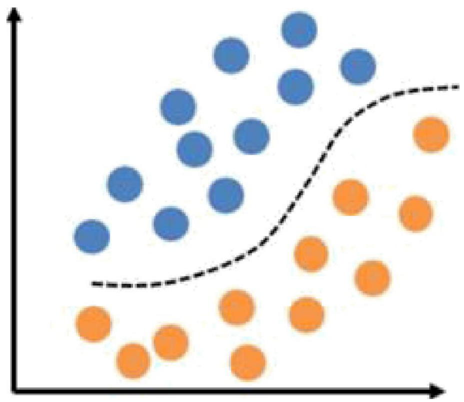
- The
END -

Converting non-linear into linear form

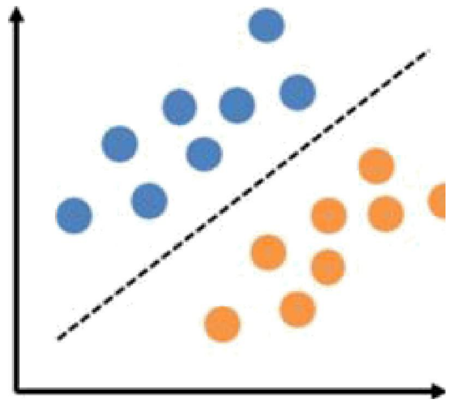
Some useful transformations to convert non-linear equations into linear form

Non-linear vs Linear

Nonlinear



Linear



#1

Non-linear form	$Y = aX^b$
Transformation	Taking log on both sides, $\log Y = \log(aX^b)$ $\log Y = \log a + \log(X^b)$ $\log Y = \log a + b \log X$ Let $\log Y = Y'$, Let $\log a = A$, $\log X = X'$
Linear form	$Y' = A + bX'$

#2

Non-linear form	$Y = ab^X$
Transformation	Taking log on both sides, $\log Y = \log(ab^X) = \log a + \log b^X = \log a + X \log b$; $\log Y = \log a + (\log b) X$
Linear form	$Y' = A + BX$

#3

Non-linear form	$$\frac{1}{Y} = a + bX$$
Transformation	Tranform $\frac{1}{Y}$ only...
Linear form	$$Y' = a + bX$$

#4

Non-linear form	$Y = \dfrac{1}{a + bX}$
Transformation	Take Reciprocal / Tranform Y only... $\frac{1}{Y} = a + bx$ $Y' = a + bx$
Linear form	$Y' = a + bX$

#5

Non-linear form	$\frac{1}{Y} = a + \frac{b}{1+X}$
Transformation	Change $\frac{1}{Y}$ and $\frac{1}{1+X}$ Let $\frac{1}{Y} = Y'$ Let $X' = \frac{1}{1+X}$ $Y' = a + bX'$
Linear form	$Y' = a + bX'$

#6

Non-linear form	$Y = a + b\sqrt{X}$
Transformation	Let $\sqrt{X} = X'$ So $Y = a + bX'$
Linear form	$Y = a + bX'$

