Lecture 1 Intro to Excel.pptx

lecture 2 Centeral Tendency.pptx

Lecture 3 Dispersion(1).pptx

lecture 4 Graphs.pptx

lecture 5 SPC Charta.pptx

lec 6.ppt

lecture no.7 computation.pptx

lecture 9.pptx

Lec10.ppt

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Software's

- MS EXCEL 2010
- SPSS 19

Lecture 1

Contents

- Starting MS EXCEL
- Opening a file,
- Naming and defining variables,
- Data display,
- Entering data,
- Help system,
- tables ,
- Saving files in Excel ,
- Leaving Excel,
- Practical exercise in Data Preparation.

Introduction to Microsoft Excel

Objectives:

- To define spreadsheets and explain basic functionality
- To introduce the basic features of Excel
  - Vocabulary
  - Entering Data
  - Formatting Data
  - Precision vs. Display
  - Operators & Order of Precedence

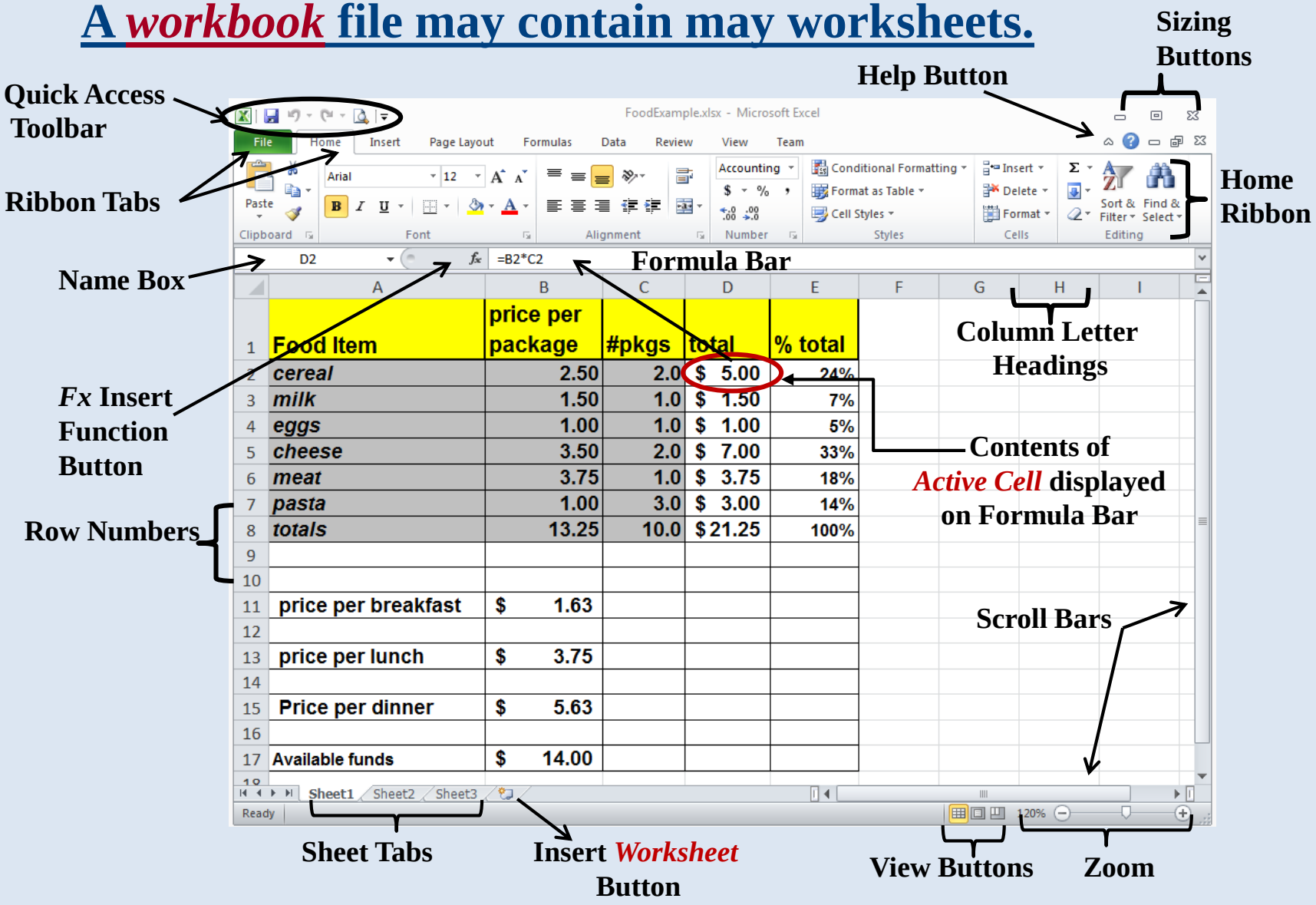
Spreadsheet: Electronic sheet of paper organized by columns & rows

The advantage of an electronic spreadsheet is it allows you to easily change data and have all "related" calculations automatically update..

	A	B	C	D	E
1	Food Item	price per package	#pkgs	total	% total
2	*cereal*	2.50	2.0	$ 5.00	24%
3	*milk*	1.50	1.0	$ 1.50	7%
4	*eggs*	1.00	1.0	$ 1.00	5%
5	*cheese*	3.50	2.0	$ 7.00	33%
6	*meat*	3.75	1.0	$ 3.75	18%
7	*pasta*	1.00	3.0	$ 3.00	14%
8	*totals*	13.25	10.0	$21.25	100%

Spreadsheets in Excel are referred to as *worksheets*.

A *workbook* file may contain may worksheets.



	Food Item	price per package	#pkgs	total	% total
2	*cereal*	2.50	2.0	$ 5.00	24%
3	*milk*	1.50	1.0	$ 1.50	7%
4	*eggs*	1.00	1.0	$ 1.00	5%
5	*cheese*	3.50	2.0	$ 7.00	33%
6	*meat*	3.75	1.0	$ 3.75	18%
7	*pasta*	1.00	3.0	$ 3.00	14%
8	*totals*	13.25	10.0	$ 21.25	100%
9					
10					
11	price per breakfast	$ 1.63			
12					
13	price per lunch	$ 3.75			
14					
15	Price per dinner	$ 5.63			
16					
17	Available funds	$ 14.00			

Each box is referred to as a "*cell*". Cells may contain *Labels, Values* or *Formulas* that result in a value or label. A cell is identified first by its column letter and then by its row number

Columns

Rows

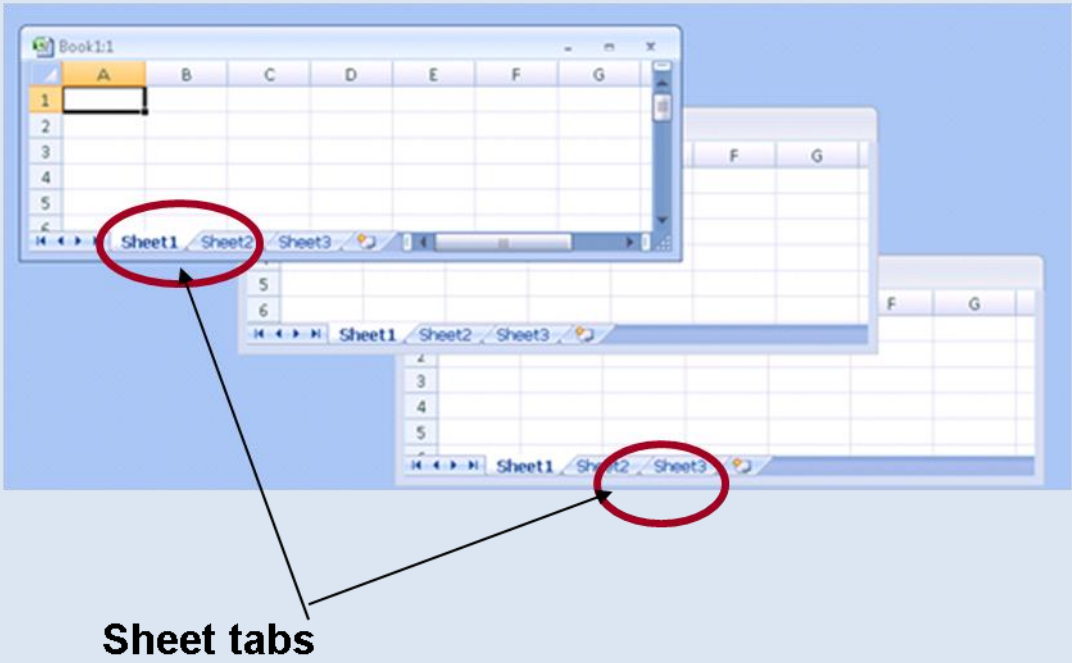
Labels

Cell D2 Contains the Formula = B2*C2

	A	B	C	D
1	Food Item	price per package	#pkgs	total
2	*cereal*	2.50	2.0	$ 5.00
3	*milk*	1.50	1.0	$ 1.50
4	*eggs*	1.00	1.0	$ 1.00
5	*cheese*	3.50	2.0	$ 7.00
6	*meat*	3.75	1.0	$ 3.75
7	*pasta*	1.00	3.0	$ 3.00
8	*totals*	13.25	10.0	$21.25

One can also write formulas that refer to cells on other worksheets – *Sheetname!*Cell-Reference

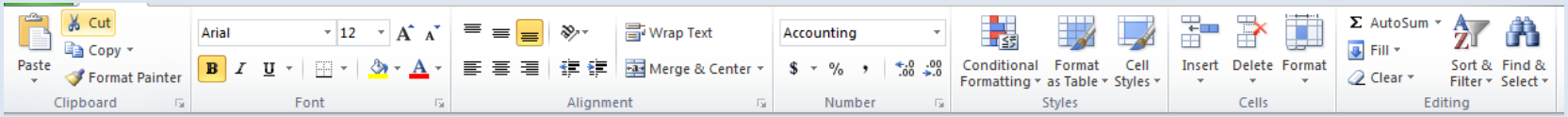
***input!*B1*input!*B3 + A1**
When referencing a cell on the same spreadsheet as the *active* cell the sheet name is not required.



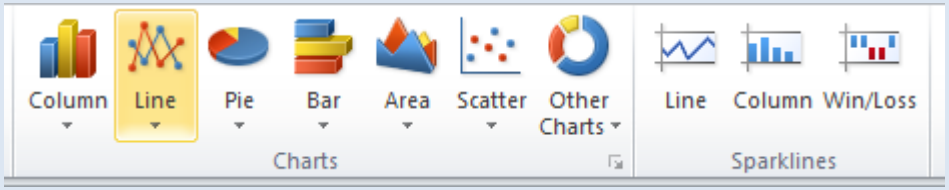
Sheets may be named and displayed with different colors tabs, The order of the worksheets may be modified as well.

File tab – opens menus for opening and saving Files, and modifying Excel Options

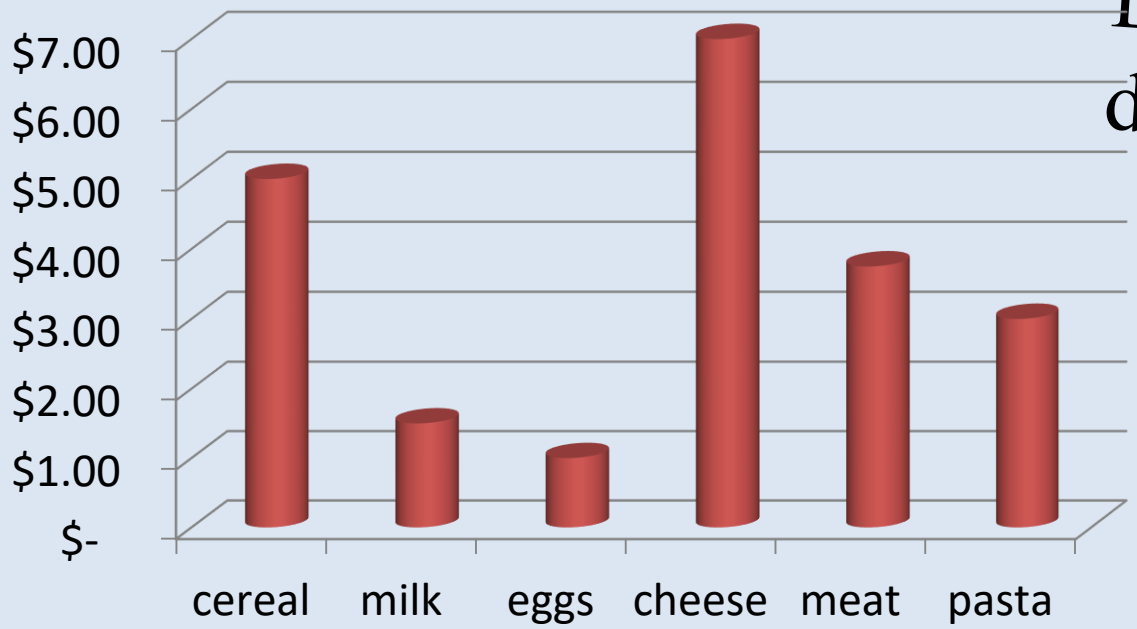
Quick Access Toolbar can be customized to include icons to frequently Used features such as Print Preview



Home Ribbon use to change fonts, justify text, insert rows etc. Ribbons are organized into ***Groups*** of similar tasks such as the Font group or the Number group. In addition, there are other ribbons containing groups/buttons for laying out pages using the review features etc.



Highlight your data, select a Chart type and Edit & its done!

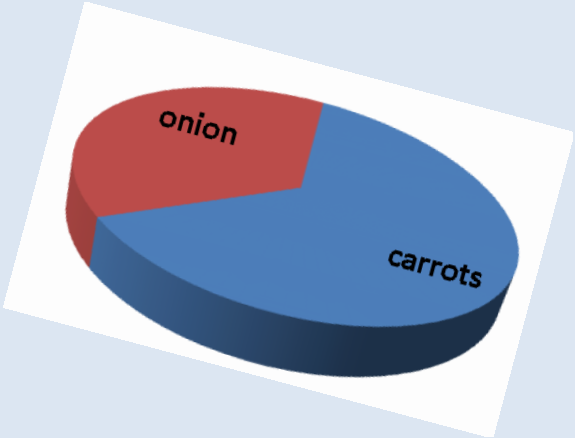


The "Power" of using Spreadsheet Applications

	A	B	C	D
1	Items	$/each	Quantity	Total
2	carrots	$ 1.00	4	$ 4.00
3	onion	$ 0.39	6	$ 2.34
4	total			$ 6.34
5				

=B2*C2

- **Each entry can be related to other values by including cell referencing in *formulas.***
- **Formula values are automatically updated when a referenced value changes**
- **Formulas can be copied**
- **Charts can be easily generated**



Formulas

- A ***formula*** is a sequence of values, cell references and operators that produce a new value.

= E8 + 3*(E10 - E11)

- Formulas always start with an equal sign **=**
- In addition a formula can also contain built-in ***functions*** like SUM, AVERAGE, IF, COUNTIF, etc. ***=Sum(A2:A8)*2***

Things you need to know when writing formulas in Excel

- Data precision vs. cell display

0.02349	0.02

- Types of operators that can be used

/ ≤ –

- Order of precedence of operators

=B2+B3*B1/B8^2

In order to write Excel formulas we also need to use the correct Operator Symbols

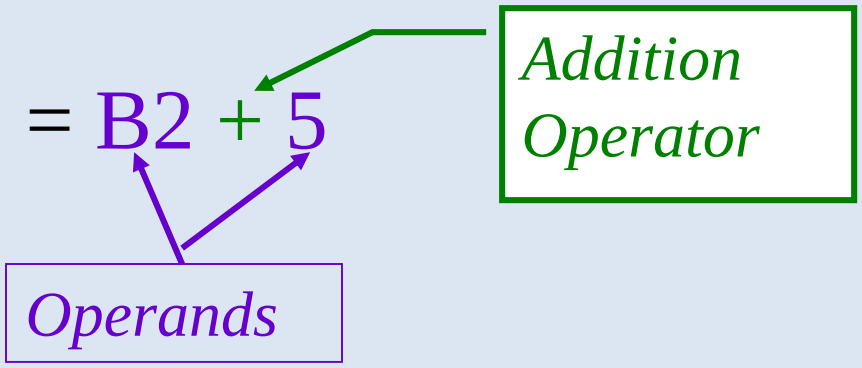
Formulas contain two types of components:

- ***Operators***: Operations to be performed

  Arithmetic operators: * / + - ^

  Relational operators: >, <, <=, >=,< >,=

- ***Operands***: Values to be operated on



Precedence of Operators

- *( ) Parenthesis* is a special operator that forces evaluation of the expression inside it first
- *Exponentiation* (2^3 →8)
- *Arithmetic operators*: Multiplication & Division
  - Multiplication & Division have equal precedence and are evaluated from left to right
- *Arithmetic operators:* Addition & Subtraction
  - Addition & Subtraction have equal precedence and are evaluated from left to right
- *Relational operators* have a lower precedence than arithmetic operators

Precision: number of decimal places stored in the computer.

Formatted Display: number of decimal places that appear in a cell

Type in a cell : =**1/8**

display in cell

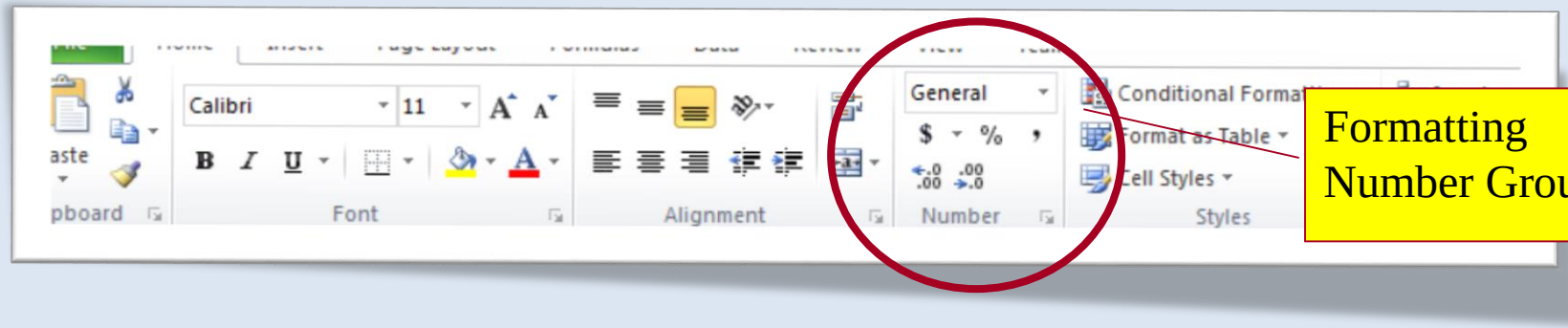
	A	B
1	0 places	0
2	1 decimal	0.1
3	2 decimal	0.13
4	3 decimal	0.125
5		

What value results for each - if multiplied by 1000?

Does the addition appear to be correct in col B?

	A	B
1	21.4	21
2	51.3	51
3	98.1	98
4	170.8	171

Formatting affects display not the precise value:



Percent %

Currency $

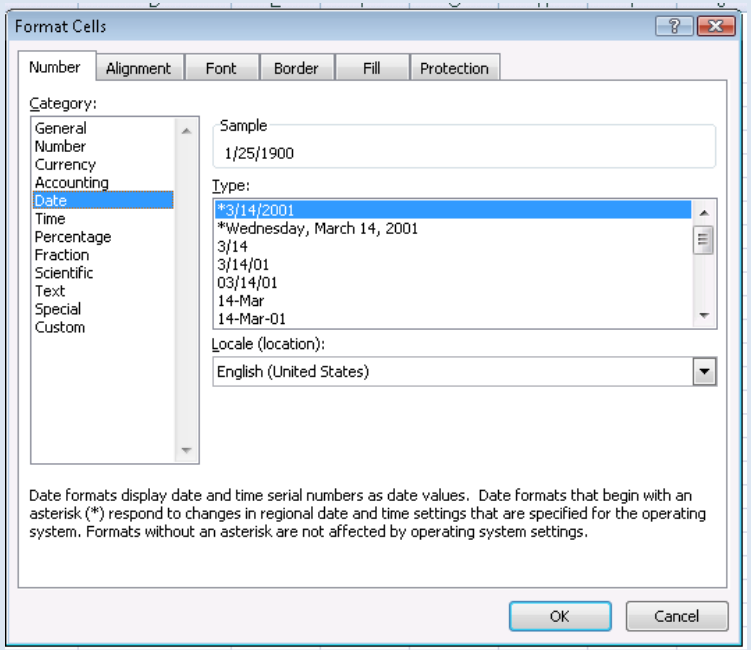
Decimal Display .0 .00 .00 .0

Commas ,

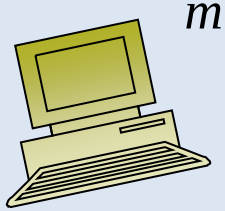
Values can also be used to display dates

	A	B
1	1/27/2013	Friday, February 01, 2013

- **Dates are *values* that can be entered in several *formats*: January 27, 2013 or 1/27/2013**
- **Excel converts these dates to a numerical representation (1/22/2013 → 41301)**
- **Thus dates may be used in formulas: =A1–B1 will result in the value *5***



Note: To do arithmetic calculations with dates if you type =1/27/2013-1/22/2013 directly in a cell it does not interpret it a date – cell references must be used.



Walkthrough: Building a Simple Spreadsheet

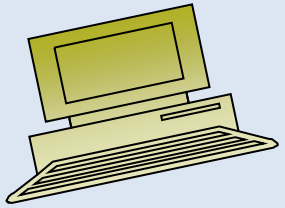
- **Entering labels and values**
- **Formatting cells**
  - –font, size, style, color, borders, alignment
  - –Numeric Format, Currency, Decimal Places
  - –text wrap, center titles
  - –Column widths, row height
- **Inserting/Deleting rows and columns and sheets**
- **Writing a simple formula & modify decimal display**
- **Create a simple chart**
- **Sheet tabs**
  - –Creating a new worksheets in a workbook (“new sections in a document”, Naming Sheets

Microsoft Excel Vocabulary

- **Workbook** - an Excel file with one or more sheets or pages
- **Worksheet** - page in the workbook (*spreadsheet*)
- **Ribbon** – Tabbed section containing command icons
- **Row -** Horizontal (Row Number)
- **Column** - Vertical (Column Letter)
- **Cell** - Column/Row combination (ex: C3)
- **Values** - Numeric Entries used in calculations
- **Labels** - text that describes the data
- **Active Cell** - cell currently in use (highlighted)
- **Formula Bar** - top of spreadsheet where Excel displays the value or formula for that cell

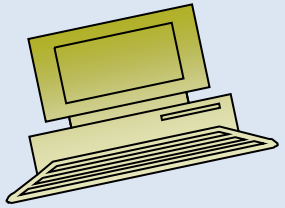
Practical Exercise in Data Preparation

Lecture 2

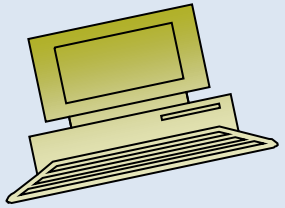


Contents

- Measure of Central Tendency
- Mean
- Median
- Mode
- Grouping Data
- Frequency Distribution
- Group Mean
- Practical exercise in Data Preparation.