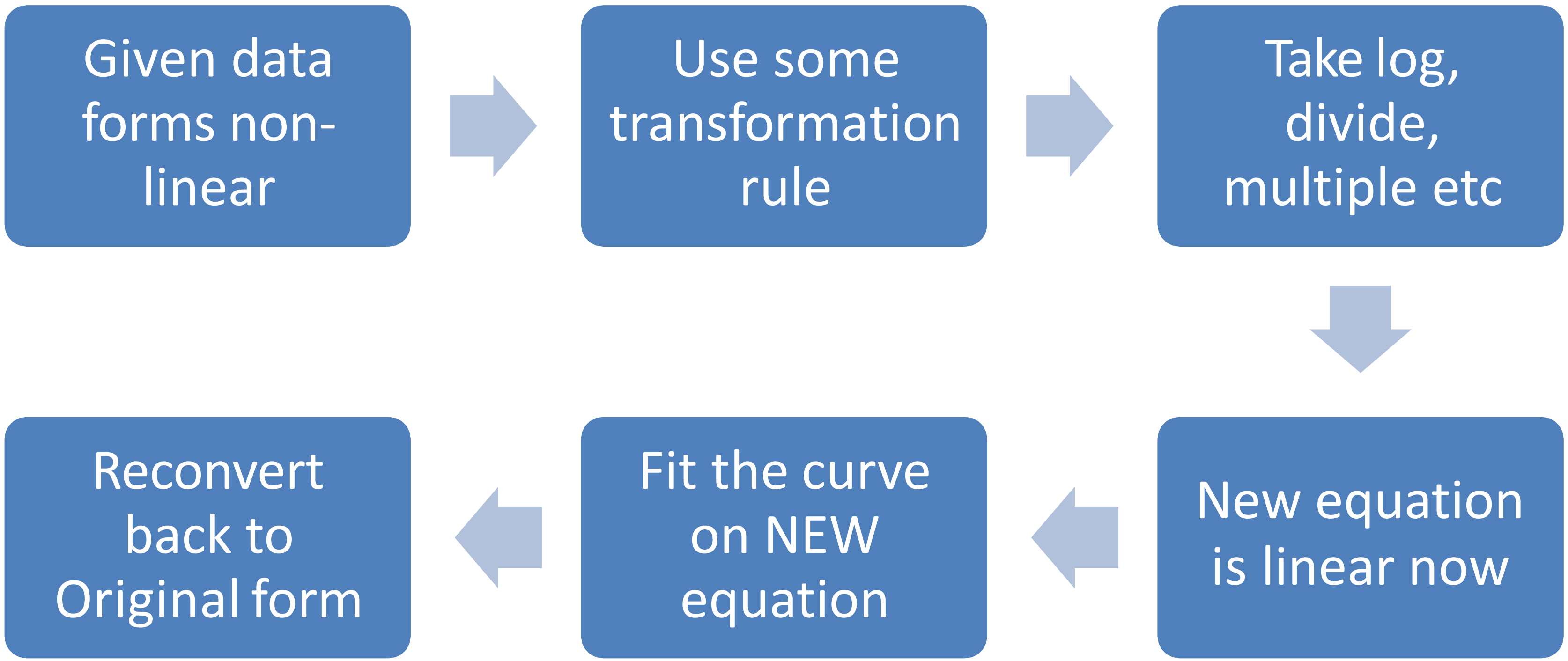
#7

Non-linear form	$Y = aX^2 + bX$
Transformation	Divide all by X
Linear form	$Y' = aX + b$

Summary

Non-linear form	Let/Suppose/Transformation	Linear form
$Y = aX^b$	$Y' = \log Y, A = \log a, X' = \log X$	$Y' = A + bX'$
$Y = ab^X$	$Y' = \log Y, A = \log a, B = \log b$	$Y' = A + BX$
$Y = \frac{1}{a+bX}$	$Y' = \frac{1}{Y}$	$Y' = a + bX'$
$\frac{1}{Y} = a + bX$	$Y' = \frac{1}{Y}$	$Y' = a + bX'$
$\frac{1}{Y} = a + \frac{b}{1+X}$	$Y' = \frac{1}{Y}, X' = \frac{1}{1+X}$	$Y' = a + bX'$
$Y = a + b\sqrt{X}$	$X' = \sqrt{X}$	$Y' = a + bX'$
$Y = aX^2 + bX$	$Y' = \frac{Y}{X}$	$Y' = aX + b$

Recap


Given data forms non-linear
Use some transformation rule
Take log, divide, multiple etc
New equation is linear now
Fit the curve on NEW equation
Reconvert back to Original form

- The
END -

Regression &Correlation Analysis

How to fit an Exponential Curve? Lecture 14

Procedure and step by step calculation with example

How to fit an Exponential Curve

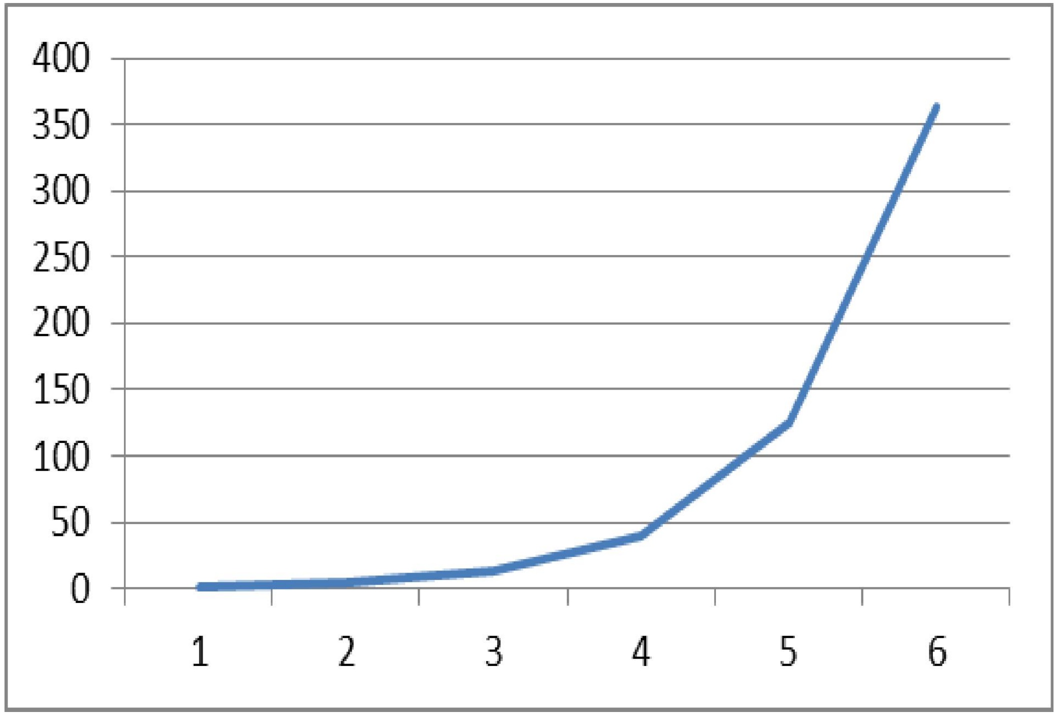
Example:

Fit an exponential curve $Y = ae^{bX}$ to data given below:

X	Y
1	1.6
2	4.5
3	13.8
4	40.2
5	125.0
6	363.0

Before we start ...

- What information you get from this graph?



Transformation of given equation

$$Y = ae^{bX}$$

Taking log on both sides...

$$\log Y = \log(ae^{bx})$$

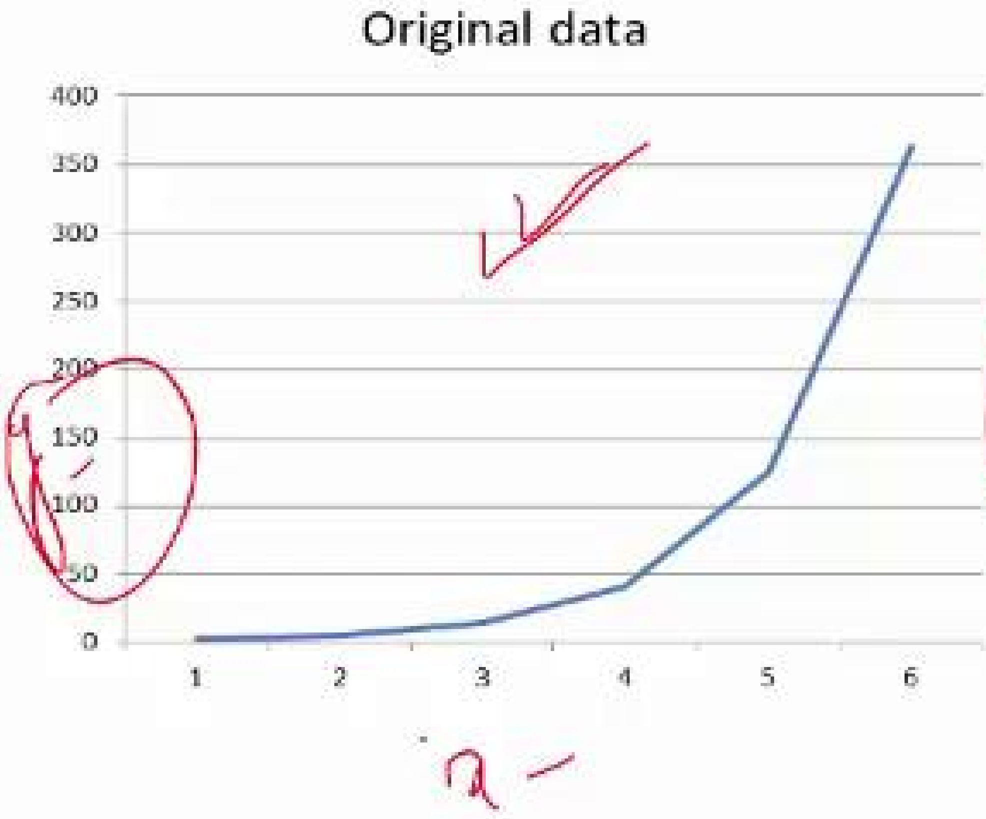
$$= \log a + \log e^{bx}$$

$$= \log a + bx \log e$$

$$\log y = \log a + (b \log e) x$$

$$Y = A + Bx$$

Just for fun … Graph of this equation



Transformation of given equation

Non-linear form ⟷ Linear form

$$Y = ae^{bX} \longrightarrow Y' = A + BX$$

where $Y' = \log Y$

$A = \log a$

$B = b \log e$

Now it is linear form …

$$Y = a + bX$$

$$Y' = A + BX$$

$$b = \frac{n(\Sigma XY) - (\Sigma X)(\Sigma Y)}{n\Sigma X^2 - (\Sigma X)^2}$$

$$B = \frac{n\Sigma XY' - (\Sigma X)(\Sigma Y')}{n\Sigma X^2 - (\Sigma X)^2}$$

$$a = \bar{Y} - b\bar{X}$$

$$A = \bar{Y'} - B\bar{X}$$

Required Calculation

X	Y	Log Y	(Log Y)^2	(X)(Log Y)	(X)(X)
1	1.6	0.2041	0.0417	0.2041	1
2	4.5	0.6532	0.4267	1.3064	4
3	13.8	1.1399	1.2993	3.4196	9
4	40.2	1.6042	2.5735	6.4169	16
5	125.0	2.0969	4.3970	10.4846	25
6	363.0	2.5599	6.5531	15.3594	36
21	-	8.2582	15.2914	37.1911	91