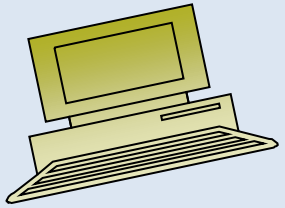
How to Use Excel to Find the Mean, Median & Mode



Objectives

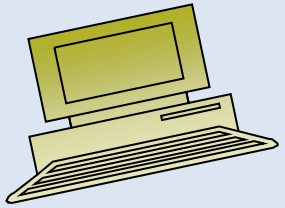
After completing this lecture, you should be able to:

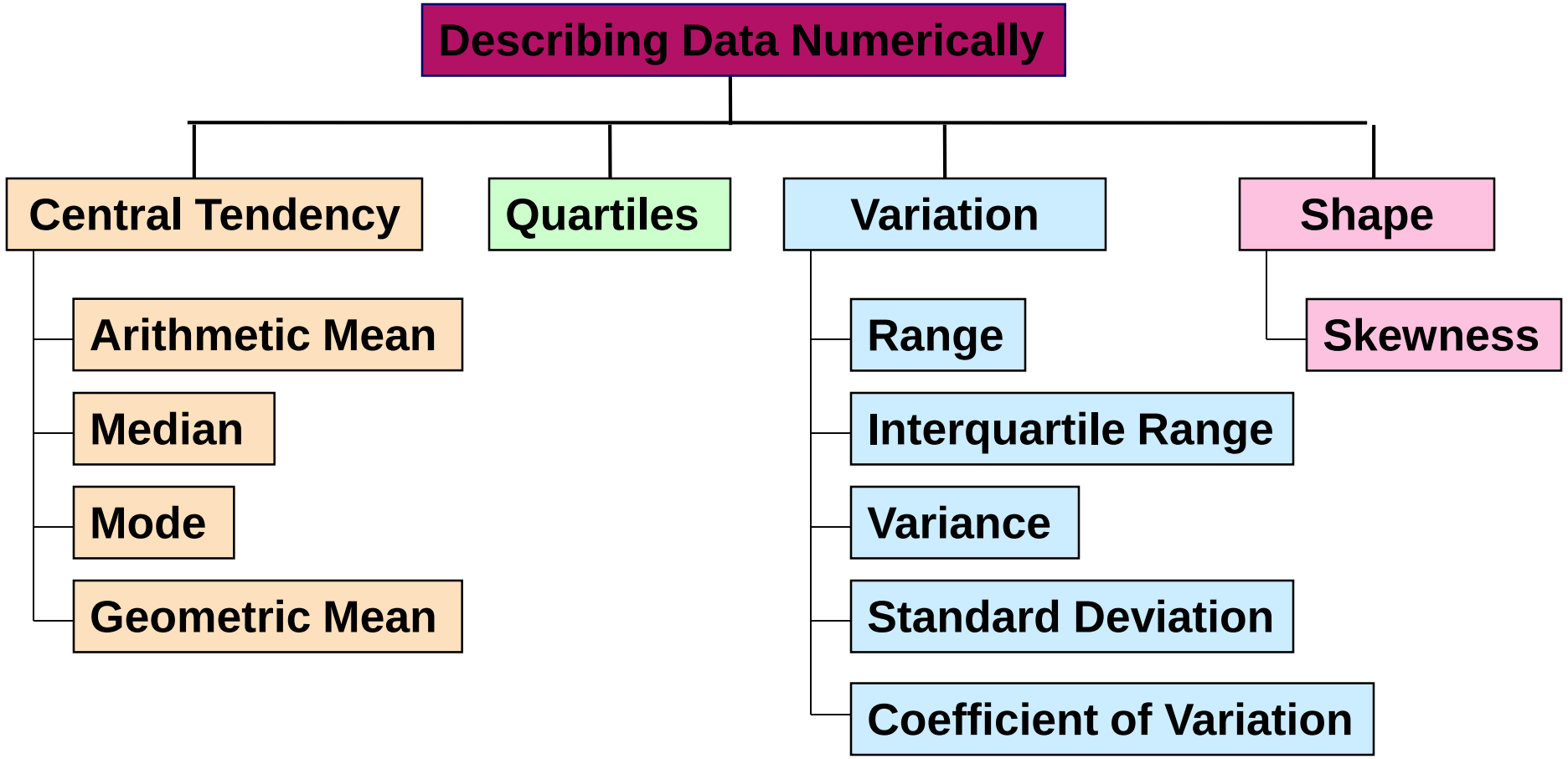
- Compute and interpret the mean, median, and mode for a set of data.
- Group data and frequency distribution.



Central Tendency:

- In statistics a central tendency (or, more commonly, a measure of central tendency) is a central or typical value for a probability distribution. It may also be called a center or location of the distribution.
- The most common measures of central tendency are the arithmetic mean, the median and the mode.
- A central tendency can be calculated for either a finite set of values




Describing Data Numerically
Central Tendency
Arithmetic Mean
Median
Mode
Geometric Mean
Quartiles
Variation
Range
Interquartile Range
Variance
Standard Deviation
Coefficient of Variation
Shape
Skewness

Measures of Central Tendency

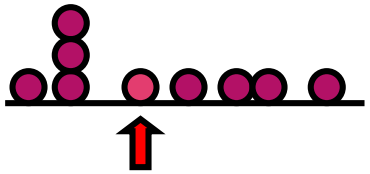
Overview

Central Tendency

Arithmetic Mean

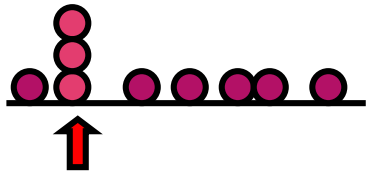
$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

Median



Midpoint of ranked values

Mode



Most frequently observed value

Geometric Mean

$$\bar{X}_G = (X_1 \times X_2 \times \cdots \times X_n)^{1/n}$$

Arithmetic Mean

- The arithmetic mean (mean) is the most common measure of central tendency
  - For a sample of size n:

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

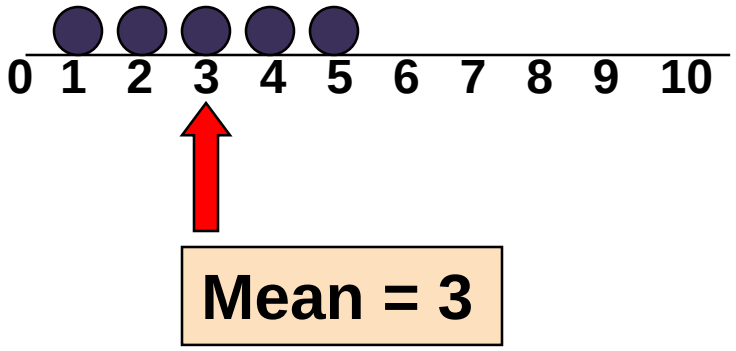
Sample size

Observed values

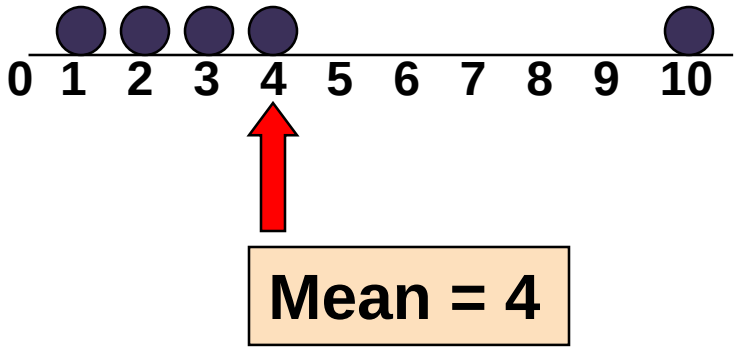
Arithmetic Mean

(continued)

- The most common measure of central tendency
- Mean = sum of values divided by the number of values
- Affected by extreme values (outliers)



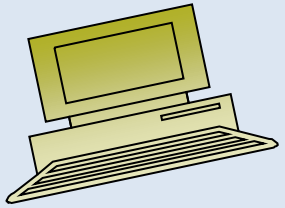
$$\frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$$



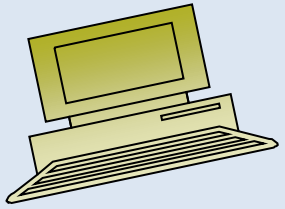
$$\frac{1+2+3+4+10}{5} = \frac{20}{5} = 4$$

Introduction with excel:

- Microsoft Excel 2010 is designed to store numerical inputs and permit calculation on those numbers, making it an ideal program if you need to perform any numerical analysis such as computing the mean, median, mode and range for a set of numbers.
- Each of these four mathematical terms describes a slightly different way of looking at a set of numbers .
- Excel has a built-in function to determine each of them.



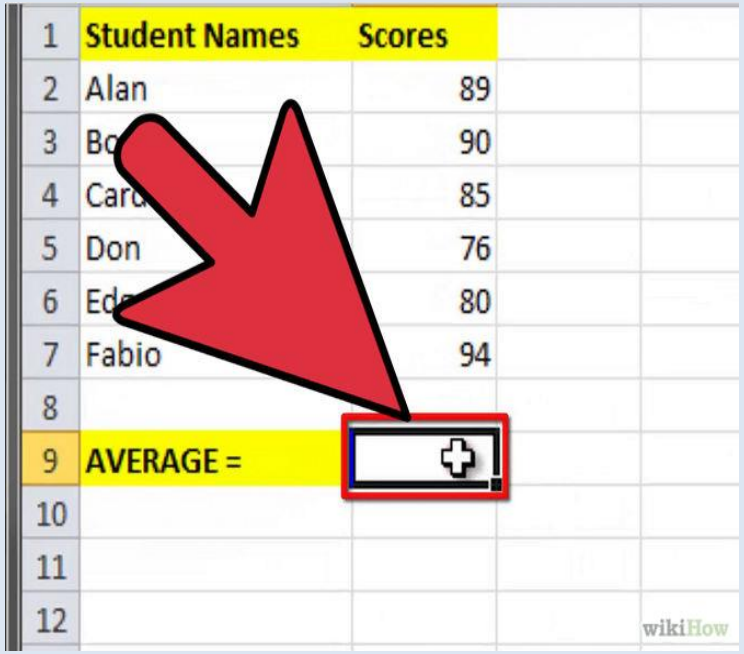
- Microsoft Excel includes a number of statistical functions, including the ability to figure the mean, median and mode of a data sample.
- While mean, the average of a group of numbers,
- Median, the midpoint number of a data group, are used
- Mode, the most frequently appearing number in a data group, can be useful as well, such as using the most frequent numeric grade score to provide feedback on the effectiveness of a teaching method.

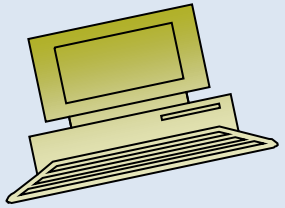


Mean in Excel:

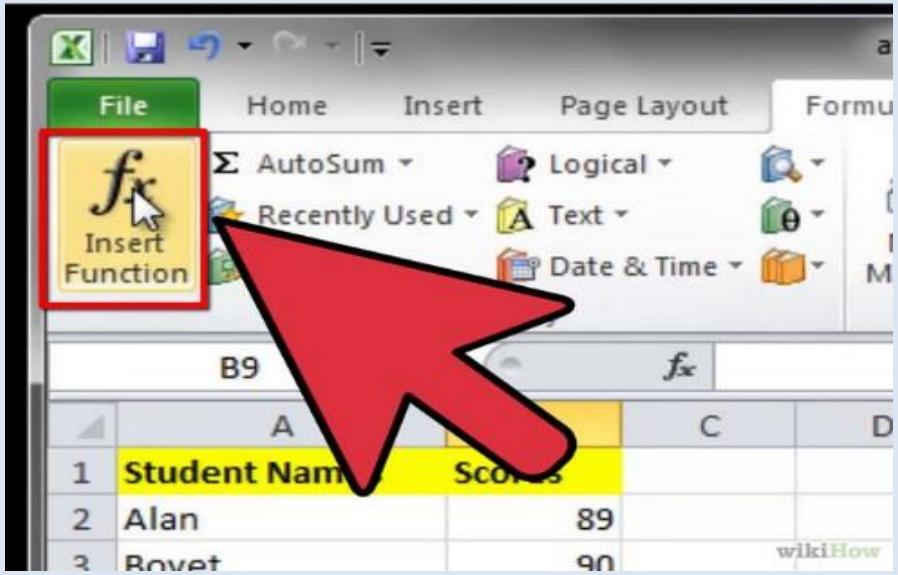
Use the "AVERAGE" function in Excel to find the mean of a set of numbers.

- Enter the range of numbers in your Excel spreadsheet.
- Click where you want the mean (Average).



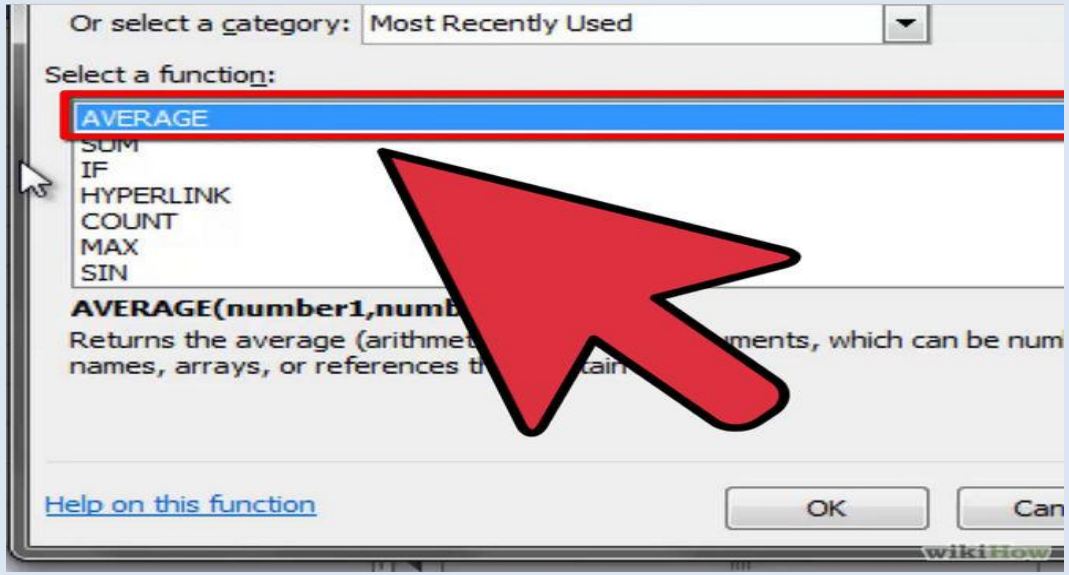


- Click "Formulas" and select the "Insert Function" tab.
- Enter the numbers in your Excel spreadsheet in either a row or a column.



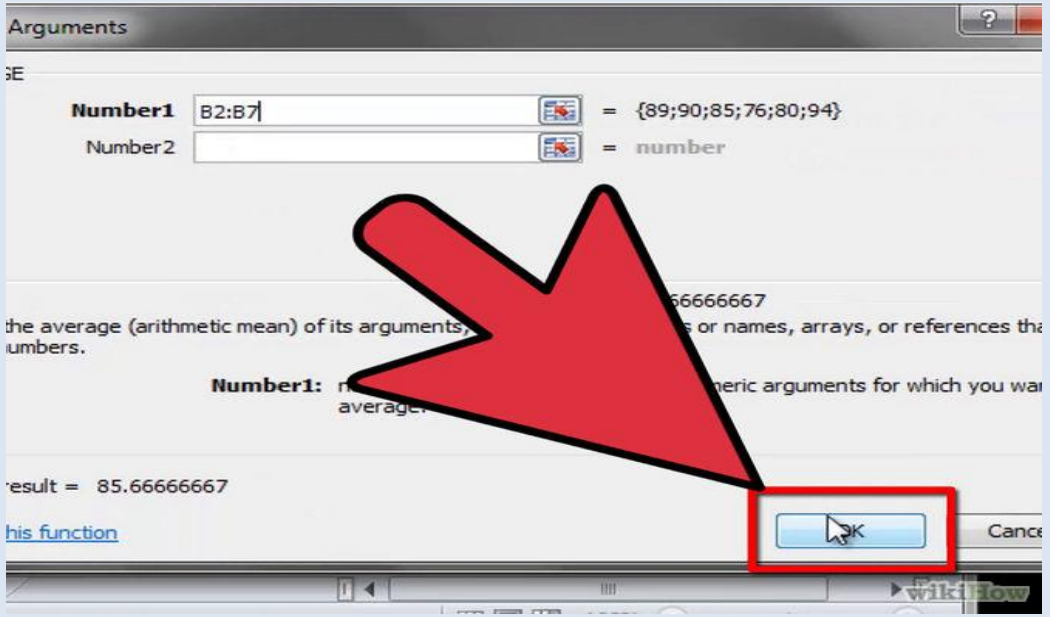
Select Formula:

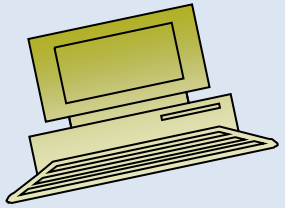
- **Scroll down and select the "Average" function.**



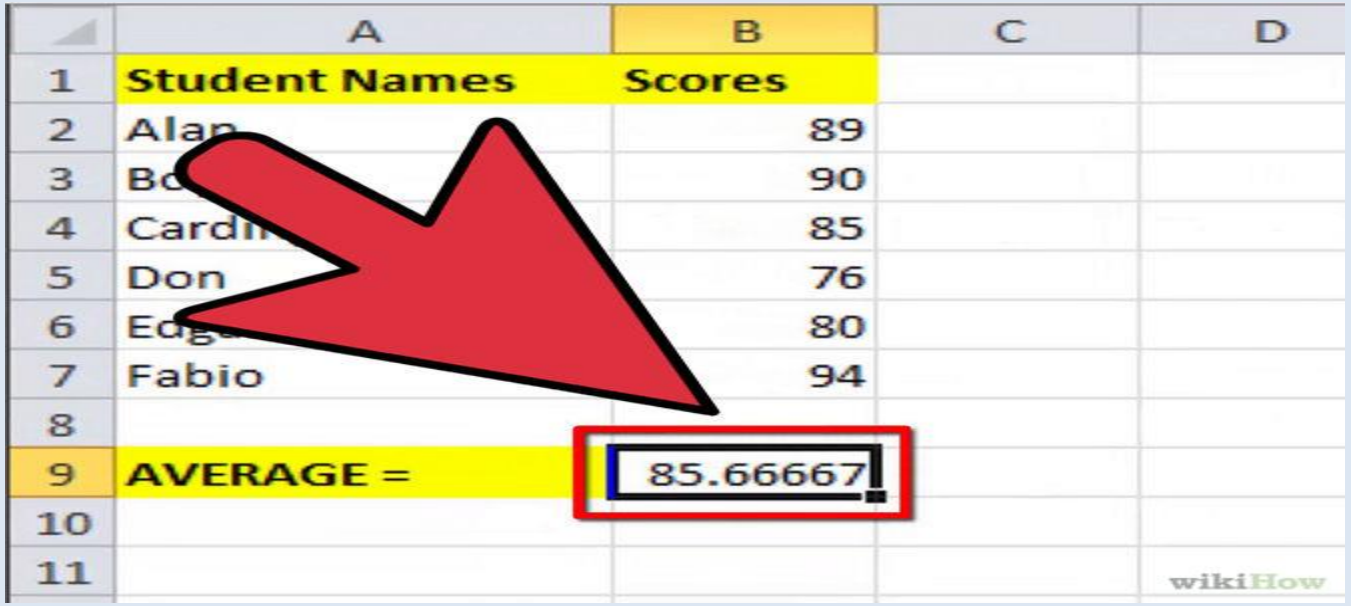
Select Data

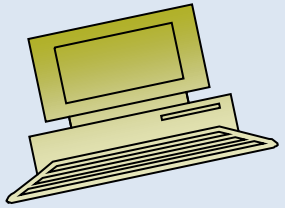
- Select the range of data to find the value of mean.
- Enter the cell range for your list of numbers in the number 1 box, for instance B2:B7and click "OK“.





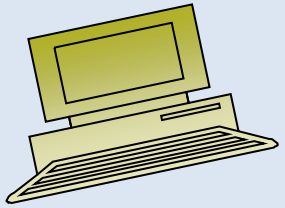
- **The mean (average) for the list will appear in the cell you selected.**





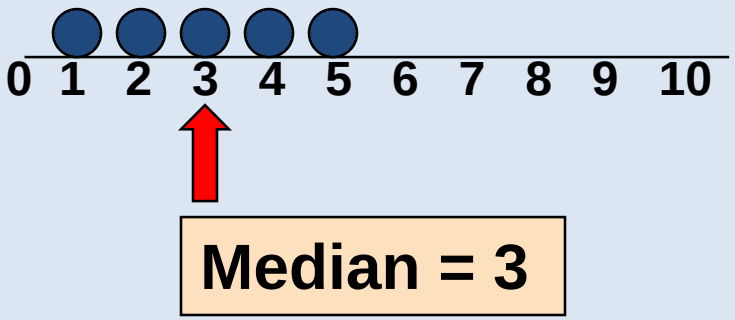
Median

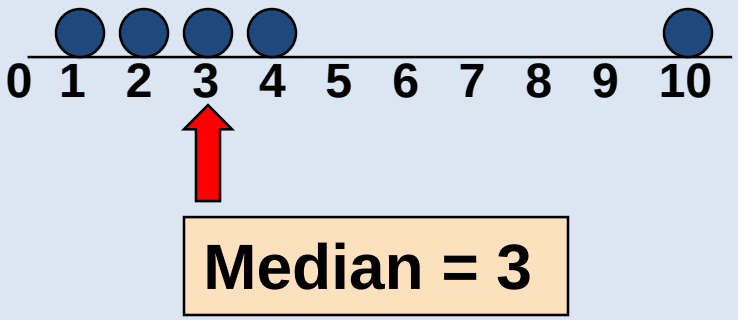
- To find the **median**, list the values of the data set in numerical order and identify which value appears in the middle of the list.



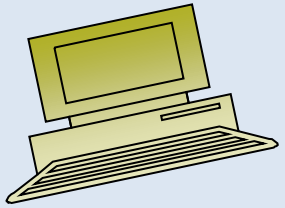
Median

- In an ordered array, the median is the “middle” number (50% above, 50% below)





- Not affected by extreme values



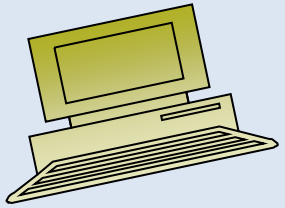
Finding the Median

- The location of the median:

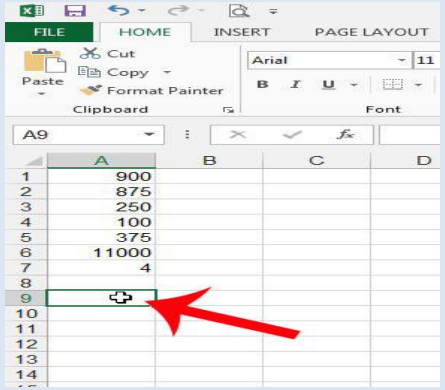
$$\text{Median position} = \frac{n+1}{2} \text{ position in the ordered data}$$

  - If the number of values is odd, the median is the middle number
  - If the number of values is even, the median is the average of the two middle numbers

- Note that $\frac{n+1}{2}$ is not the value of the median, only the position of the median in the ranked data



Median in Excel

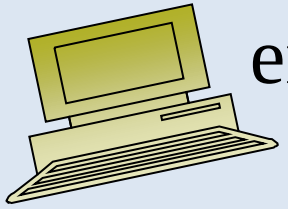


- **Step 1**

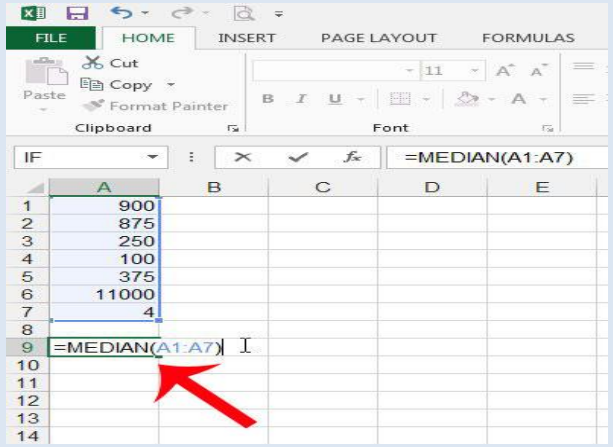
Open a new Microsoft Excel 2010 spreadsheet by double-clicking the Excel icon.