$n=6$, $\Sigma x=21$, $\Sigma y'$, $\Sigma y'^2$, $\Sigma xy'$, Σx^2

B formula

$$B = \frac{n(\Sigma XY') - (\Sigma X)(\Sigma Y')}{n\Sigma X^2 - (\Sigma X)^2}$$

$$= \frac{6(37.1911) - (21)(8.2582)}{6(91) - (21)^2}$$

$$= \frac{49.72}{105}$$

$$B = 0.47$$

A formula

$$A = \overline{Y}' - B\overline{X}$$

$= 1.3764 - (0.4735)(3.5)$

$A = -0.2811$

$\overline{Y}' = \frac{\Sigma Y'}{n}$

$\overline{X} = \frac{\Sigma X}{n}$

$Y' = A + BX$

$Y' = -0.2811 + 0.4735X$

Converting back …

$A = \log a$

$\Rightarrow a = anti - \log(A)$

$= anti\text{-}\log(-0.2811)$

$a = 0.5242$

$B = b(\log e)$

$\because e = 2.7183$

$0.4735 = b \log(2.7183)$

$0.4735 = b(0.4343)$

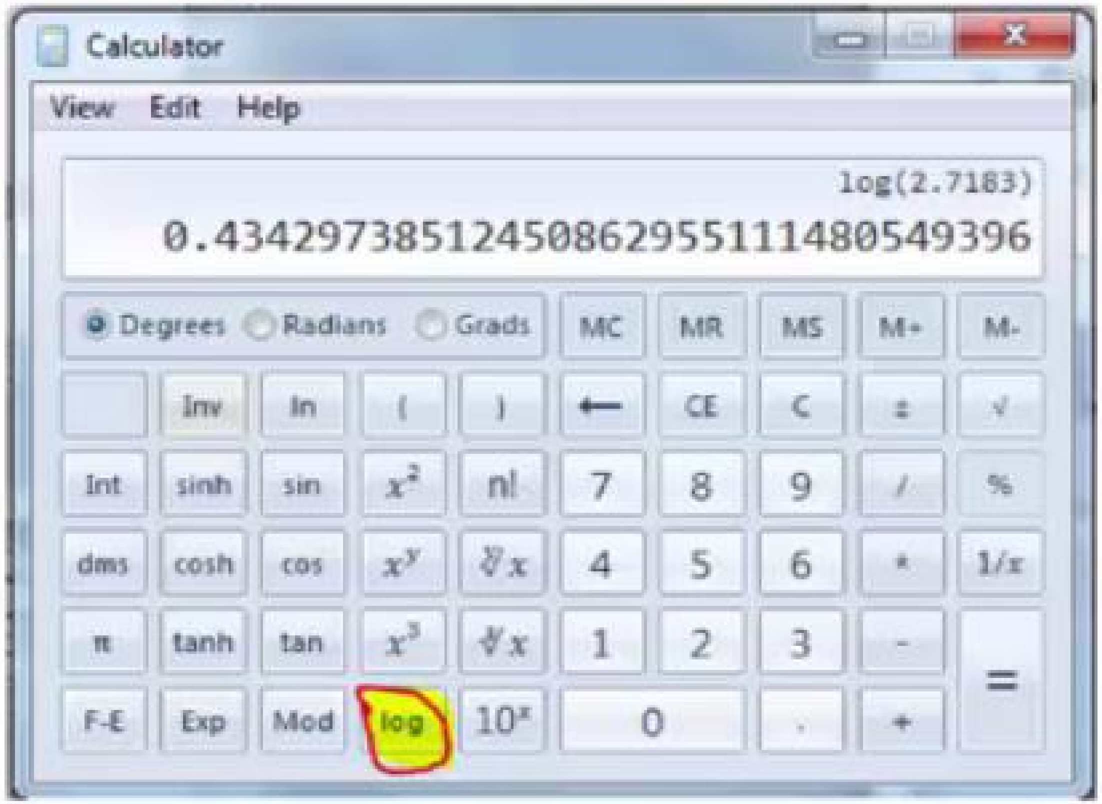
$\frac{0.4735}{0.4343} = b$

$b = 1.09$

How to Use Windows Calculator

- For **Anti – log**
  - Write value
  - Then Press 10ˣ

- For **log**
  - Write value
  - Then Press log



Hence the original equation …

$$Y = a e^{bX}$$

Putting values,

$$Y = 0.5242\, e^{(1.09)X}$$

The required Equation

Recap

Given Data

Make suitable Transformation

Get new equation

Final Equation

Converted back

Do Table Calculations

- The
END -

Regression &Correlation Analysis

How to fit an Exponential Curve?

Procedure and step by step calculation with example II

How to fit an Exponential Curve

Example:

Fit an exponential curve $Y = aX^b$ to data given below:

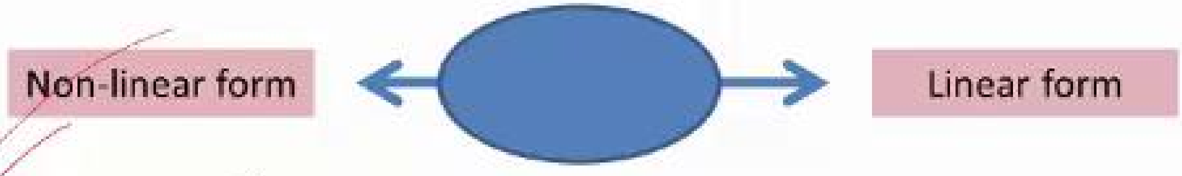
X	Y
1	2.98
2	4.26
3	5.21
4	6.10
5	6.80
6	7.50

Y

How .