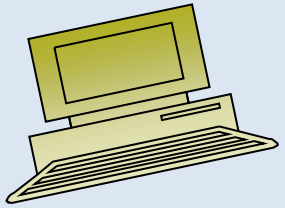
- **Step 2**

Click on cell A1 and enter the first number in the set of numbers that you are investigating. Press "Enter" and the program will automatically select cell A2 for you. Enter the second number into cell A2 and continue until you have entered the entire set of numbers into column A.



- Step 3: Type **=MEDIAN(AA:BB)** into the cell, where **AA** is the cell location of the first cell of your range, and **BB** is the cell location of the last cell. In my example image below, the formula would be**=MEDIAN(A1:A7)**. You can then press **Enter** on your keyboard to calculate the formula.

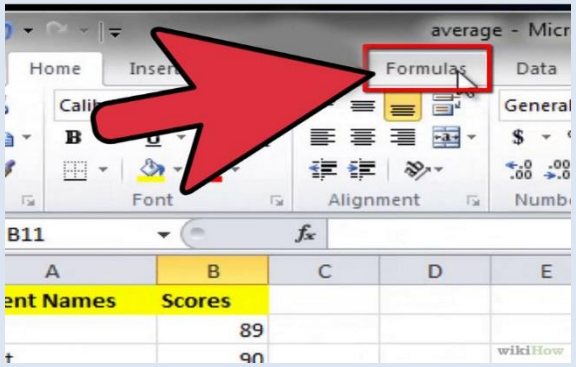


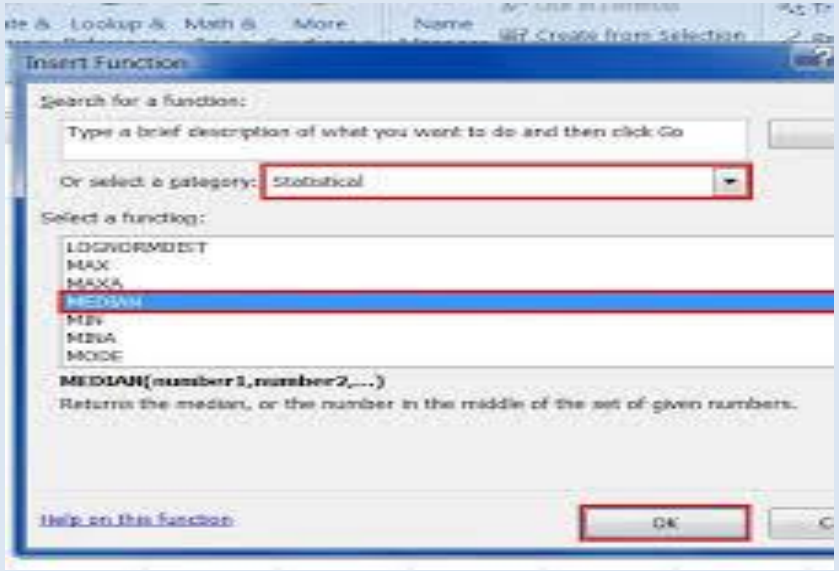


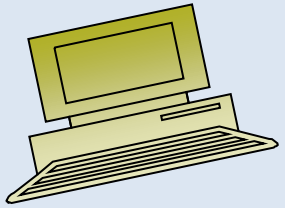
How to select function

Use the “Median" function in Excel to find the median of a set of numbers.

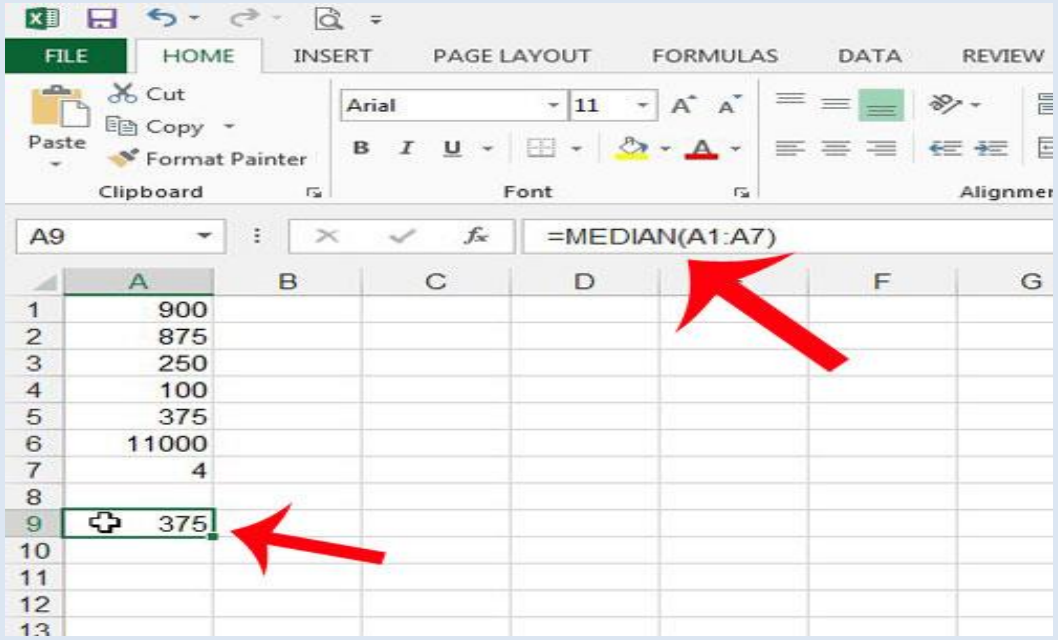
- Enter the range of numbers in your Excel spreadsheet.
- Click where you want the median.

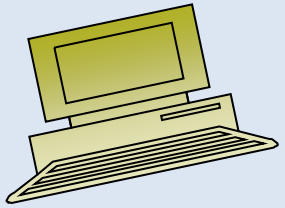






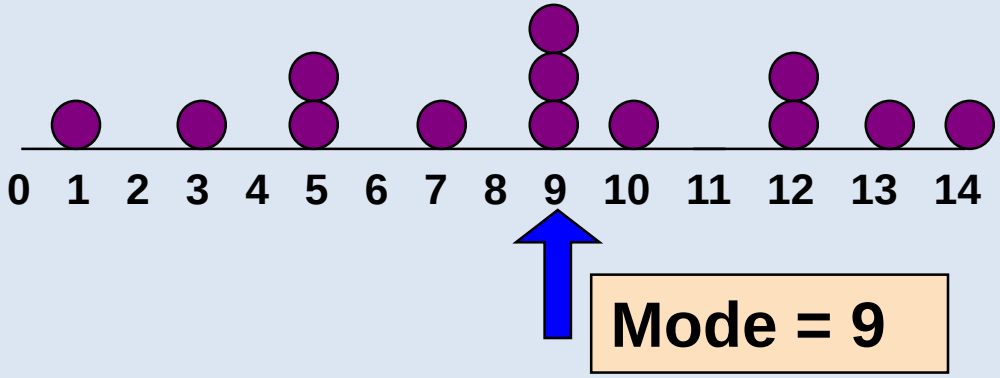
- Note that the median will be displayed in the cell, but that you can still view the formula by clicking on the cell, then looking at the **Formula Bar** above the spreadsheet.

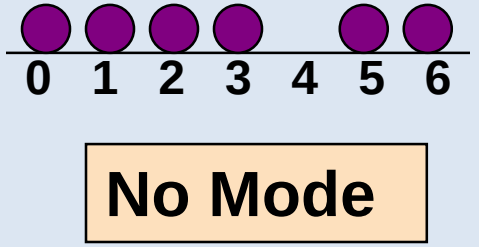


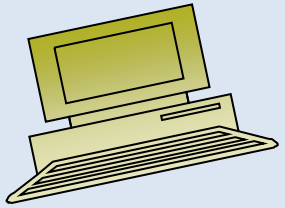


Mode

- A measure of central tendency
- Value that occurs most often
- Not affected by extreme values
- Used for either numerical or categorical data
- There may may be no mode
- There may be several modes

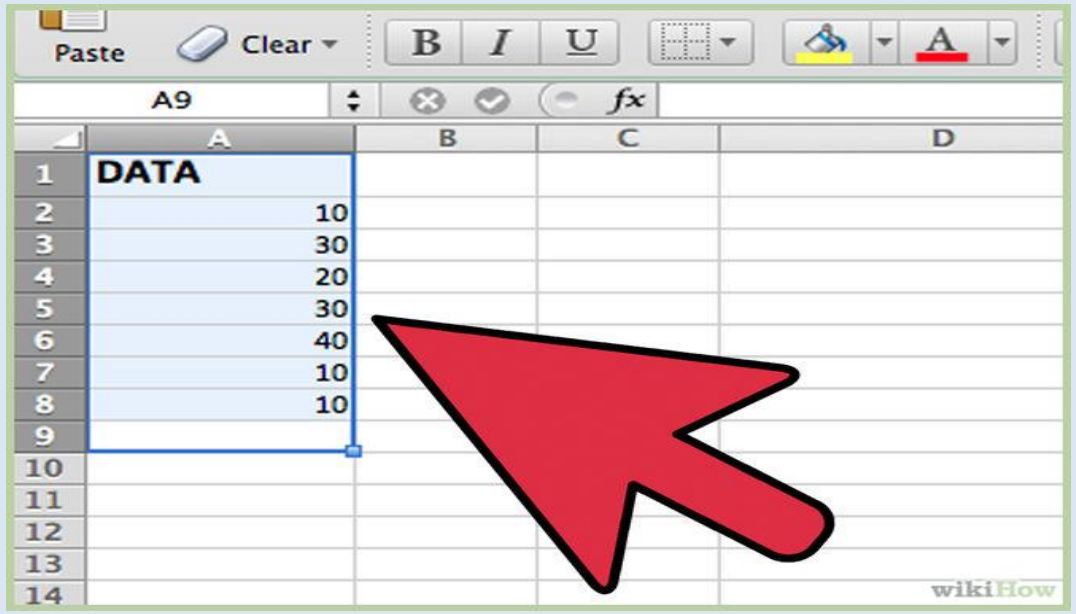


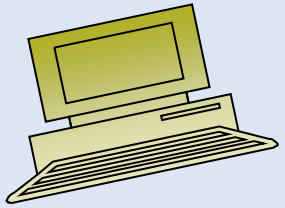




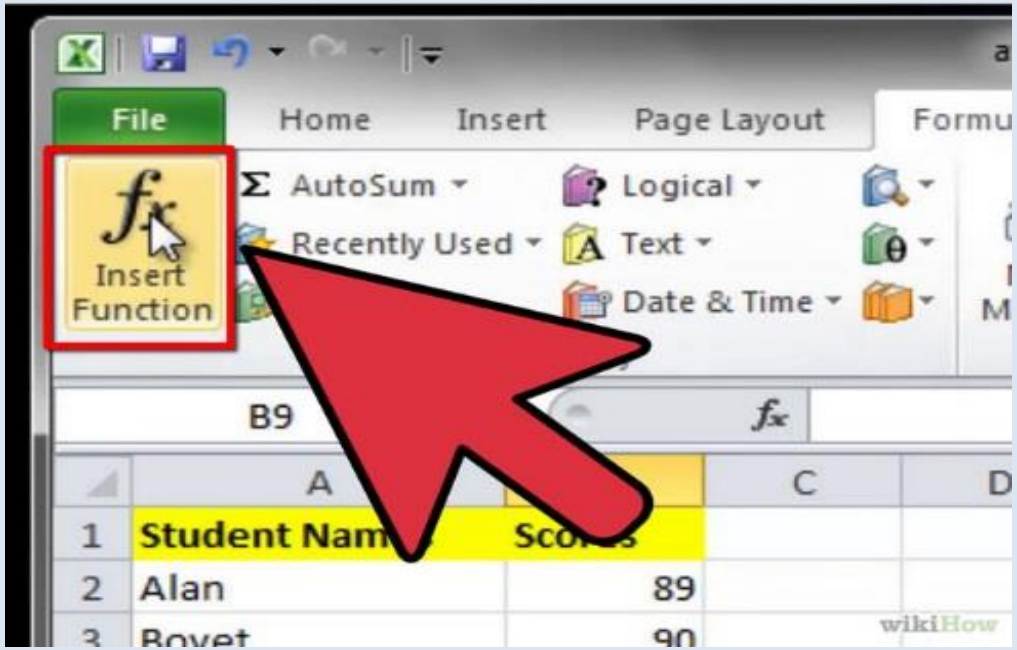
How to calculate mode using Excel.

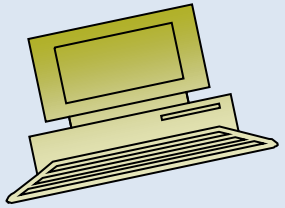
- **Enter each number in the data set into its own cell.** For consistency, it helps to enter the number in consecutive cells in either a row or column, and for readability, a column is better.





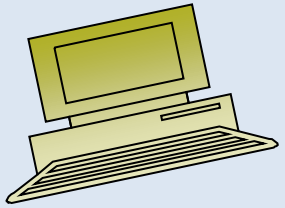
- Click "Formulas" and select the "Insert Function" tab.
- Enter the numbers in your Excel spreadsheet in either a row or a column.



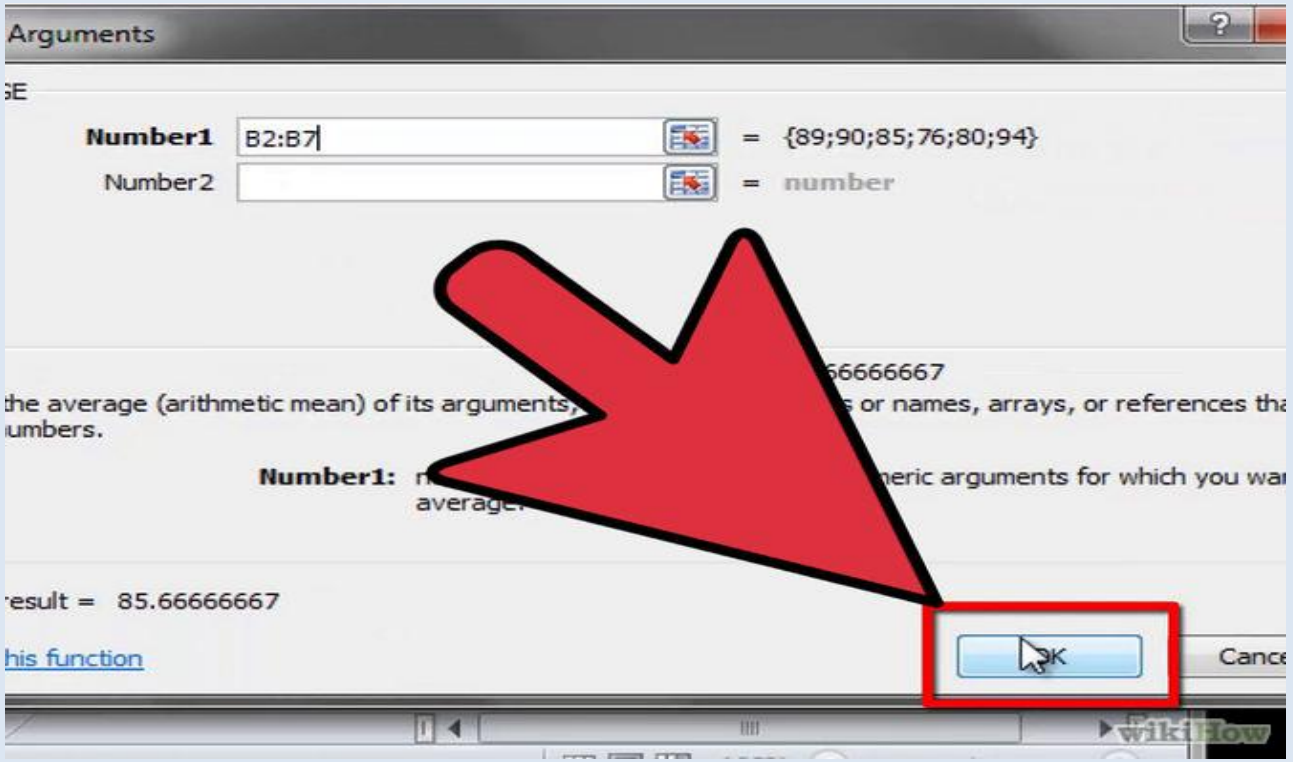


Mode

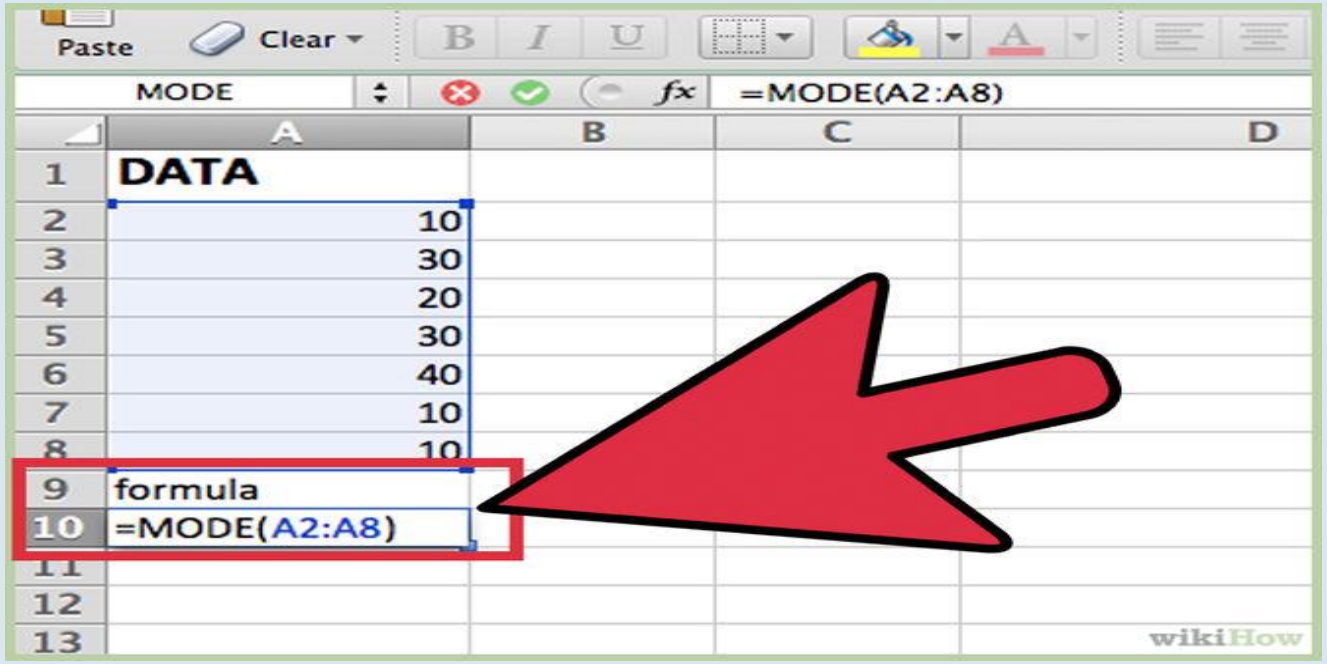
- **Enter the MODE function into the cell in which you wish to display the result.** The MODE function's format is "=MODE(Cx:Dy)," where C and D represent the letter of the column of the first and last cell in the range, and x and y represent the number of the first and last row in the range. (Although different letters are used in this example, you will use the same column letter for both the first and last cell if you entered the data in a column of cells or the same row number for both the first and last cell if you entered the data in a row of cells.)
- You can also specify each cell individually, up to 255 cells,
- as in "=MODE(A1, A2, A3)," but this is not advisable unless you have only a very small dataset and do not plan to add to it. You can also use the function with constants, for example, "=MODE(4,4,6)," but this requires editing the function each time you wish to search for a different mode.
- You may want to format the cell in which the mode will display with bolding or italics to distinguish it from the numbers in the dataset.

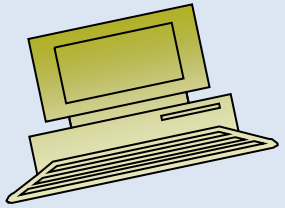


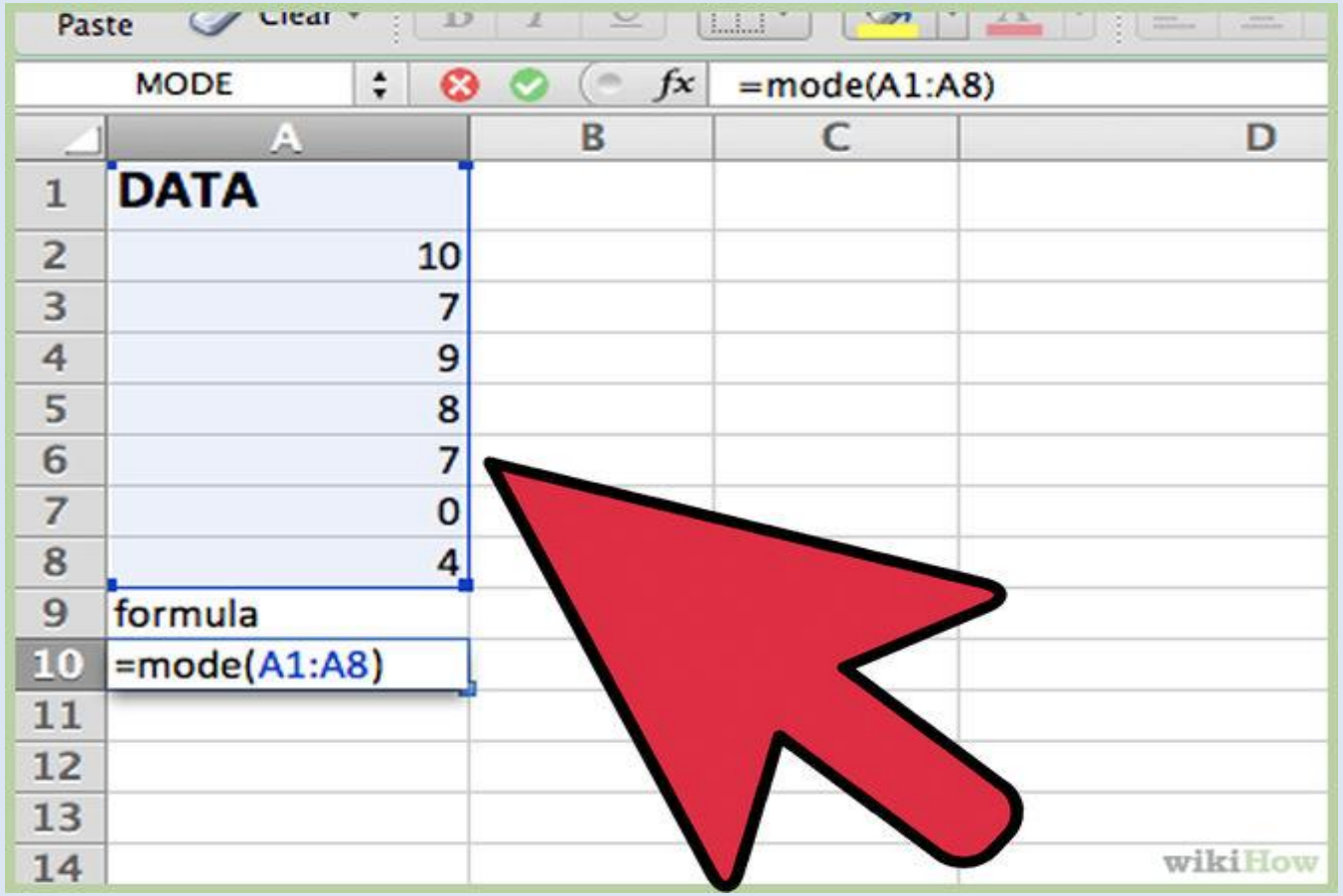
- **Enter the cell range for your list of numbers in the number 1 box, for instance A2:A8 and click "OK“.**



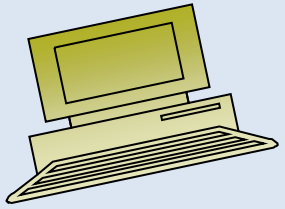
- **Enter the MODE function into the cell in which you wish to display the result.**





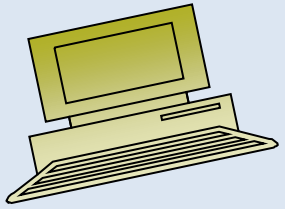

Paste
MODE
=mode(A1:A8)
A
B
C
D
1 DATA
2 10
3 7
4 9
5 8
6 7
7 0
8 4
9 formula
10 =mode(A1:A8)
11
12
13
14
wikiHow

- **Calculate and display the result.**
- For a dataset of 10, 7, 9, 8, 7, 0 and 4 entered in cells 1 through 8 of Column A, the function =MODE(A1:A8) will deliver a result of 7, because 7 appears more often in the data than any other number.



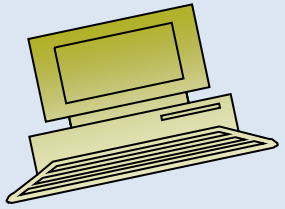
More than one mode

- If the data set contains more than one number that qualifies as the mode (such as 7 and 9 each appearing twice and every other number appearing only once), whichever mode number is listed first in the data set will be the result. If none of the numbers in the data set appear more often than any other, the MODE function will display the error result #N/A.
- The MODE function is available in all versions of Excel, including Excel 2010, which includes it for compatibility with spreadsheets created in earlier versions.
- Excel 2010 uses the MODE.SNGL function, which except for syntax (=MODE.SNGL(Cx:Dy)) works essentially the same as the MODE function in earlier versions of Excel.

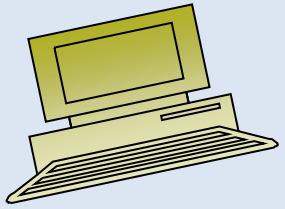


- Mean is generally used, unless extreme values (outliers) exist

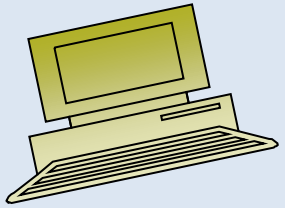
- Then median is often used, since the median is not sensitive to extreme values.



Practical Exercise in Data Preparation



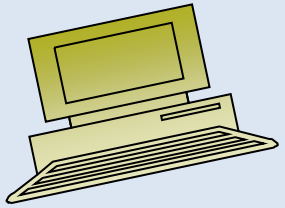
Lecture 3



Contents

Measure of Variation

- Range,
- Standard deviation,
- Variance,
- Inter Quartile Range,
- Coefficient of Variation,
- Box Plot ,
- 5-number summary,
- Real Statistics Add-in ,
- Practical exercise.

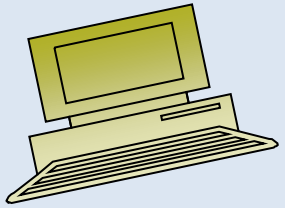


Objectives

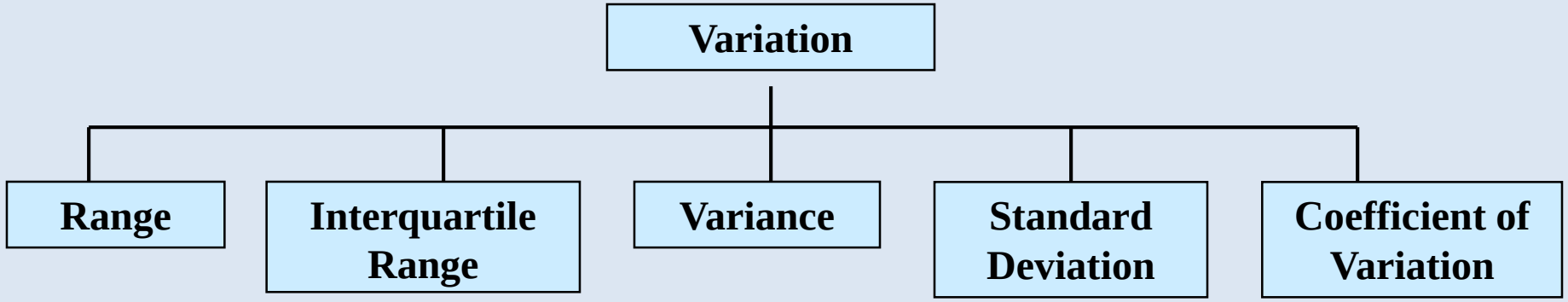
After completing this lecture, you should be able to:

Compute and interpret :

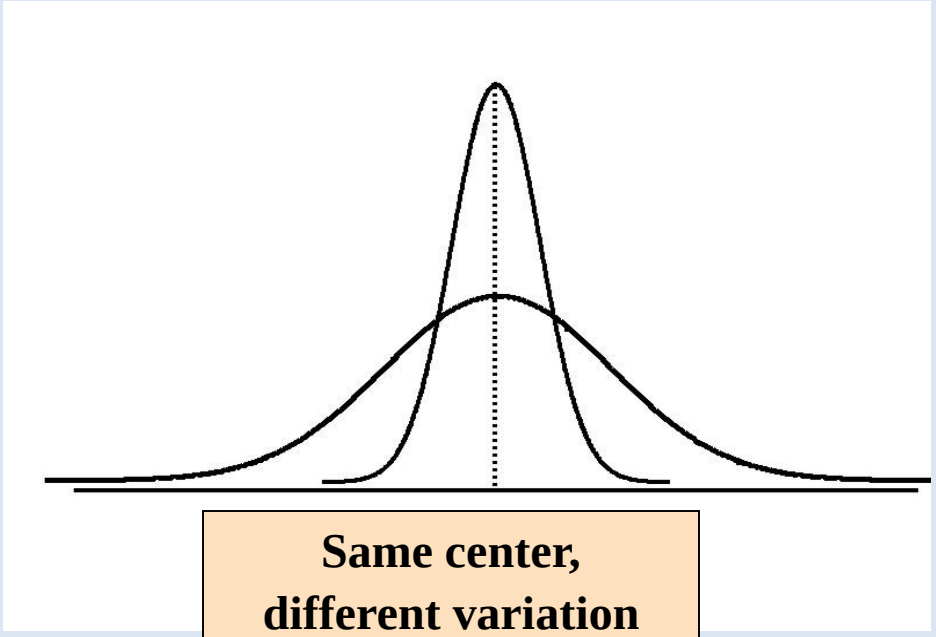
- Range,
- Standard deviation,
- Variance,
- Inter Quartile Range,
- Coefficient of Variation,
- Box Plot.

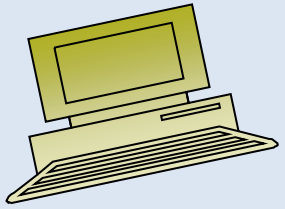


Measures of Variation:



- Measures of variation give information on the **spread** or **variability** of the data values.



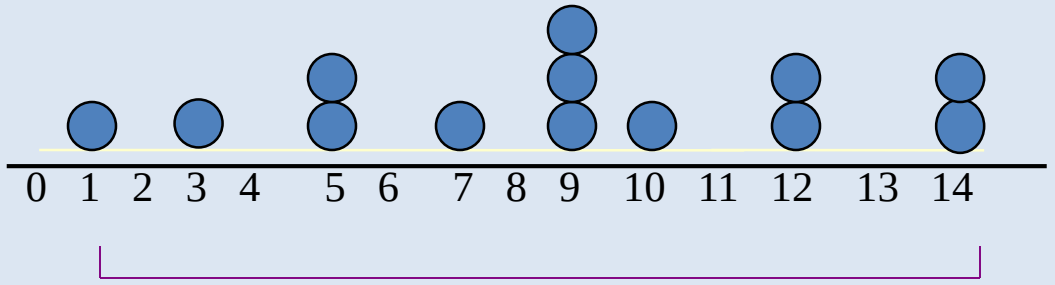


Range:

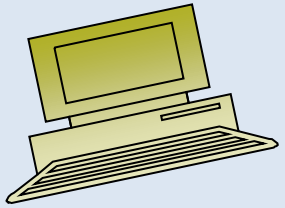
- Simplest measure of variation
- Difference between the largest and the smallest observations:

$$\text{Range} = X_{\text{largest}} - X_{\text{smallest}}$$

Example:

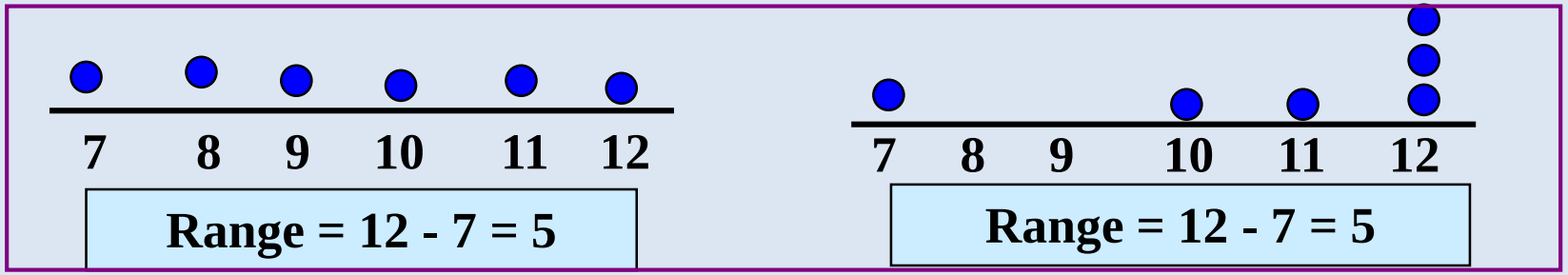


Range = 14 - 1 = 13



Disadvantages of the Range

- Ignores the way in which data are distributed



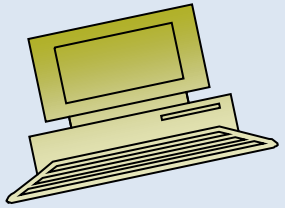
- Sensitive to outliers

1,1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,3,3,3,3,4,5

Range = 5 - 1 = 4

1,1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,3,3,3,3,4,120

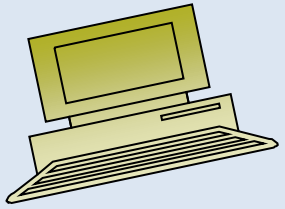
Range = 120 - 1 = 119



Interquartile Range

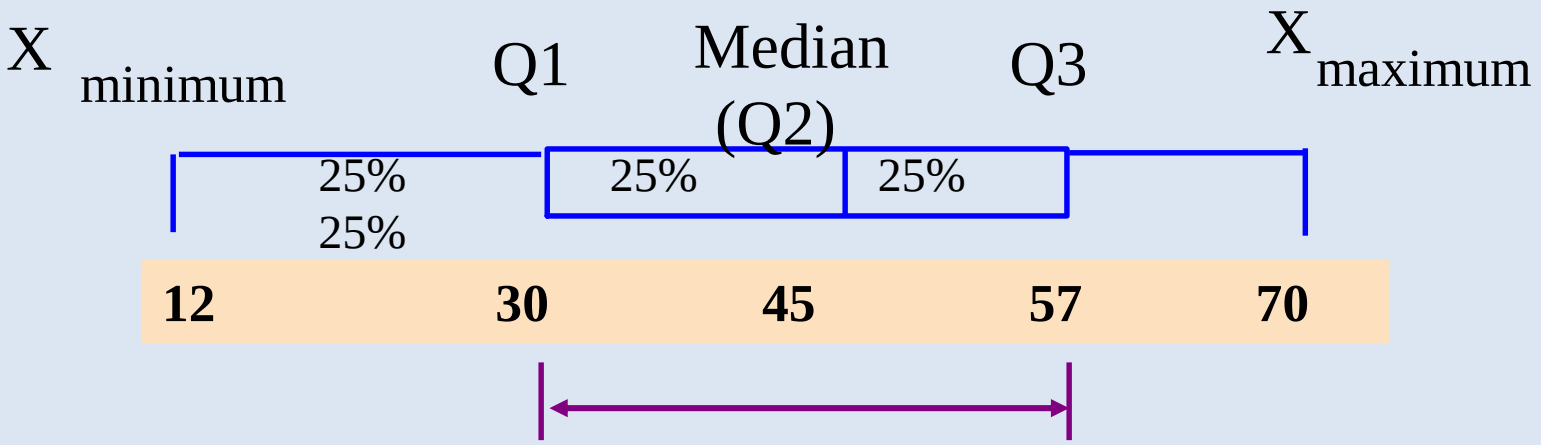
- Can eliminate some outlier problems by using the **interquartile range**
- Eliminate some high- and low-valued observations and calculate the range from the remaining values
- Interquartile range = 3^{rd} quartile – 1^{st} quartile

  $= Q_3 - Q_1$

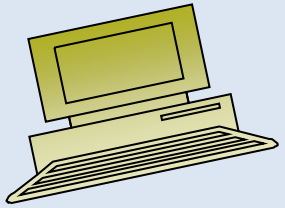


Interquartile Range

Example:



Interquartile range
= 57 – 30 = 27



Variance

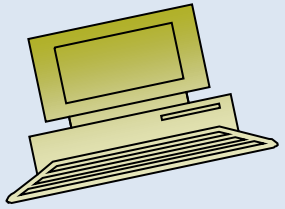
- Average (approximately) of squared deviations of values from the mean
  - Sample variance:

$$s^2 = \frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}$$

Where $\overline{X}$ = arithmetic mean

n = sample size

X_i = i^{th} value of the variable X

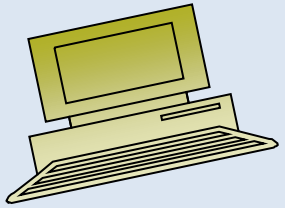


Standard Deviation

- Most commonly used measure of variation
- Shows variation about the mean
- Has the same units as the original data

– Sample standard deviation:

$$s = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}}$$



Calculation Example: Sample Standard Deviation

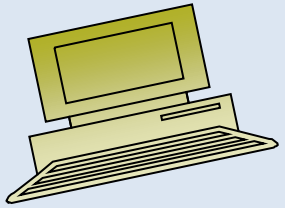
Sample Data (X_i) : **10 12 14 15 17 18 18 24**

n = 8 Mean = $\overline{X}$ = 16

$$S = \sqrt{\frac{(10-\overline{X})^2 + (12-\overline{X})^2 + (14-\overline{X})^2 + \cdots + (24-\overline{X})^2}{n-1}}$$

$$= \sqrt{\frac{(10-16)^2 + (12-16)^2 + (14-16)^2 + \cdots + (24-16)^2}{8-1}}$$

$$= \sqrt{\frac{130}{7}} = 4.309$$



Population Variance

- Average of squared deviations of values from the mean

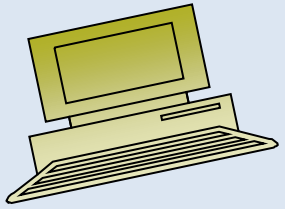
  – Population variance:

$$\sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}$$

Where μ = population mean

N = population size

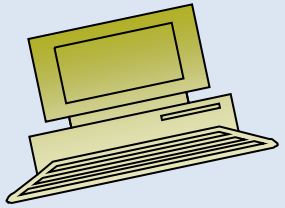
X_i = i^{th} value of the variable X



Population Standard Deviation

- Most commonly used measure of variation
- Shows variation about the mean
- Has the same units as the original data

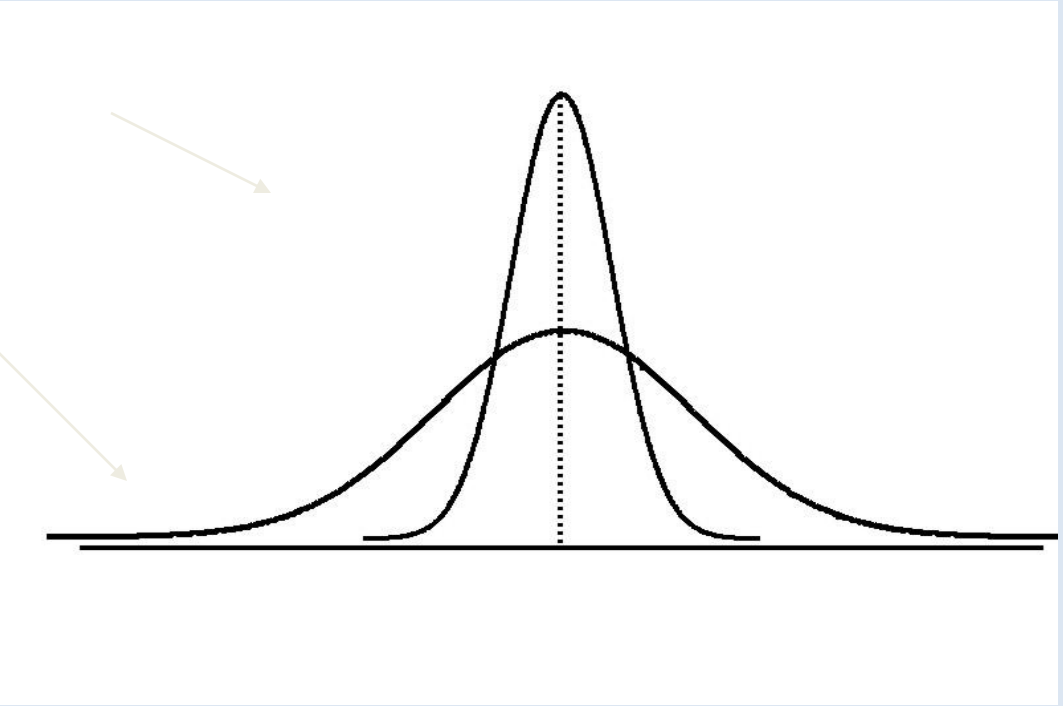
$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}}$$

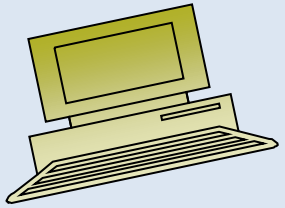


Measuring variation

Small standard deviation

Large standard deviation





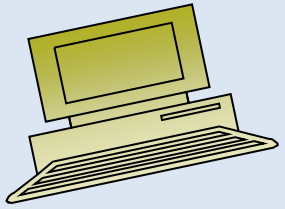
Enter Data:

- **Step 1**

Open a new Microsoft Excel 2010 spreadsheet by double-clicking the Excel icon.

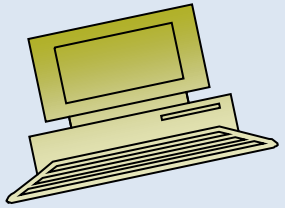
- **Step 2**

Click on cell B1 and enter the first number in the set of numbers that you are investigating. Press "Enter" and the program will automatically select cell B2 for you. Enter the second number into cell B2 and continue until you have entered the entire set of numbers into column B.