Takin

Transformation of given equation



$$Y = aX^b$$

How .... ?

Taking log on both sides...

$$\log y = \log(ax^b)$$
$$= \log a + \log x^b$$
$$\log y = \log a + b \log x$$
$$Y' = A + bX'$$

Let
$$\log y = Y'$$
$$\log a = A$$
$$\log x = X'$$

N

1st normal eq

(Apply Σ on b

Now it is linear form ...

$$Y' = A + bX'$$

A, b

Normal Equation

1st normal equation:

(Apply Σ on both sides)

$$\sum Y' = nA + b\sum X'$$

2nd normal equation:

(Multiply with X′ and then Apply Σ on both sides)

$$X'Y' = AX' + bX'^2$$

$$\sum X'Y' = A\sum X' + b\sum X'^2$$

X
1
2
3
4
5
6
Total

Required Calculation

X	Y	Log Y	Log X	(Log X)^2	(Log X)*(Log Y)
1	2.98	2.98	0.0000	0.0000	0.0000
2	4.26	4.26	0.3010	0.0906	0.1895
3	5.21	5.21	0.4771	0.2276	0.3420
4	6.10	6.10	0.6021	0.3625	0.4728
5	6.80	6.80	0.6990	0.4886	0.5819
6	7.50	7.50	0.7782	0.6055	0.6809
Total	-	4.3133	2.8574	1.7749	2.2672

5

Solving Normal Eqs…find b

$$\frac{6}{2.8574} = 2.0998$$

$$\Sigma Y' = nA + b\Sigma X' - - - - (1)$$

$$\Sigma X'Y' = A\Sigma X' + b\Sigma (X')^2 - - - - (2)$$

$$4.3133 = 6A + b(2.8574) \longrightarrow (1)$$

$$2.2672 = (2.8574)A + (1.7749)b \longrightarrow (2)$$

Now multiply Eq (2) with 2.0998,

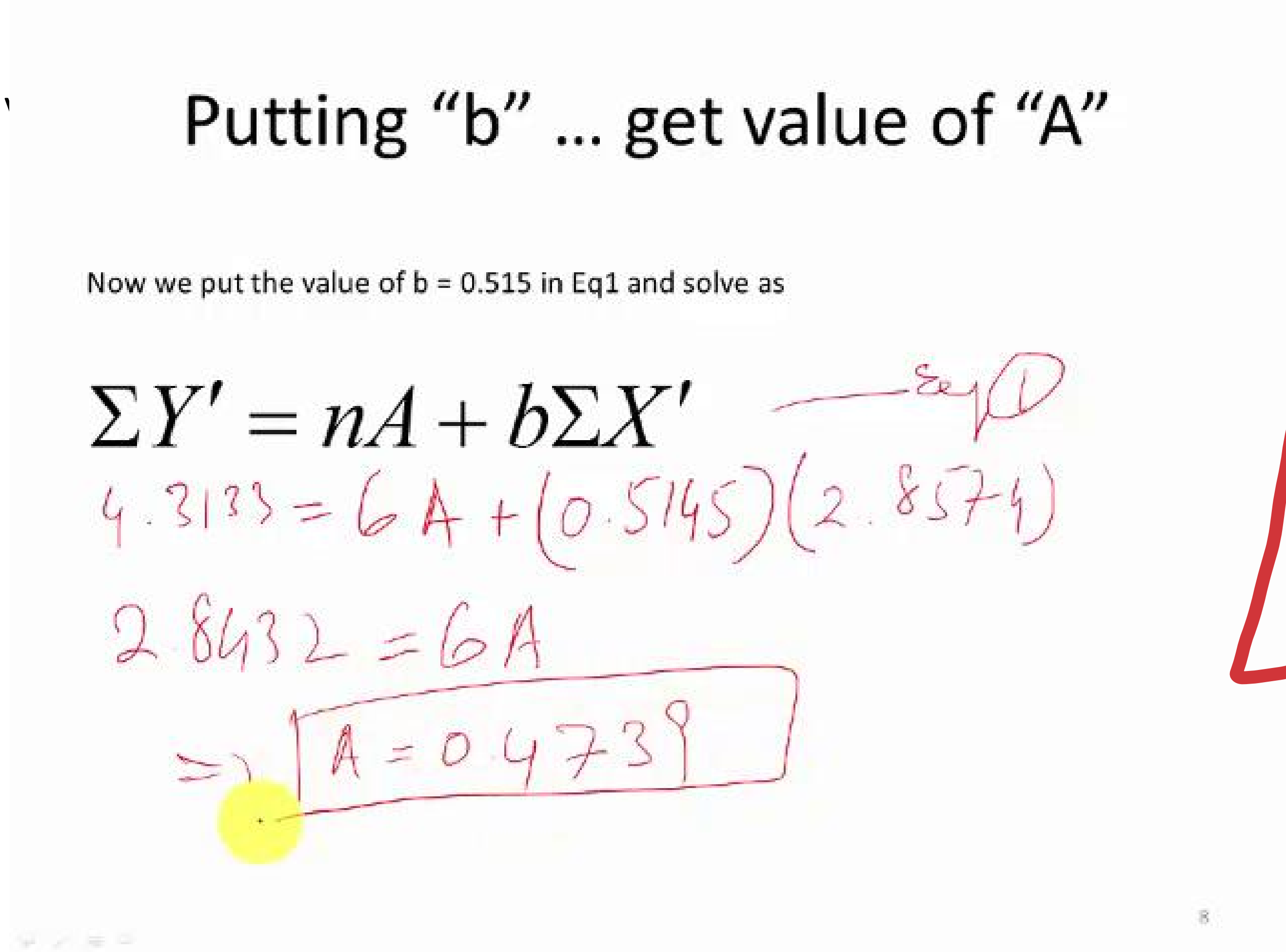
$$4.7607 = 6A + 3.7270b \longrightarrow (3)$$

Subtract Eq (1) from Eq (3)

$$0.4474 = 0 + 0.8696b \Rightarrow b = .05145$$

Now we put the

$$\Sigma Y' =$$



Converting back …

$$A = \log a$$

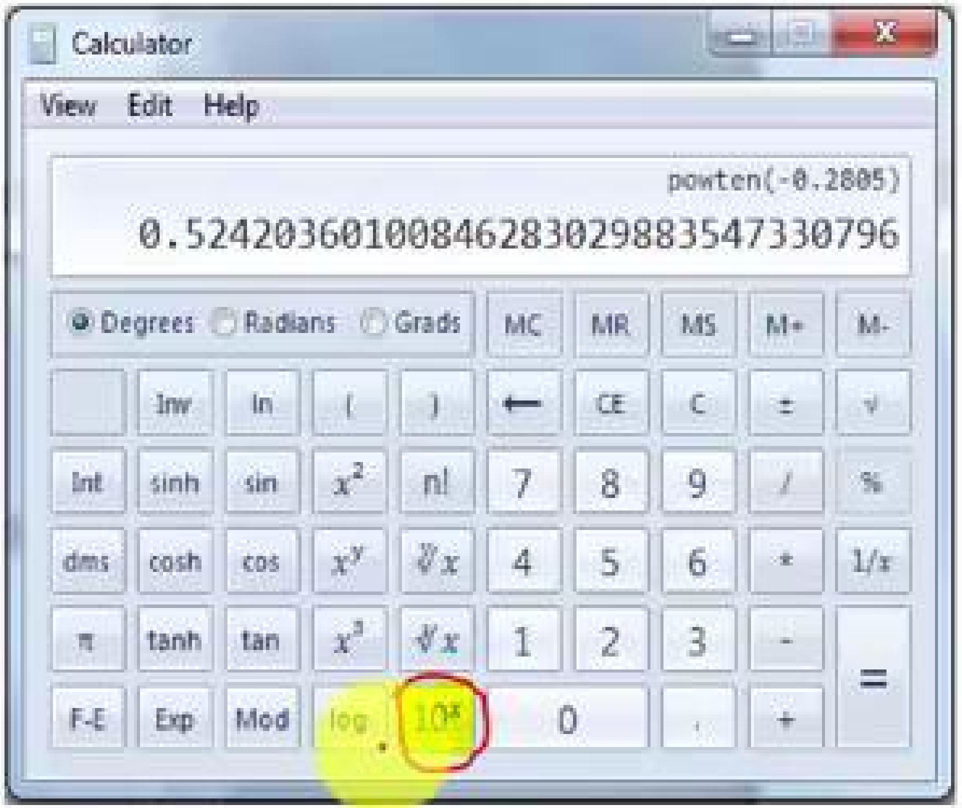
$$\Rightarrow a = anti - \log(A)$$

$$= anti\_log(0.4739)$$

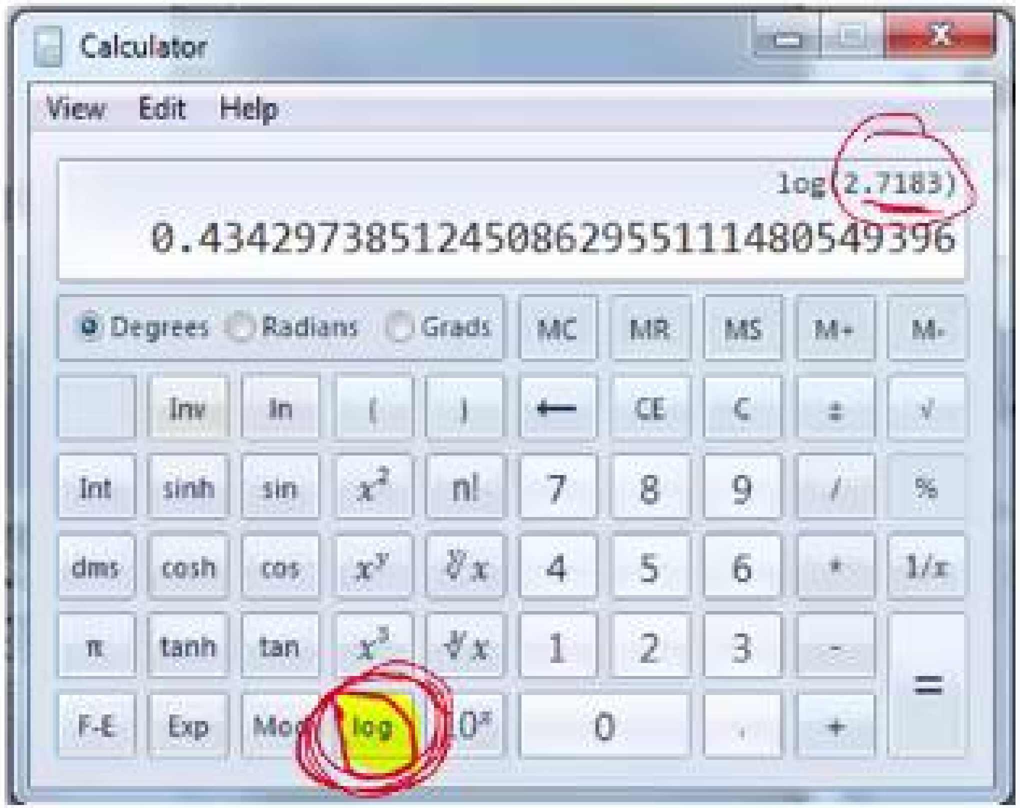
$$a = 1.61$$

How to Use Windows Calculator

- For **Anti – log**
  - Write value
  - Then Press 10ˣ



- For **log**
  - Write value
  - Then Press log



Hence the fitted equation becomes…

$$Y = aX^{b}$$

$$Y = 161X^{0.5145}$$

This is the required Equation.

Recap

Given Data

Make suitable Transformation

Get new equation

Final Equation

Converted back

Solve Normal Eqs and Do Table Calculations

- The
END -

How to fit a Reciprocal Curve? Lecture 15

Procedure and step by step calculation with example

How to fit a Reciprocal Curve

Explanation by Example.

Example #1:

Fit a reciprocal curve $\frac{1}{Y} = a + bX$ to data given below:

X	Y
0	10
1	8
4	5
6	4
12	2.5
16	2

Given equation ...

$$\frac{1}{Y} = a + bX$$

Let $Y' = \frac{1}{Y}$ and make a new equation

$$Y' = a + bx$$

Now it is linear form ...

$$Y = a + bX \quad \longrightarrow \quad Y' = a + bX$$

As we already know

$$b = \frac{n(\Sigma XY) - (\Sigma X)(\Sigma Y)}{n\Sigma X^2 - (\Sigma X)^2}$$

$$a = \overline{Y} - b\overline{X}$$

New form ....

$$b = \frac{n\Sigma xy' - \Sigma x \, \Sigma y'}{n\Sigma x^2 - (\Sigma x)^2}$$

$$a = \overline{y'} - b\overline{x}$$

Required Calculation

x	y	Y' = 1/y	Xy'	xx
0	10	0.10	0	0
1	8	0.13	0.125	1
4	5	0.20	0.8	16