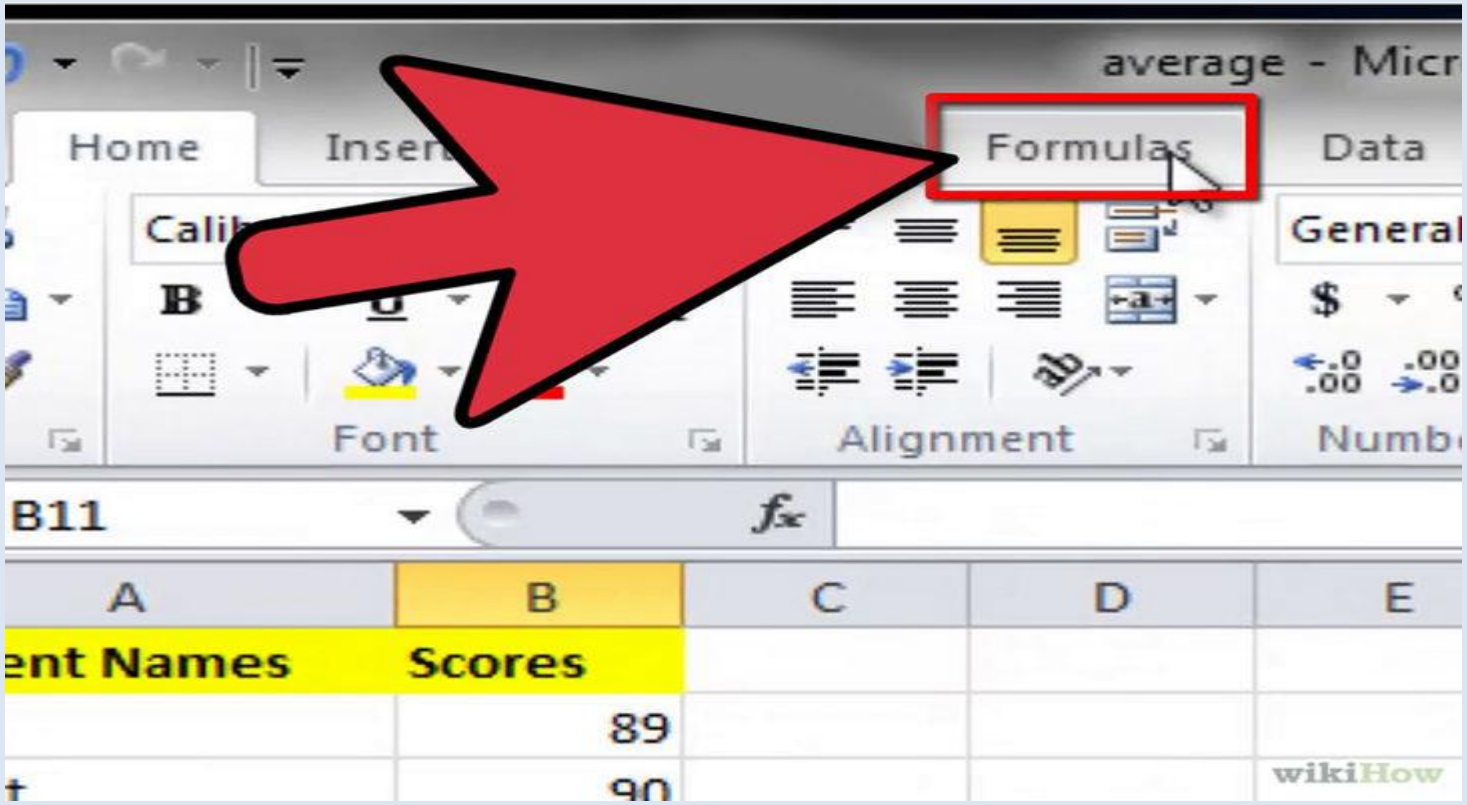


Advantages of Variance and Standard Deviation

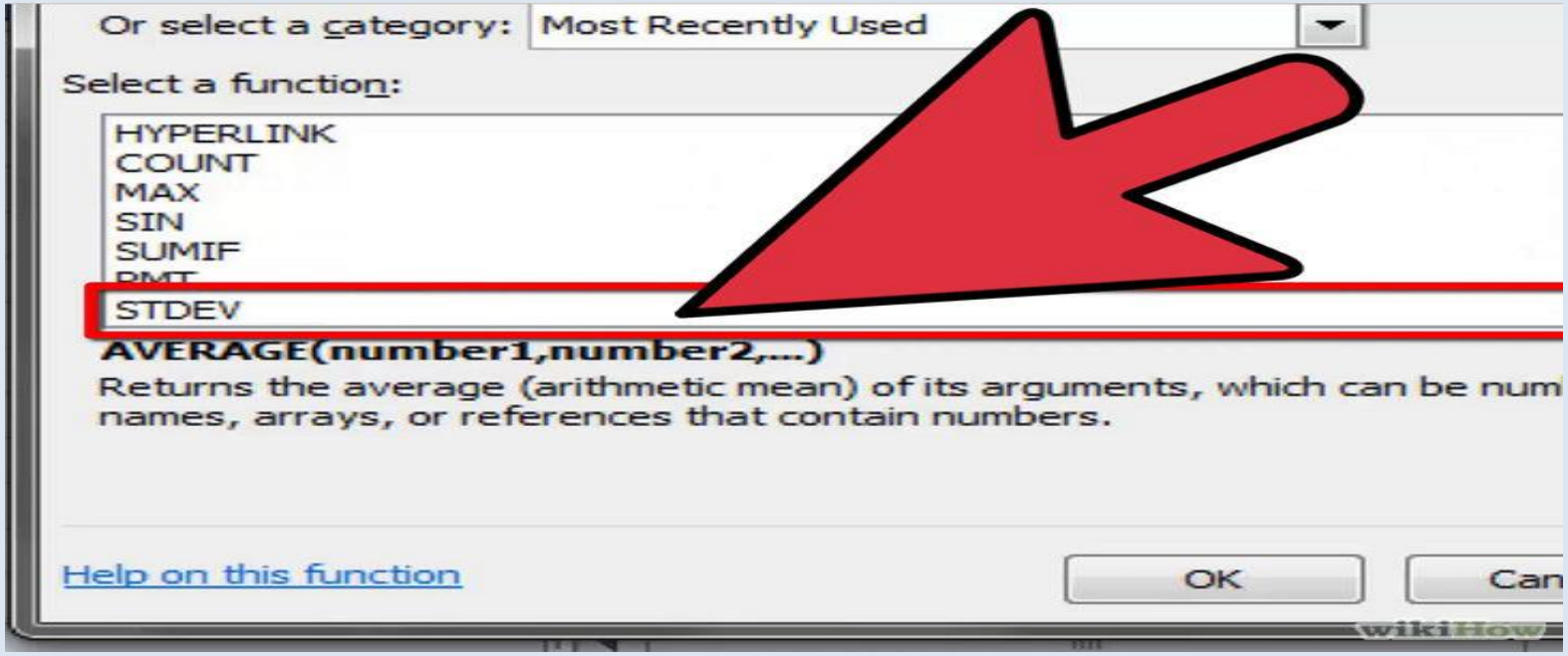
- Each value in the data set is used in the calculation
- Values far from the mean are given extra weight
  (because deviations from the mean are squared)



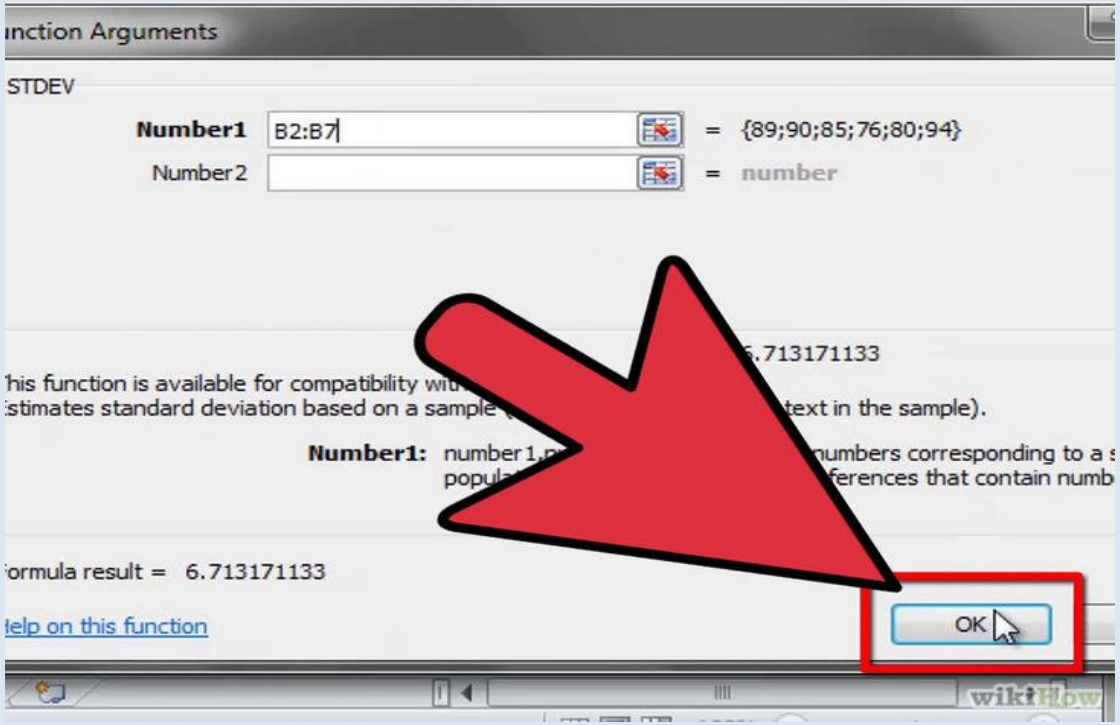
Click on "Formulas" and select the "Insert Function" (*fx*) tab again.

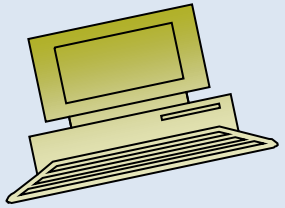


Scroll down the dialog box and select the STDEV function

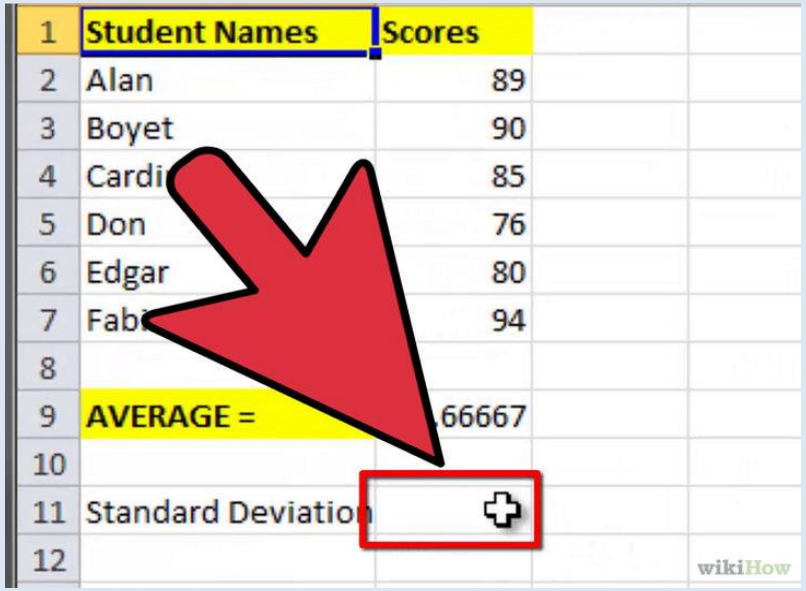


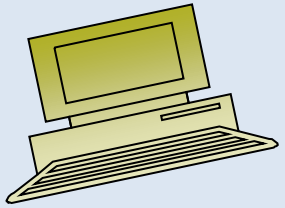
Enter the cell range for your list of numbers in the number 1 box and click OK



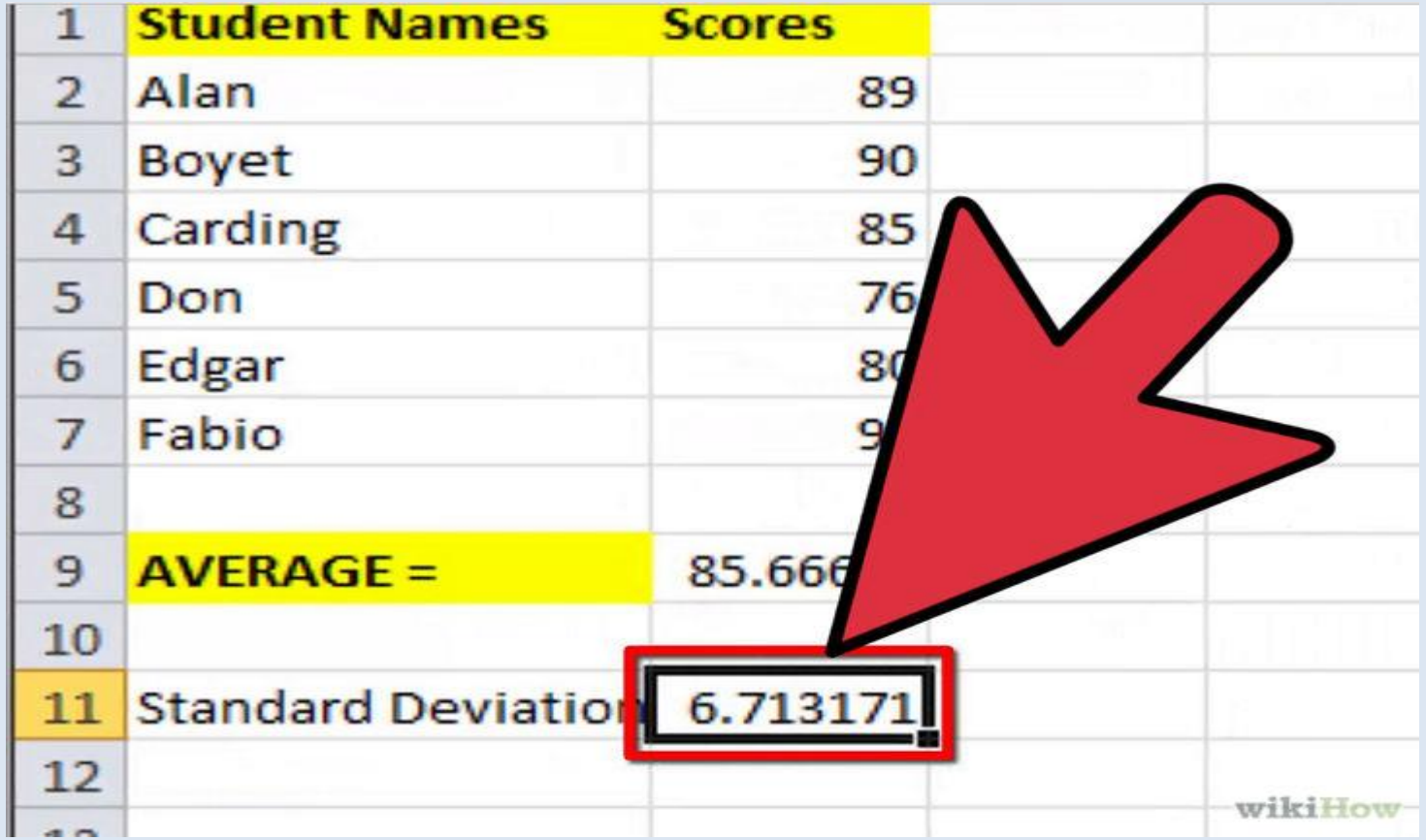


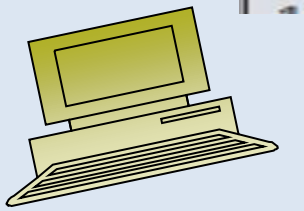
Use the STDEV function to compute the standard deviation. Place your cursor where you wish to have it appear.



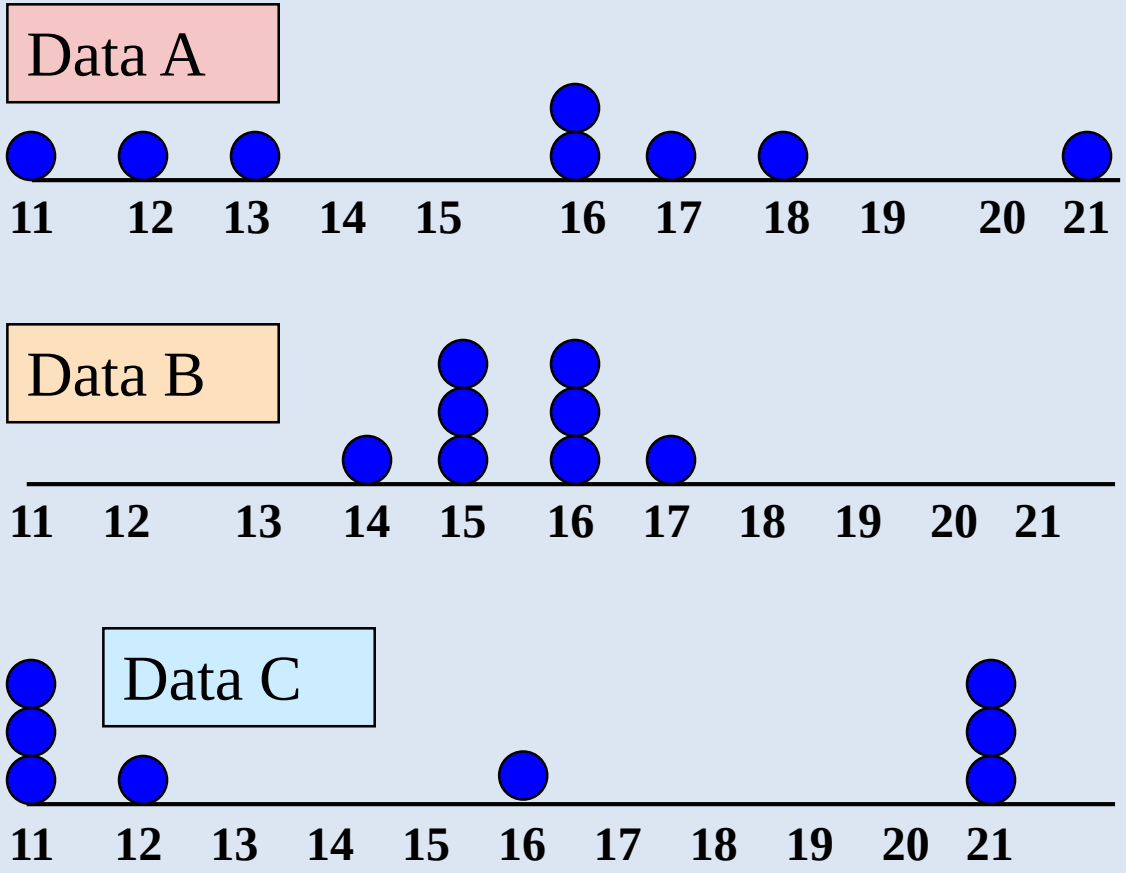


The standard deviation will appear in the cell you selected.





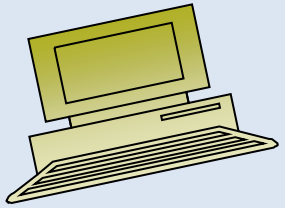
Comparing Standard Deviations:



Mean = 15.5
S = 3.34

Mean = 15.5
S = 0.92

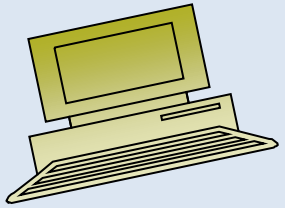
Mean = 15.5
S = 4.84



Coefficient of Variation:

- Measures relative variation
- Always in percentage (%)
- Shows variation relative to mean
- Can be used to compare two or more sets of data measured in different units

$$CV = \left(\frac{S}{\bar{X}} \right) \cdot 100\%$$



Comparing Coefficient of Variation

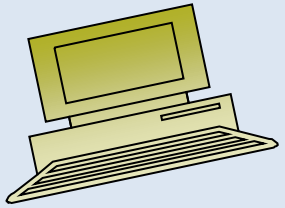
- Stock A:
  - Average price last year = $50
  - Standard deviation = $5

$$CV_A = \left(\frac{S}{\overline{X}} \right) \cdot 100\% = \frac{\$5}{\$50} \cdot 100\% = 10\%$$

- Stock B:
  - Average price last year = $100
  - Standard deviation = $5

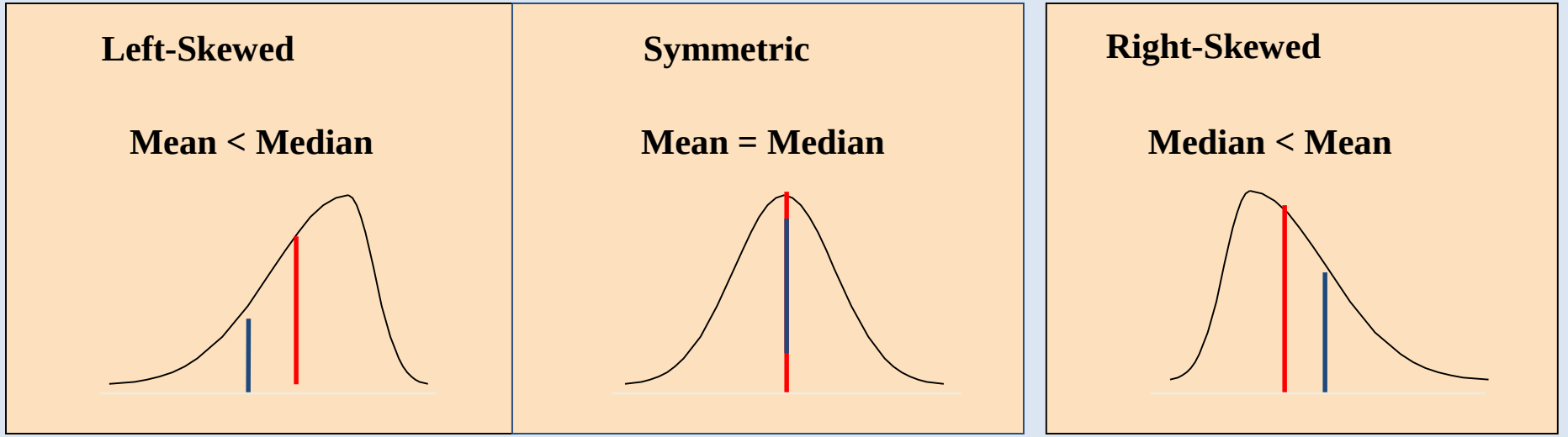
$$CV_B = \left(\frac{S}{\overline{X}} \right) \cdot 100\% = \frac{\$5}{\$100} \cdot 100\% = 5\%$$

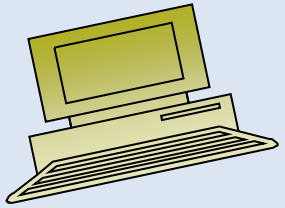
Both stocks have the same standard deviation, but stock B is less variable relative to its price



Shape of a Distribution:

- Describes how data is distributed
- Measures of shape
  - Symmetric or skewed

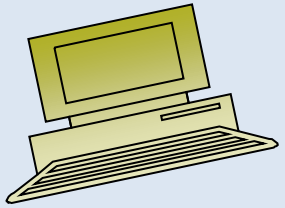




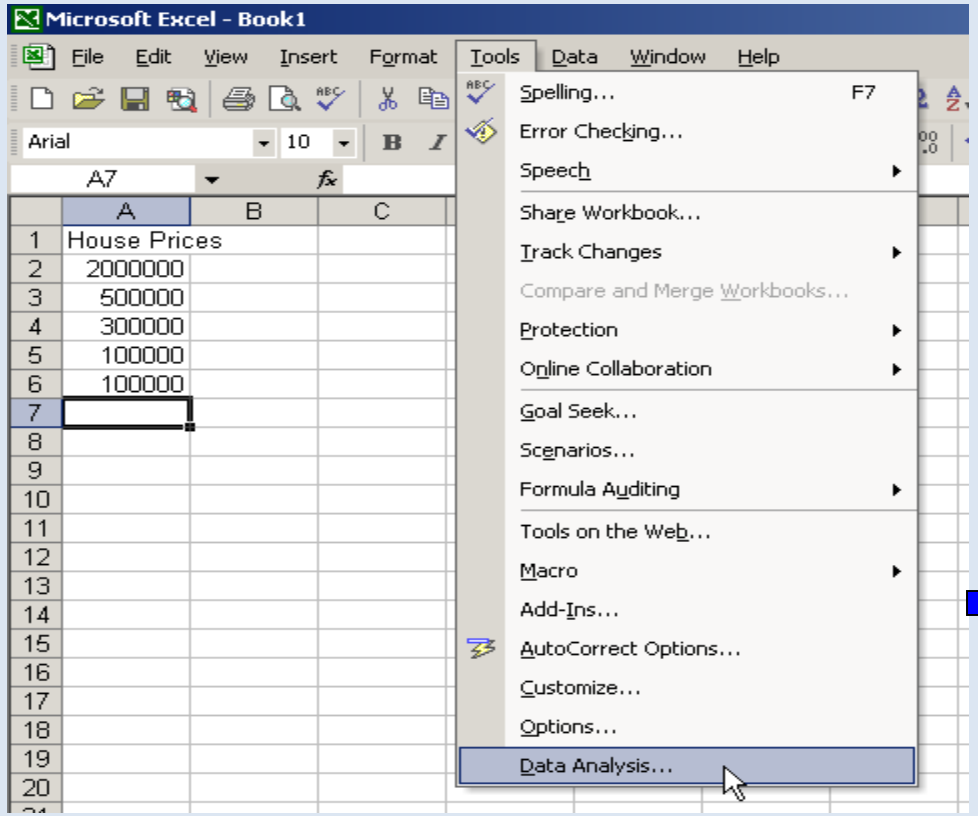
Using Microsoft Excel:

- Descriptive Statistics can be obtained from Microsoft® Excel
  - Use menu choice:

    tools / data analysis / descriptive statistics
  - Enter details in dialog box



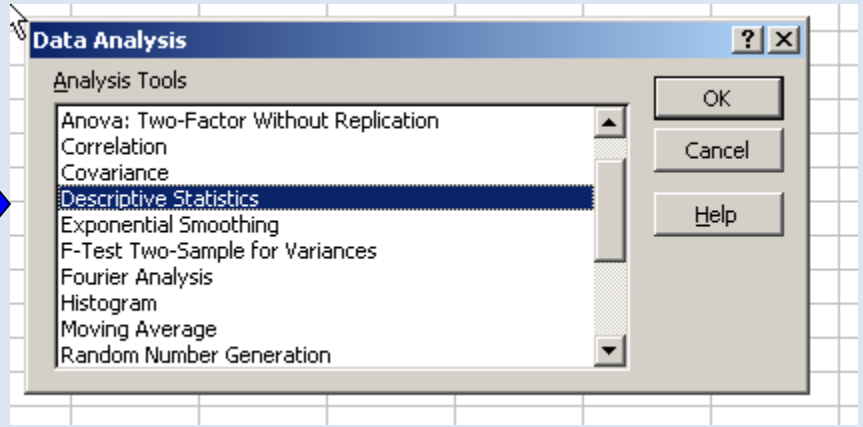
Using Excel:

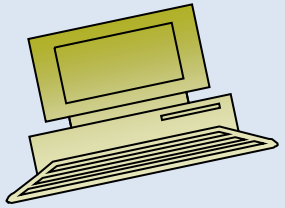


- Use menu choice:

tools / data analysis /

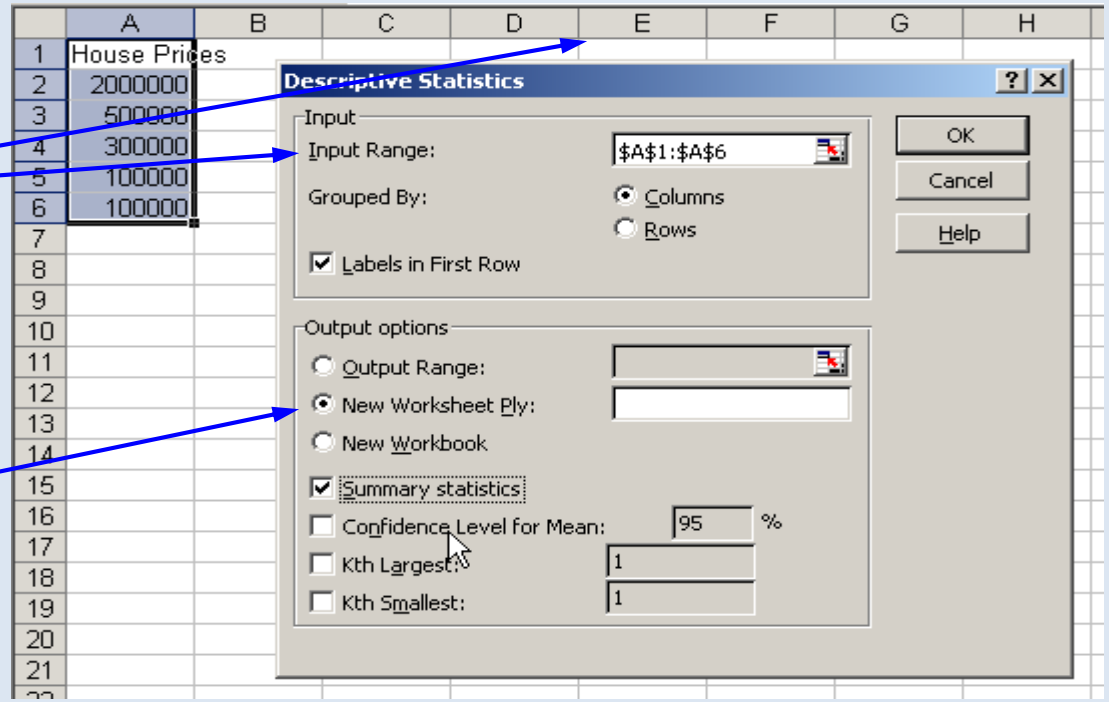
descriptive statistics

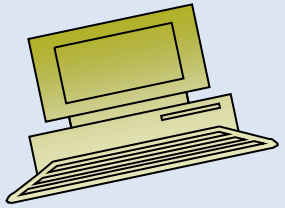




Using Excel:

- Enter dialog box details
- Check box for summary statistics
- Click OK



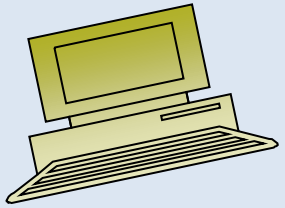


Excel output

Microsoft Excel descriptive statistics output, using the house price data:

House Prices:

$2,000,000
500,000
300,000
100,000
100,000



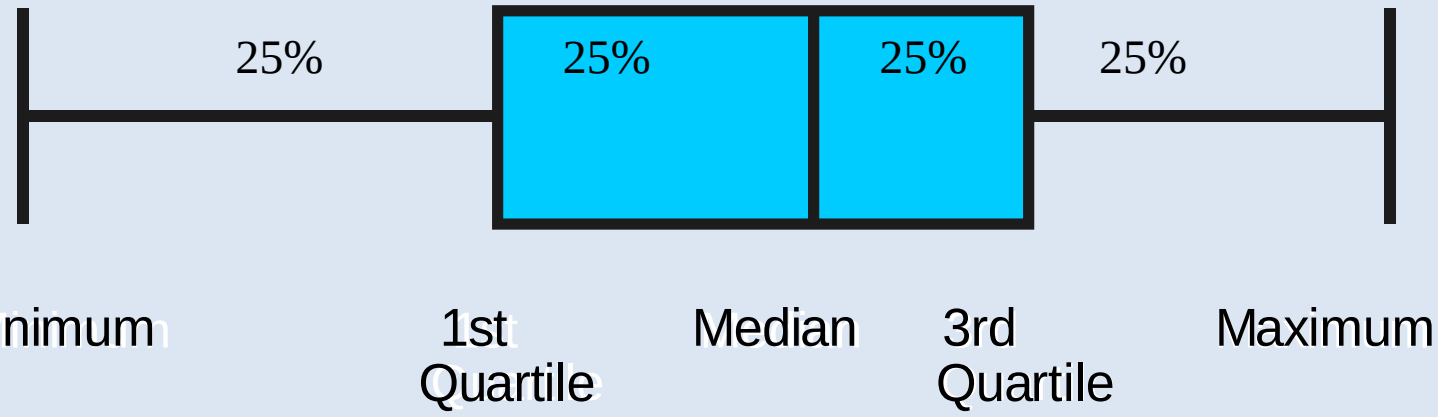
	A	B
1	*House Prices*	
2		
3	Mean	600000
4	Standard Error	357770.8764
5	Median	300000
6	Mode	100000
7	Standard Deviation	800000
8	Sample Variance	6.4E+11
9	Kurtosis	4.130126953
10	Skewness	2.006835938
11	Range	1900000
12	Minimum	100000
13	Maximum	2000000
14	Sum	3000000
15	Count	5
16		

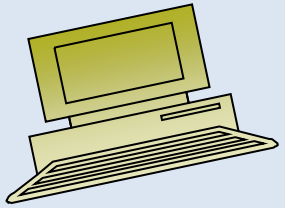
Exploratory Data Analysis:

- Box-and-Whisker Plot: A Graphical display of data using 5-number summary:

Minimum -- Q1 -- Median -- Q3 -- Maximum

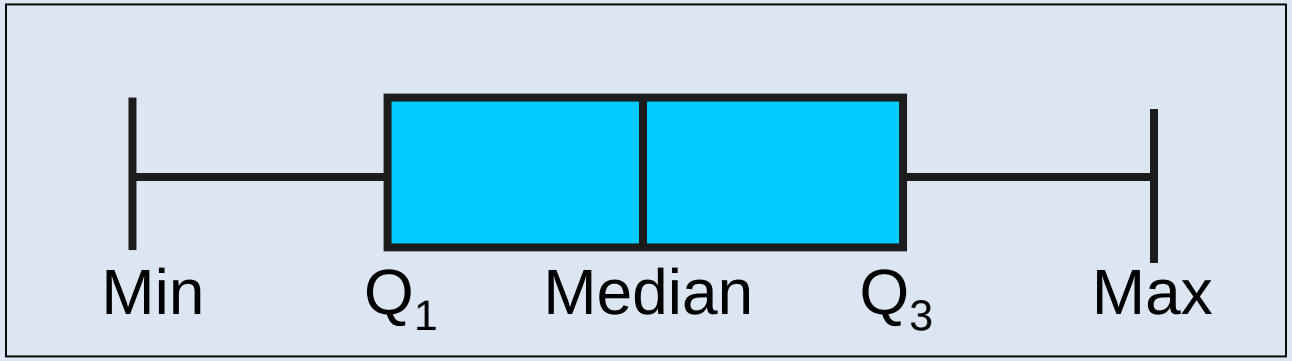
Example:



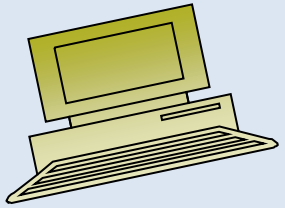


Shape of Box-and-Whisker Plots

- The Box and central line are centered between the endpoints if data are symmetric around the median

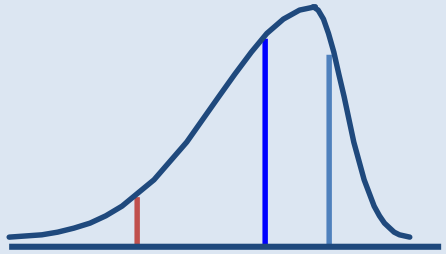


- A Box-and-Whisker plot can be shown in either vertical or horizontal format



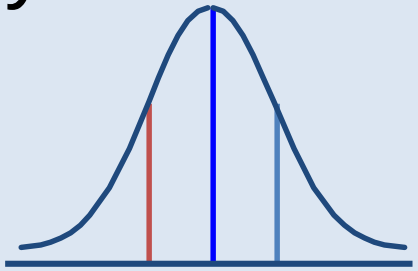
Distribution Shape and Box-and-Whisker Plot

Left-Skewed



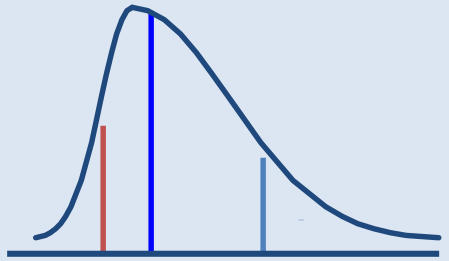
Q1 Q2 Q3

Symmetric

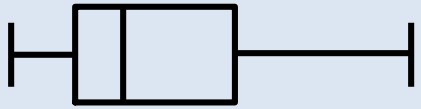


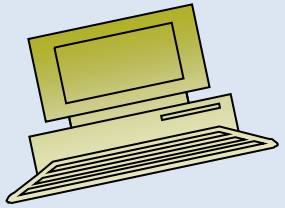
Q1 Q2 Q3

Right-Skewed

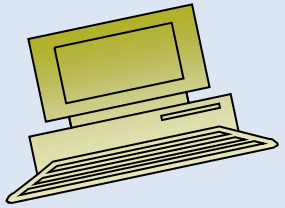


Q1 Q2 Q3

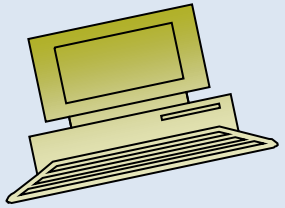




Practical Exercise



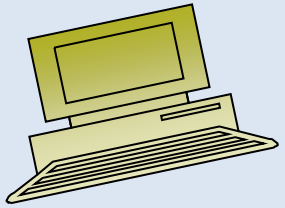
Lecture 4



Objectives

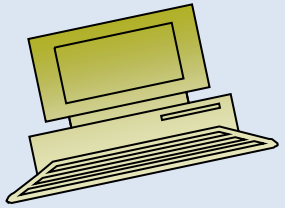
After completing this chapter, you should be able to:

- Construct and interpret a frequency distribution
- Construct a histogram
- Create and interpret bar charts, pie charts,
- Present and interpret categorical data in bar charts and pie charts



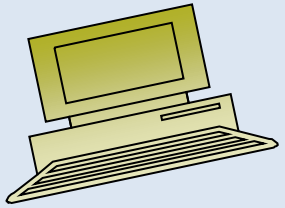
Topics to be covered

- Frequency distribution
- Histogram
- Bar charts
- Pie charts



Organizing and Presenting Data Graphically

- Data in raw form are usually not easy to use for decision making
  - Some type of organization is needed
    - Table
    - Graph
- Techniques reviewed here:
  - Frequency Distributions and Histograms
  - Bar charts and pie charts

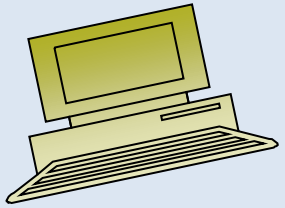


Class Intervals and Class Boundaries

- Each class grouping has the same width
- Determine the width of each interval by

$$\text{Width of interval} \cong \frac{\text{range}}{\text{number of desired class groupings}}$$

- Use at least 5 but no more than 15 groupings
- Class boundaries never overlap
- Round up the interval width to get desirable endpoints

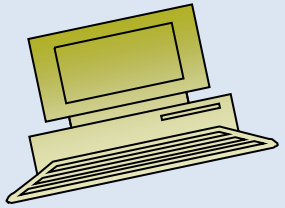


Frequency Distribution Example

Example: A manufacturer of insulation randomly selects 20 winter days and records the daily high temperature

24, 35, 17, 21, 24, 37, 26, 46, 58, 30,

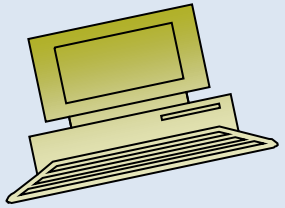
32, 13, 12, 38, 41, 43, 44, 27, 53, 27



Frequency Distribution Example

(continued)

- Sort raw data in ascending order:
- **12, 13, 17, 21, 24, 24, 26, 27, 27, 30, 32, 35, 37, 38, 41, 43, 44, 46, 53, 58**
- Find range: **58 - 12 = 46**
- Select number of classes: **5 (usually between 5 and 15)**
- Compute class interval (width): **10 (46/5 then round up)**
- Determine class boundaries (limits): **10, 20, 30, 40, 50, 60**
- Compute class midpoints: **15, 25, 35, 45, 55**
- Count observations & assign to classes

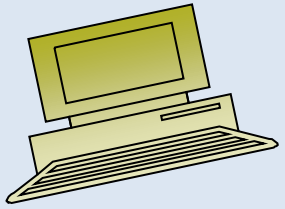


Frequency Distribution Example *(continued)*

Data in ordered array:

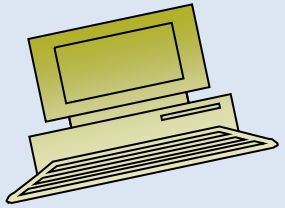
12, 13, 17, 21, 24, 24, 26, 27, 27, 30, 32, 35, 37, 38, 41, 43, 44, 46, 53, 58

Class	Frequency	Relative Frequency	Percentage
10 but less than 20	3	.15	15
20 but less than 30	6	.30	30
30 but less than 40	5	.25	25
40 but less than 50	4	.20	20
50 but less than 60	2	.10	10
Total	20	1.00	100



Graphing Numerical Data: The Histogram

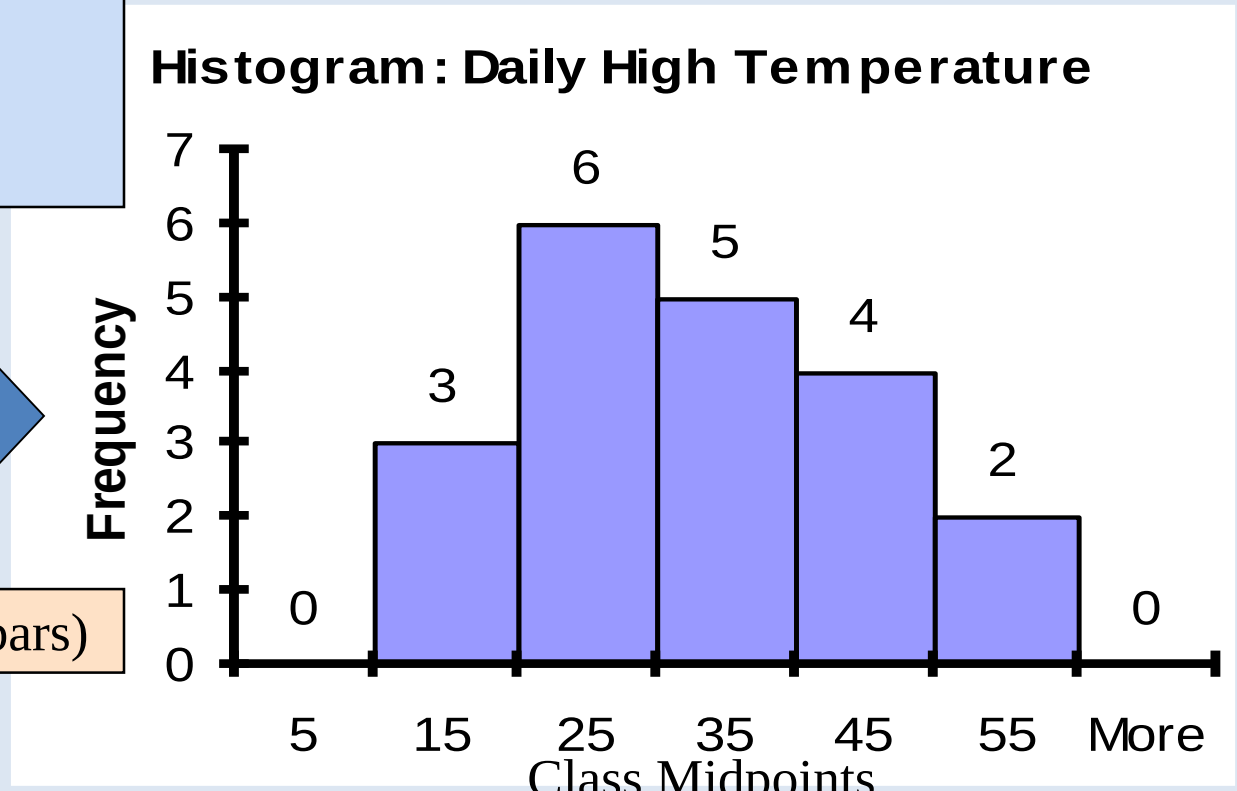
- A graph of the data in a frequency distribution is called a **histogram**
- The **class boundaries** (or **class midpoints**) are shown on the horizontal axis
- The vertical axis is either **frequency, relative frequency,** or **percentage**
- Bars of the appropriate heights are used to represent the number of observations within each class

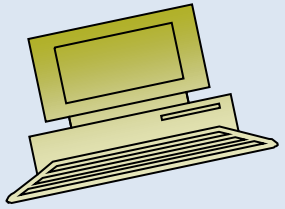


Histogram Example

Class	Class Midpoint	Frequency
10 but less than 20	15	3
20 but less than 30	25	6
30 but less than 40	35	5
40 but less than 50	45	4
50 but less than 60	55	2

(No gaps between bars)

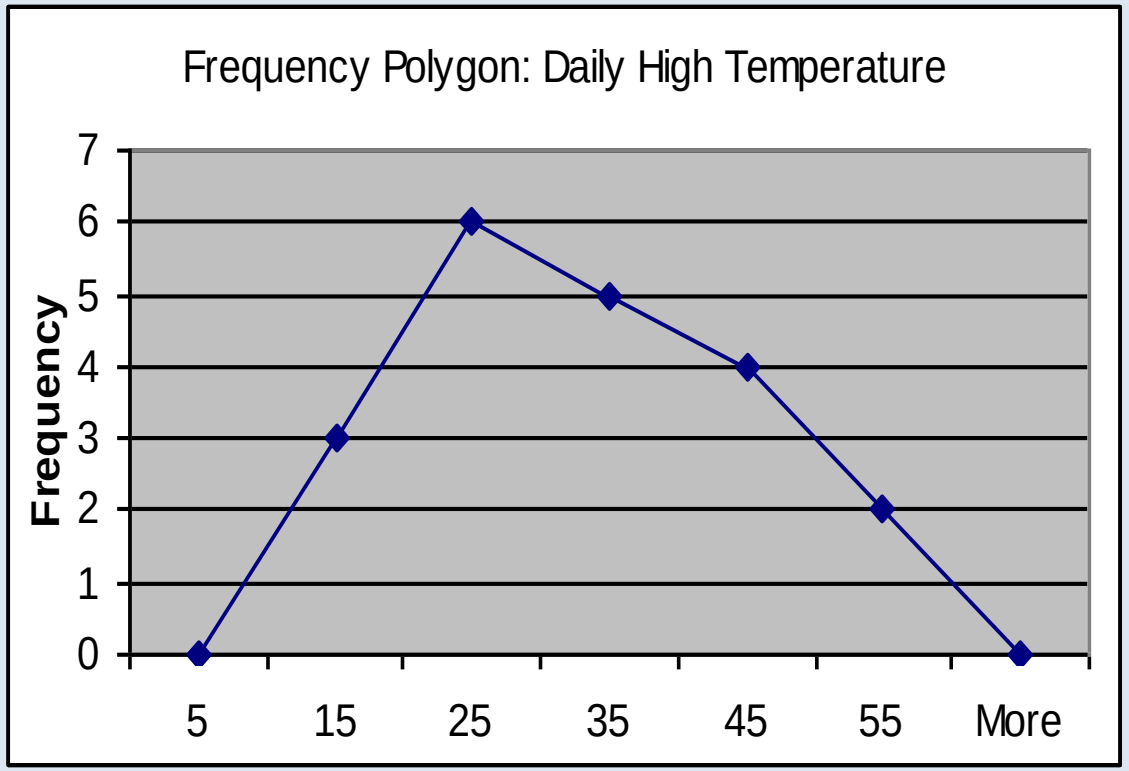


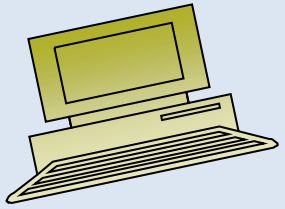


Graphing Numerical Data: The Frequency Polygon

Class	Class Midpoint	Frequency
10 but less than 20	15	3
20 but less than 30	25	6
30 but less than 40	35	5
40 but less than 50	45	4
50 but less than 60	55	2

(In a percentage polygon the vertical axis would be defined to show the percentage of observations per class)



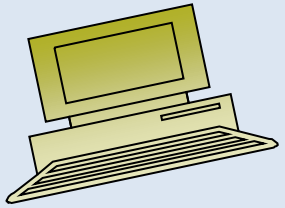


Tabulating Numerical Data: Cumulative Frequency

Data in ordered array:

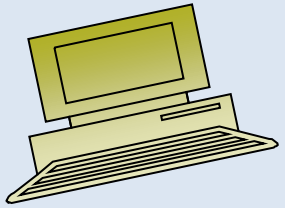
12, 13, 17, 21, 24, 24, 26, 27, 27, 30, 32, 35, 37, 38, 41, 43, 44, 46, 53, 58