6	4	0.25	1.5	36
12	2.5	0.40	4.8	144
16	2	0.50	8	256
39		1.58	15.225	453

$n = 6$, Σx $\Sigma Y'$ $\Sigma xY'$ Σx^2

b formula

$$b = \frac{n(\Sigma XY') - (\Sigma X)(\Sigma Y')}{n\Sigma X^2 - (\Sigma X)^2}$$

$$= \frac{6(15.225) - (39)(1.58)}{6(453) - (39)^2}$$

$$= \frac{29.73}{1197}$$

$$b = 0.0248$$

a formula

$$a = \bar{Y}' - b\bar{X}$$

$$= \left(\frac{1.58}{6}\right) - (0.0248)\left(\frac{39}{6}\right)$$

$$\bar{Y}' = \frac{\sum Y'}{n}$$

$$\bar{X} = \frac{\sum X}{n}$$

$$a = 0.1019$$

Hence the original equation …

$$\frac{1}{Y} = a + bX$$

Putting valueof "a" and "b" ....

$$\frac{1}{Y} = 0.1019 + 0.0248X$$

This is my final

Answer :)

Recap

- Given Data
- Make suitable Transformation
- Get new equation
- Final Equation
- Convert back, if needed
- Do Table Calculations

- The
END -

Regression &Correlation Analysis

How to fit a Reciprocal Curve? Lecture 15

Procedure and step by step calculation with example

How to fit a Reciprocal Curve

Explanation by example:

Example #2:

Fit a line of the form $Y = \frac{1}{a + bX}$ to data given below:

X	Y
0	10
1	8
4	5
6	4
12	2.5
16	2

Given equation …

$$Y = \frac{1}{a + bX}$$ → Non-linear

Transformation

Take the reciprocal on both sides

$$\frac{1}{y} = \frac{a+bx}{1}$$

Let $y' = \frac{1}{y} = a + bx$

$$y' = a + bx$$ linear form

Now it is linear form …

$$Y = a + bX$$

As we already know

$$b = \frac{n(\Sigma XY) - (\Sigma X)(\Sigma Y)}{n\Sigma X^2 - (\Sigma X)^2}$$

$$a = \overline{Y} - b\overline{X}$$

$$Y' = a + bX$$

New form ….