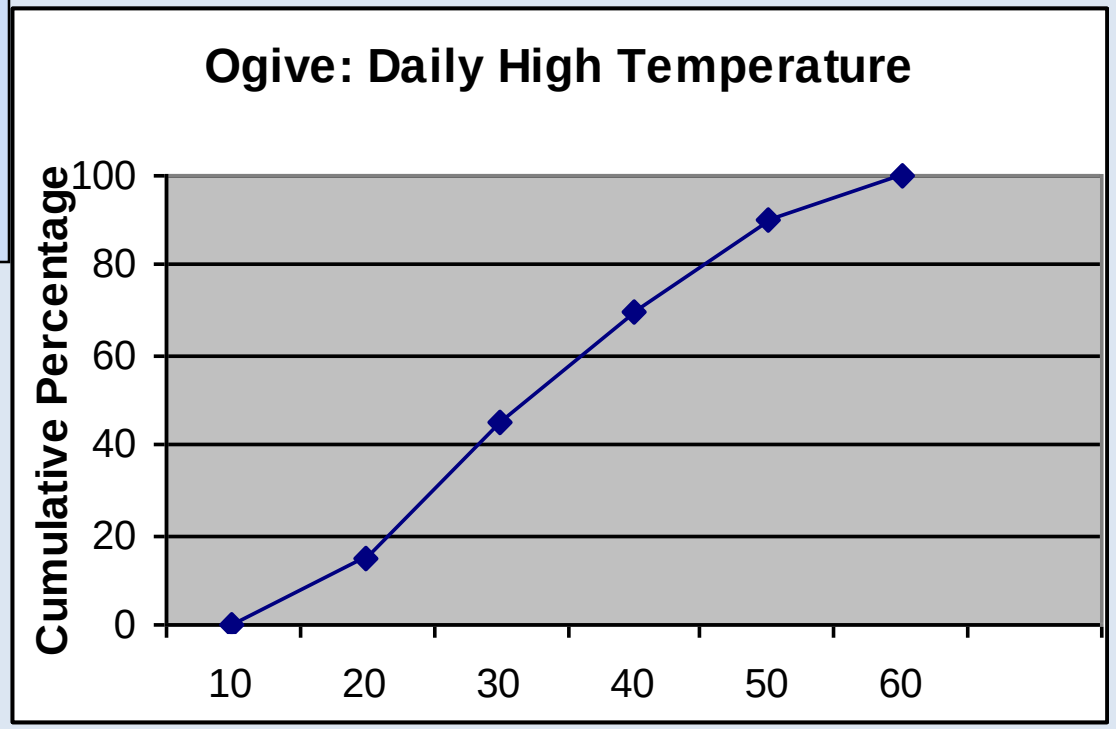
Class	Frequency	Percentage	Cumulative Frequency	Cumulative Percentage
10 but less than 20	3	15	3	15
20 but less than 30	6	30	9	45
30 but less than 40	5	25	14	70
40 but less than 50	4	20	18	90
50 but less than 60	2	10	20	100
Total	20	100		



Graphing Cumulative Frequencies:
The Ogive (Cumulative % Polygon)

Class	Lower class boundary	Cumulative Percentage
Less than 10	10	0
10 but less than 20	20	15
20 but less than 30	30	45
30 but less than 40	40	70
40 but less than 50	50	90
50 but less than 60	60	100

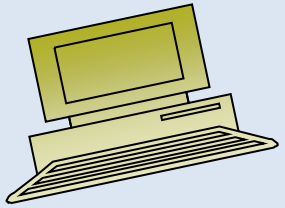




Bar and Pie Charts

- Bar charts and Pie charts are often used for qualitative (category) data

- Height of bar or size of pie slice shows the frequency or percentage for each category



Examples

	Number of Candies
Blue	23
Green	35
Red	56
Orange	54

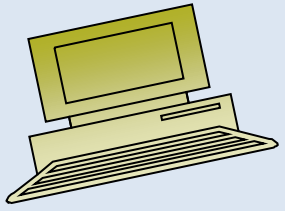
This spreadsheet has one data series in a column: Number of candies

	Bag #1	Bag #2	Bag #3
Blue	23	33	43
Green	35	22	24
Red	56	19	56
Orange	54	20	44

This spreadsheet has three data series in columns: Bag #1, Bag #2, and Bag #3

OR

This spreadsheet has four data series in rows: Blue, Green, Red, Orange.



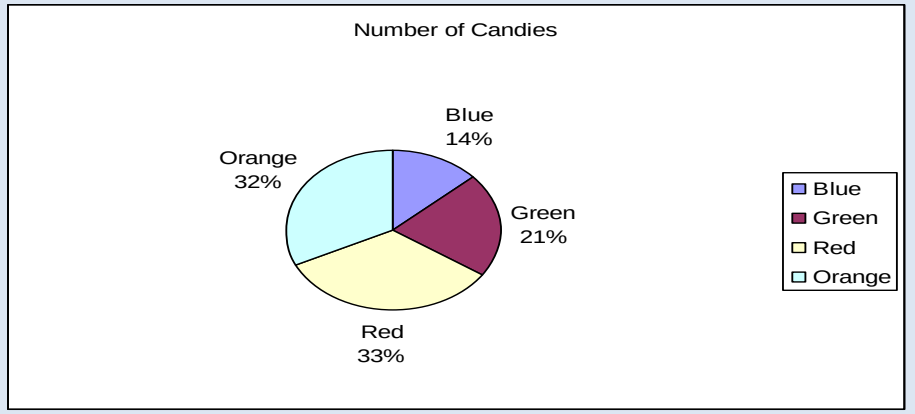
Pie Chart

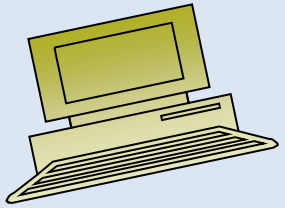
- Shows the proportional size of items that make up a data series to the sum of the items
- Pie charts have only one data series

The title comes from this cell.

Each of the numbers become a slice of the pie.

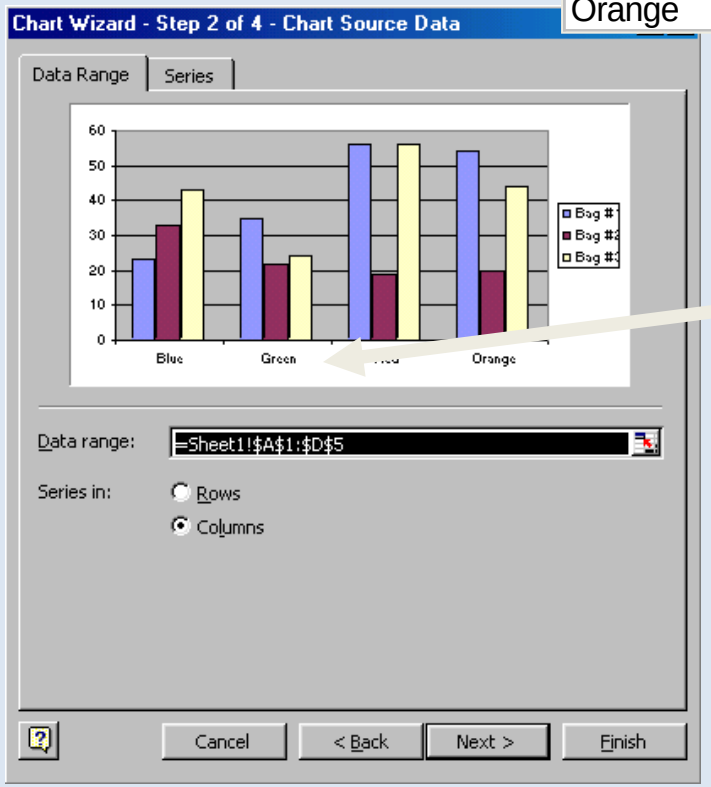
The legend and labels come from the first column.



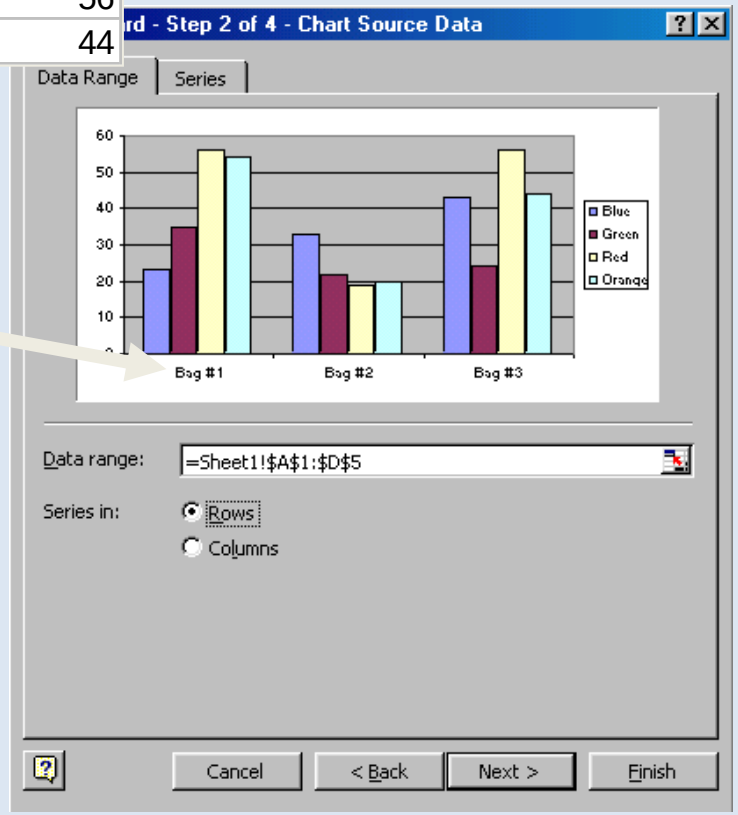


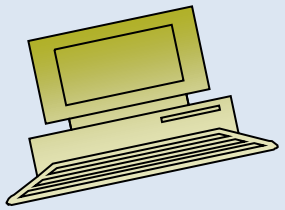
Multiple Bar Chart #1

	Bag #1	Bag #2	Bag #3
Blue	23	33	43
Green	35	22	24
Red	56	19	56
Orange	54	20	44

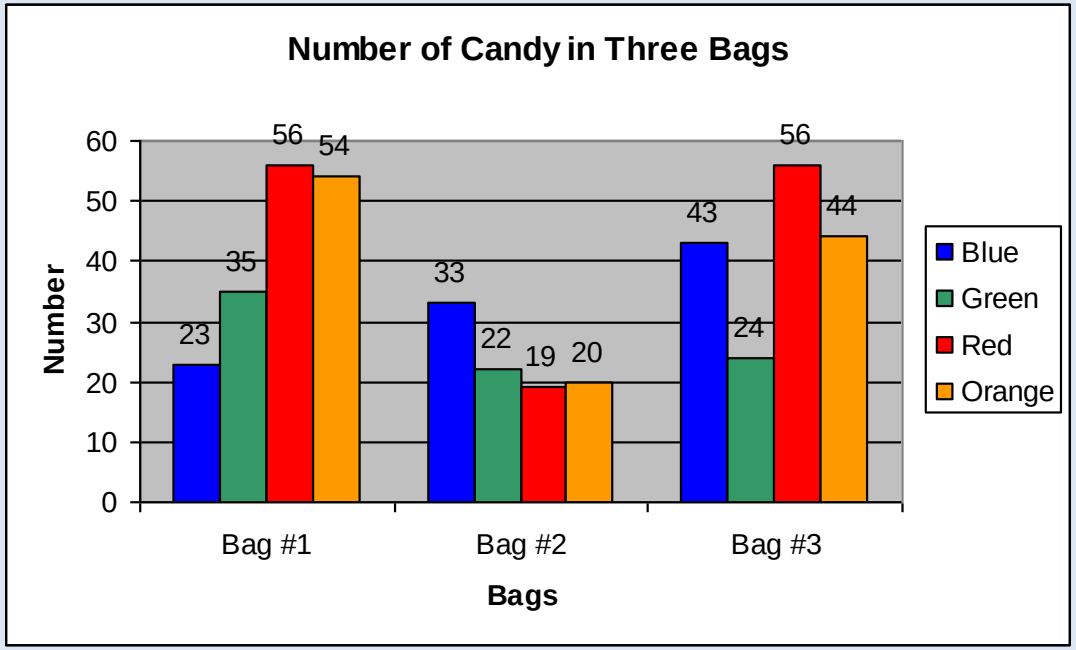


The same set of data can yield two different bar charts if the data series are in rows or columns.



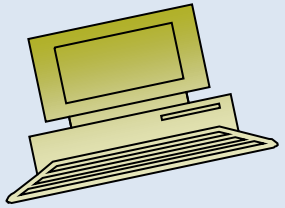


Multiple Bar Chart #2

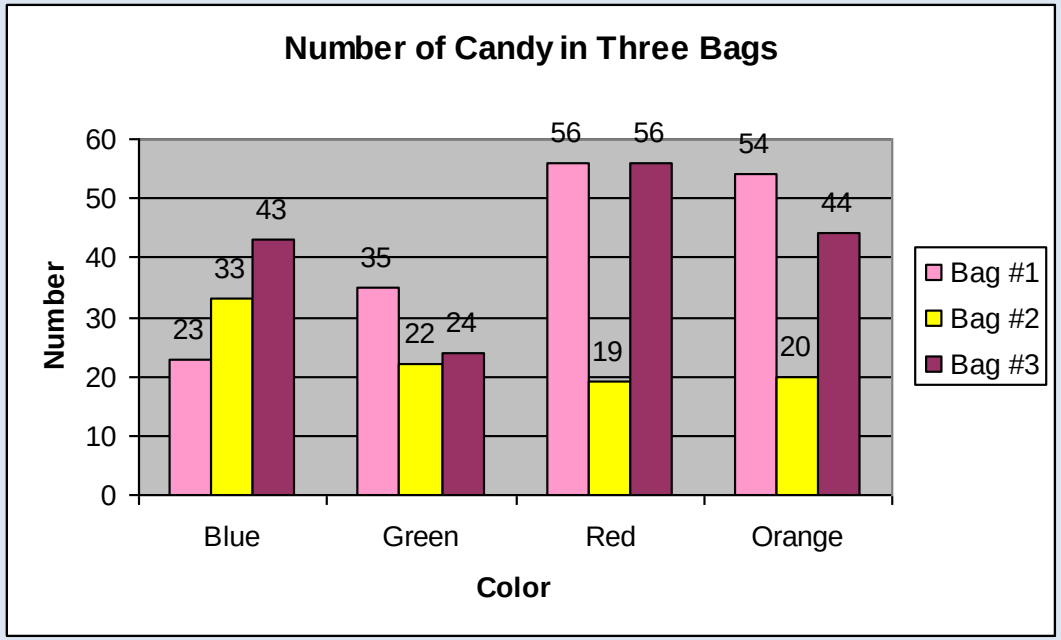


Data Set in Rows

- Each group of columns represents one bag.
- Each column in the group is a color.
- The colors are the DATA SET. They appear in the legend.

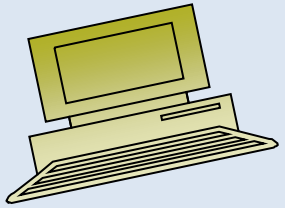


Multiple Bar Chart #3



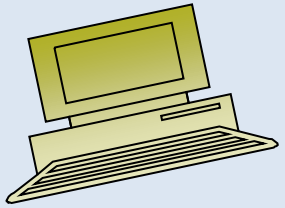
Data Set in Columns

- Each group of columns represents one color.
- Each column in the group is a bag.
- The bags are the DATA SET. They appear in the legend.

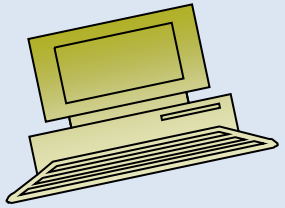


Lecture Summary

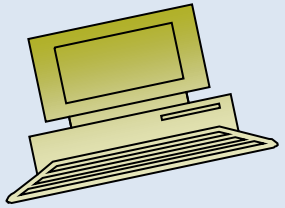
- Data in raw form are usually not easy to use for decision making -- Some type of organization is needed:
  - ♦ Table
  - ♦ Graph
- Techniques reviewed in this chapter:
  - Frequency distributions and histograms
  - Percentage polygons and ogives
  - Scatter diagrams for bivariate data
  - Bar charts, pie charts,



Practical Exercise in Data Preparation



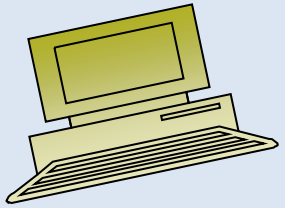
Lecture 5



Control Charts

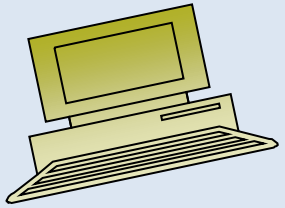
originally developed by

Walter A. Shewhart



History - beginnings

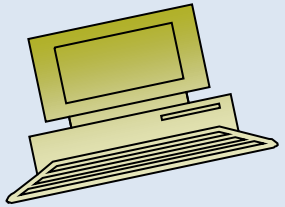
- The control chart - invented by Walter Shewhart
- How to improve the reliability of telephony transmission systems?
- Shewhart framed the problem in terms of common and special causes of variation
- In 1924, May 16, in an internal memo Shewhart introduced the control chart as a tool for distinguishing between the two causes of variation.
- Shewhart's boss, George Edwards: "Dr. Shewhart prepared a little memorandum only about a page in length… all of the essential principles and considerations which are involved in what we know today as process quality control."



Control Charts

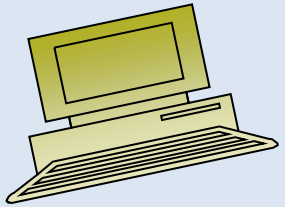
The *control chart* is a statistical quality control tool used in the monitoring variation in the characteristics of

a product or service



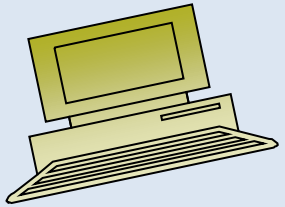
Control Charts

The control chart focuses on the time dimension and the nature of the variability in the system.



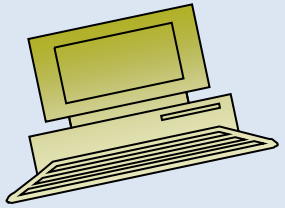
Control Charts

The control chart may be used to study past performance and/or to evaluate present conditions.

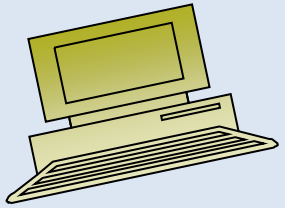


Control Charts

Data collected from a control chart may form the basis for process improvement.

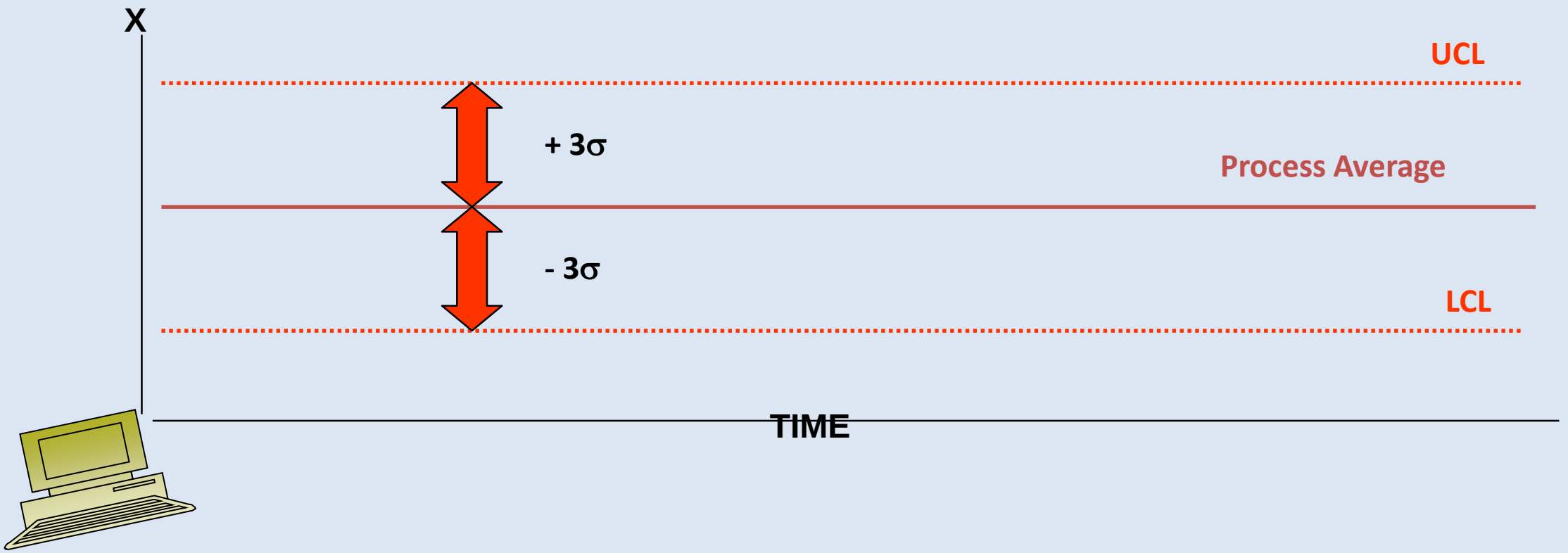


- Control chart - a graph used in SPC that shows whether a sample of data falls within the normal range of variation.
- Control chart: central line (CL), upper (UCL) and lower control limits (LCL).
  - Control limits separate common from assignable causes of variation
- Control charts for variables
  - monitor characteristics that can be measured
- Control charts for attributes
  - monitor characteristics that can be counted



Control Charts

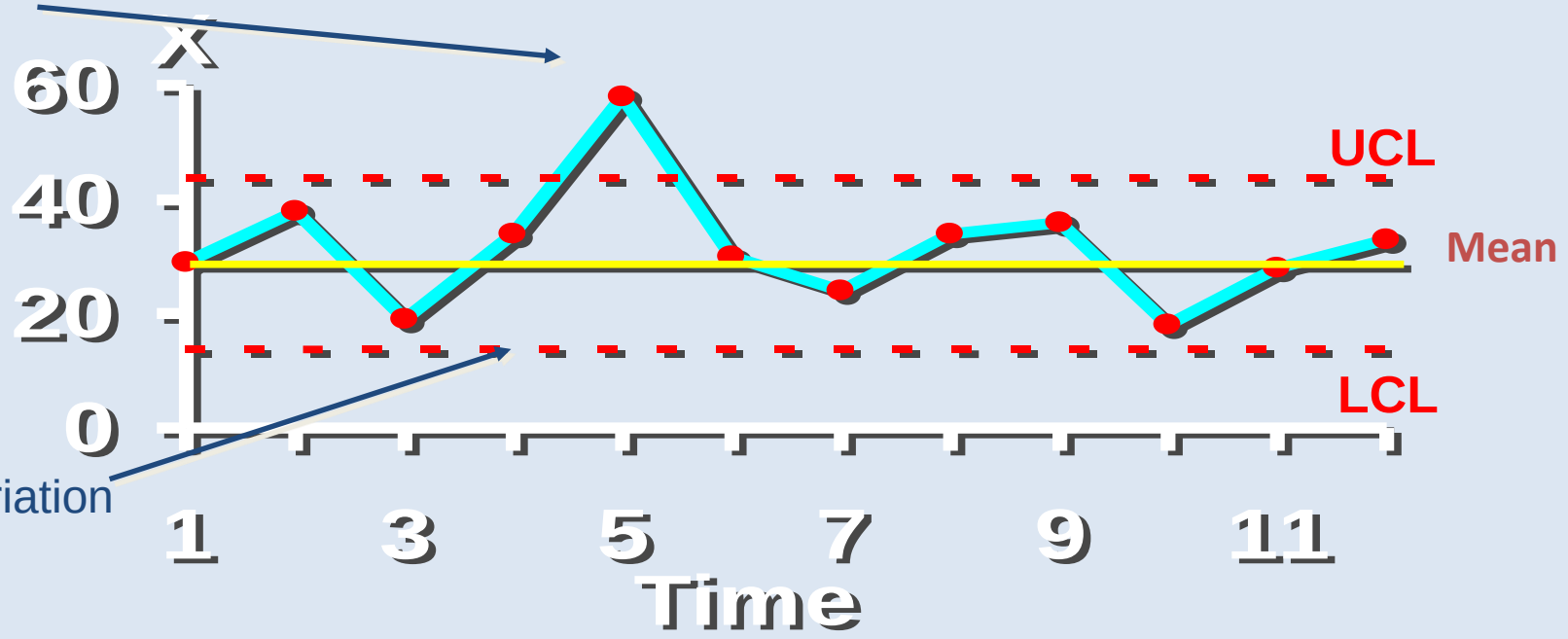
- UCL = Process Average + 3 Standard Deviations
- LCL = Process Average - 3 Standard Deviations