$b =$

$a =$

Required Calculation

x	y	Y' = 1/y	Xy'	xx
0	10	0.10	0	0
1	8	0.13	0.125	1
4	5	0.20	0.8	16
6	4	0.25	1.5	36
12	2.5	0.40	4.8	144
16	2	0.50	8	256
39		1.58	15.225	453

b formula

$$b = \frac{n(\Sigma XY') - (\Sigma X)(\Sigma Y')}{n\Sigma X^2 - (\Sigma X)^2}$$

$$= \frac{6(15.225) - (39)(1.58)}{6(453) - (39)^2}$$

$$= \frac{29.73}{1197} = 0.0248$$

a formula

$$a = \bar{Y}' - b\bar{X}$$

$= \frac{1.58}{6} - (0.0248)\left(\frac{39}{6}\right)$

$a = 0.1019$

$\bar{Y} = \frac{\Sigma y}{n} = \frac{1.58}{6}$

$\bar{X} = \frac{\Sigma x}{n} = \frac{39}{6}$

Hence the original equation …

$$\frac{1}{Y} = a + bX$$

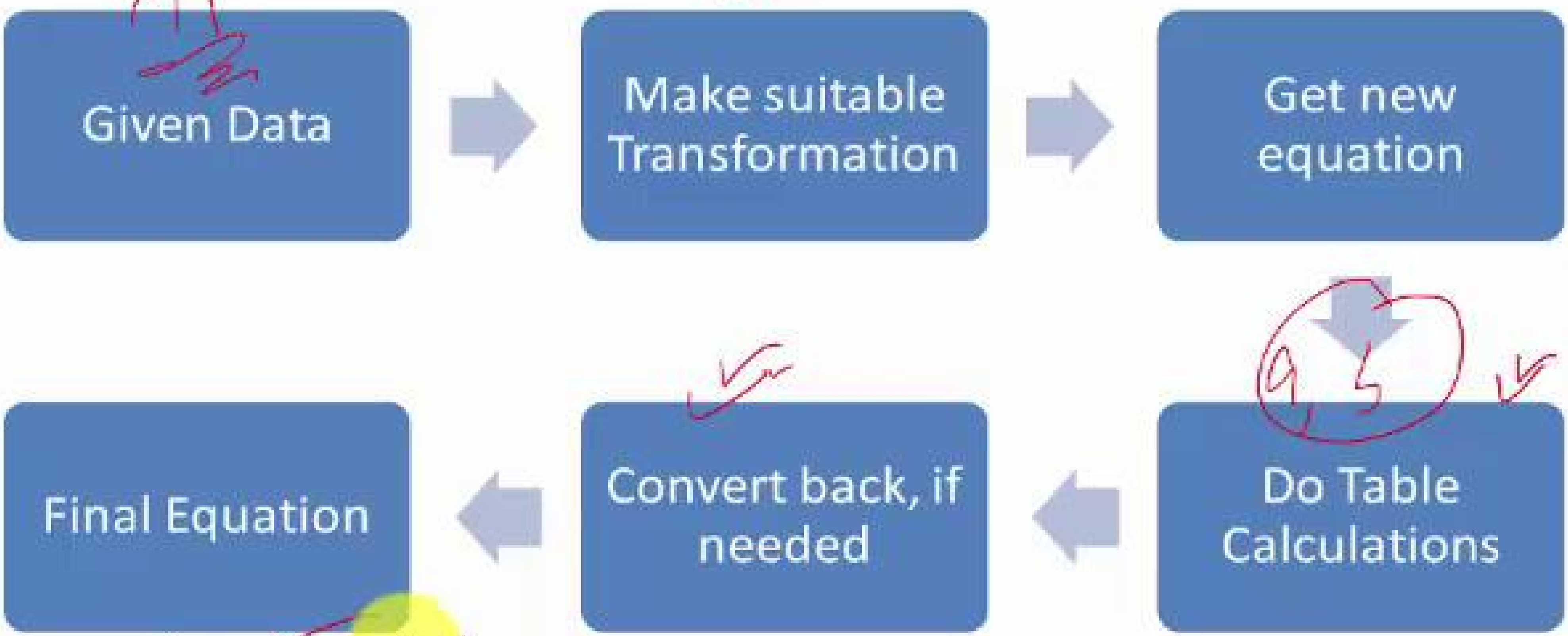
Putting value of "a" and "b" ....

$$\frac{1}{Y} =$$

$$Y = \frac{1}{a+bx}$$

$$Y = \frac{1}{0.1019 + 0.0248x}$$

Recap


Given Data
Make suitable Transformation
Get new equation
Do Table Calculations
Convert back, if needed
Final Equation

- The
END -