Control Charts

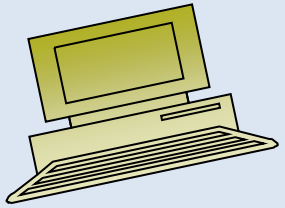
- Graph of sample data plotted over time

Assignable Cause Variation

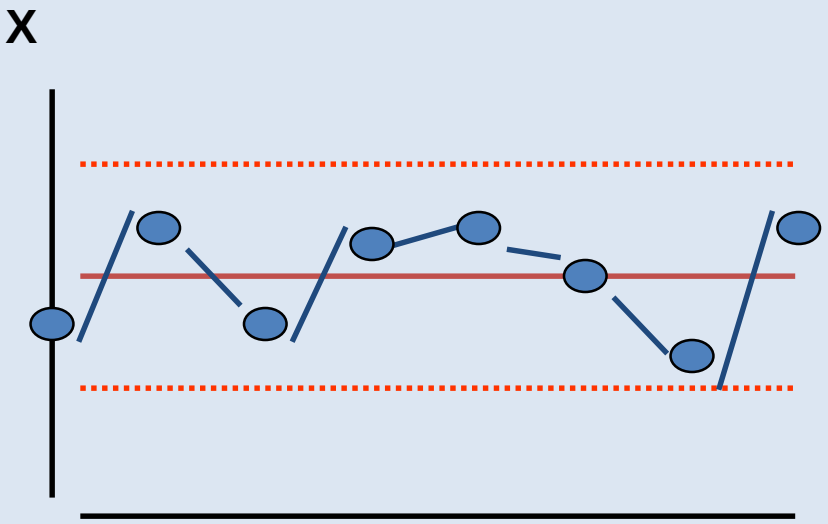


Process Average ±3σ

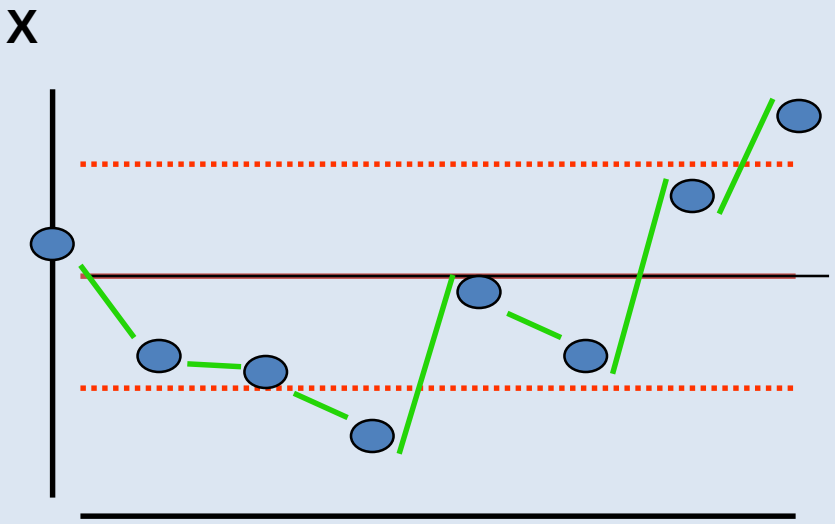
Random Variation



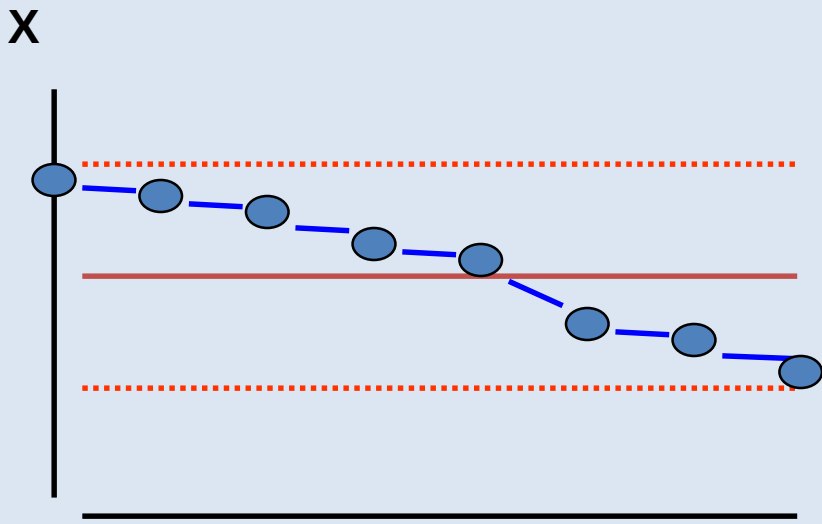
Control Charts



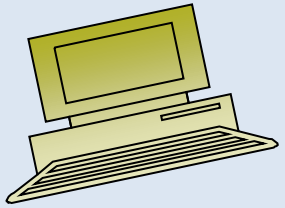
Common Cause Variation: no points outside control limit



Special Cause Variation: two points outside control limit

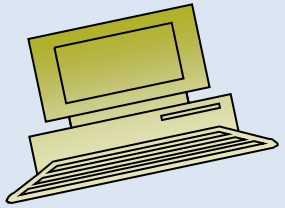


Downward Pattern: no points outside control limit; however, eight or more points in trend



Control Charts

- **Attribute charts**
- **Variables charts**



Control Charts

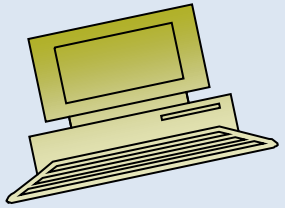
Charts may

be used for

categorical

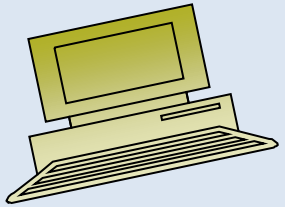
variables.

[i.e.: attributes]



Control Charts

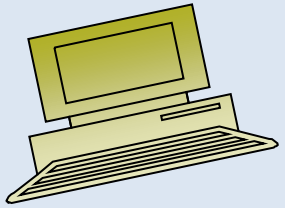
Whenever a character of interest is measured on a nominative or an interval, i.e.: categorical, scale…an *attributes* control chart is used to monitor a process.



Control Charts

- **Attributes Control Charts**

counts	[c-chart]
proportion	[p-charts]



Control Charts

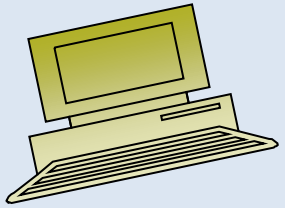
- **Attributes Control Charts**

when sample size are not constant

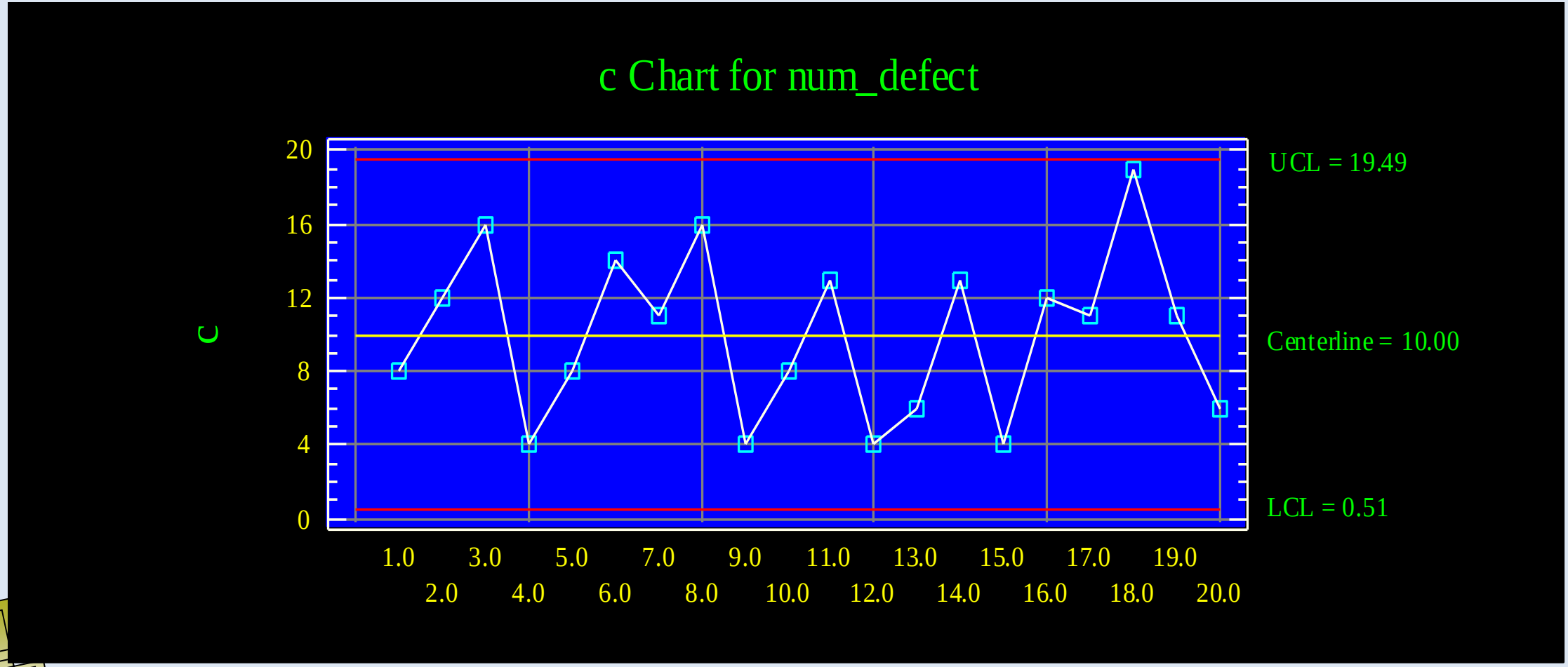
and/or are unknown

use counts charts

[c-charts]



Control Charts



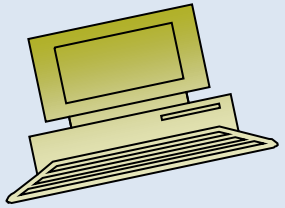
Control Charts

- **Attributes Control Charts**

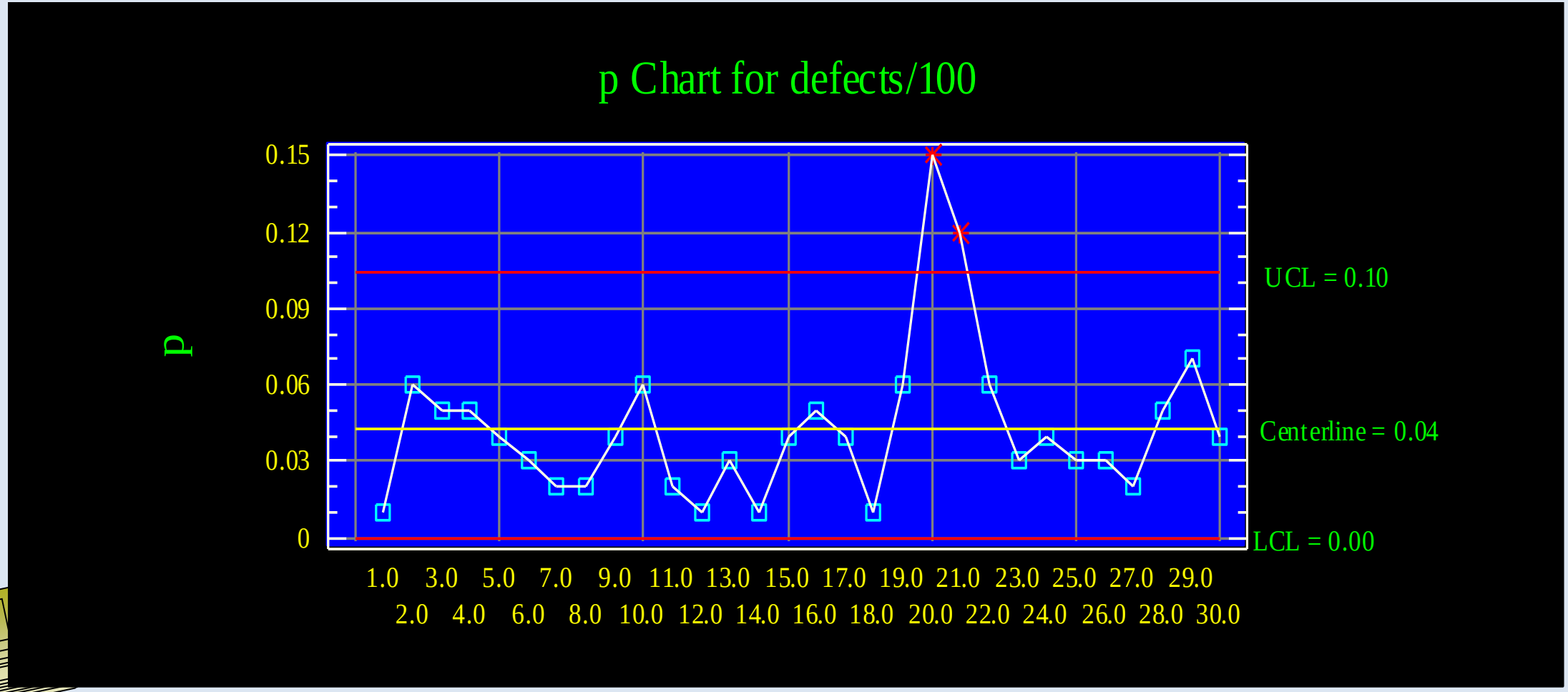
when sample size are constant and are known

use proportion charts

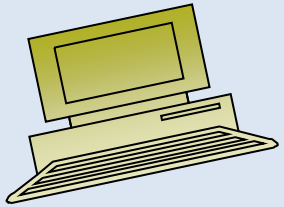
[p-charts]



Control Charts



Control Charts

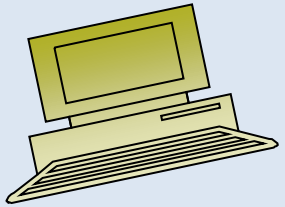


Charts may

be used for interval or ratio data [i.e.: *variables*]

Control Charts

Whenever a character of interest is measured on an interval or a ratio scale, a *variables* control chart is used to monitor a process.

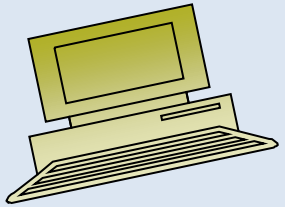


Control Charts

- **Variables Control Charts**

  Mean and Range charts

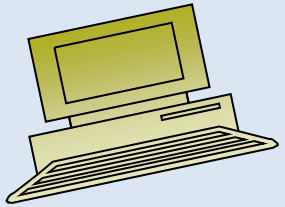
  [x-bar & R charts]



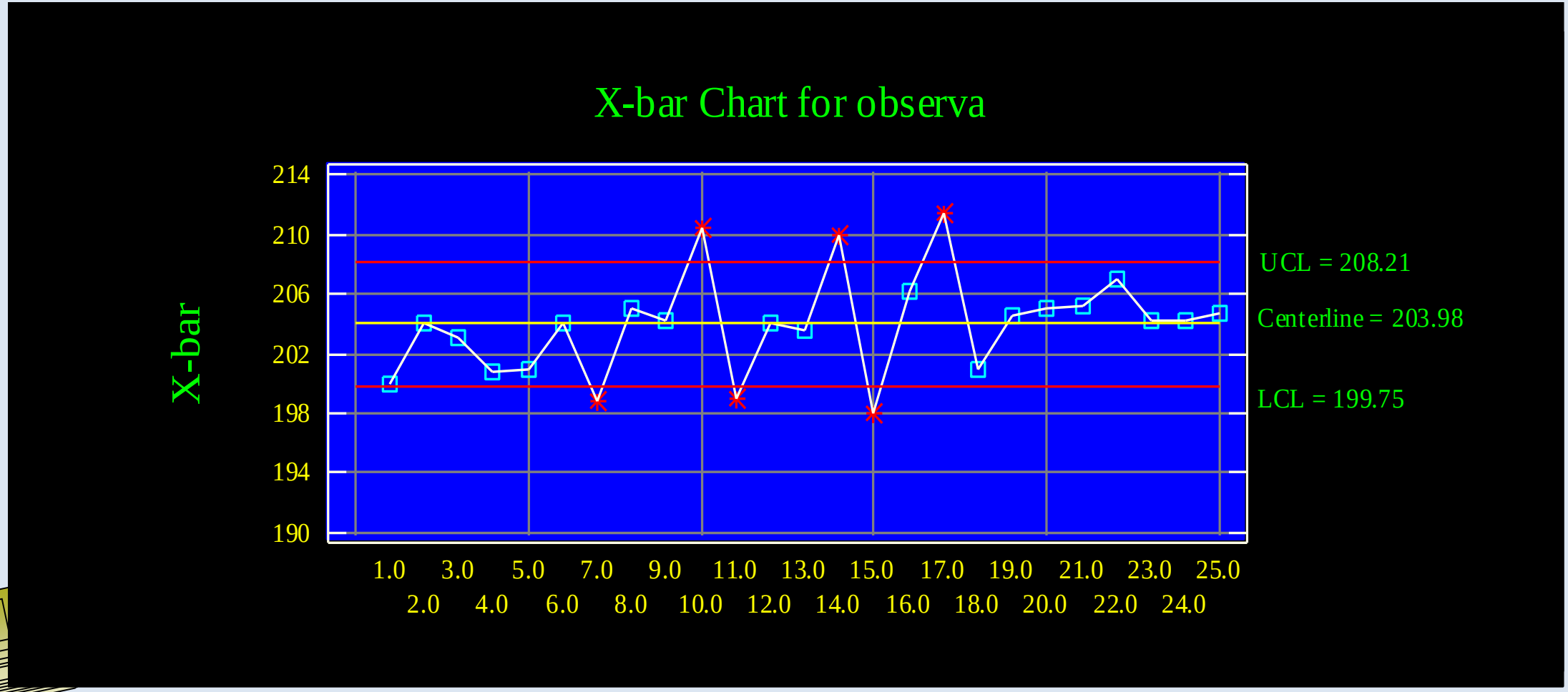
Control Charts

Variables **control charts are typically used in pairs.**

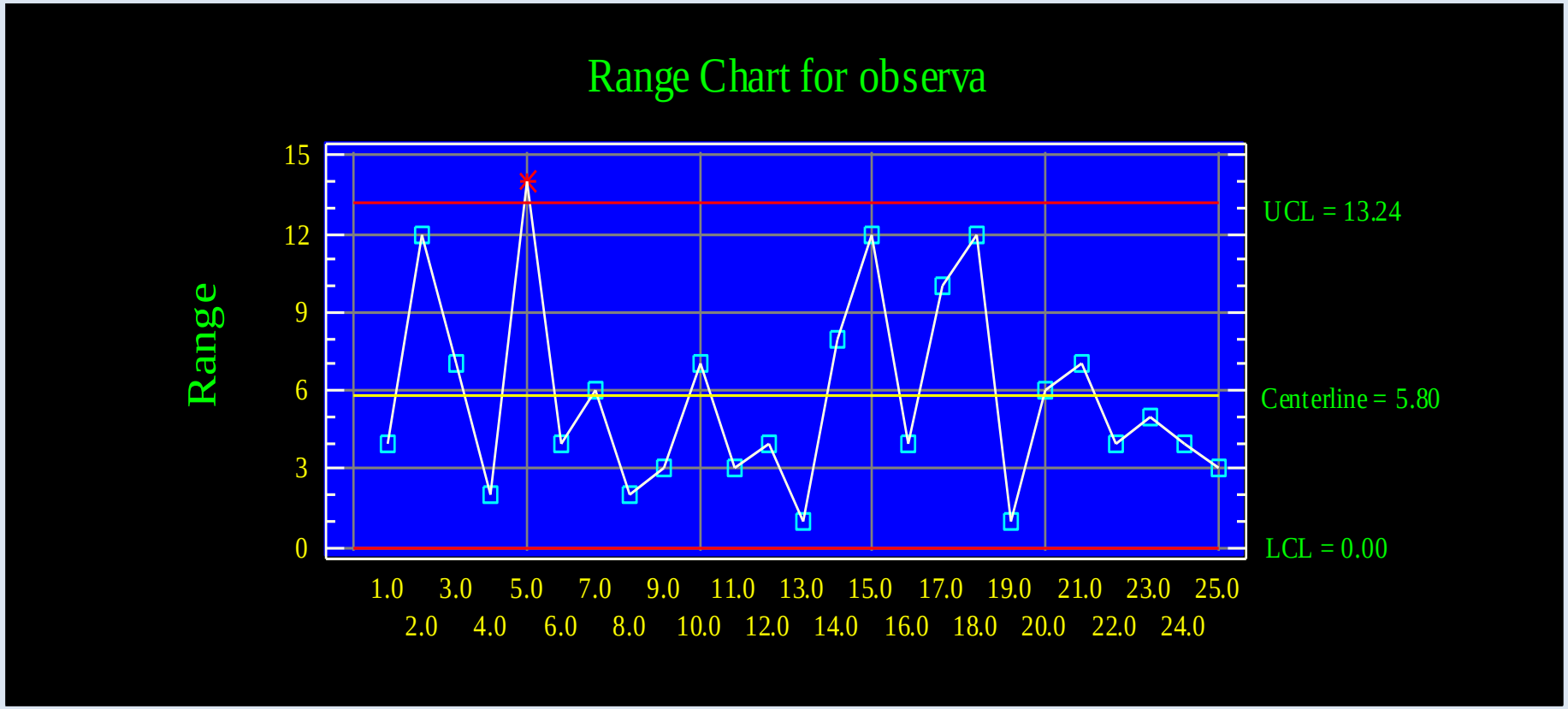
…… one chart monitors *process average* while the other monitors the *variation* in a process.

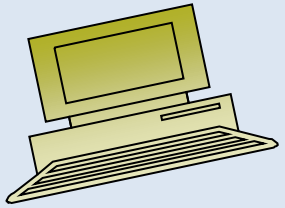


Control Charts



Control Charts

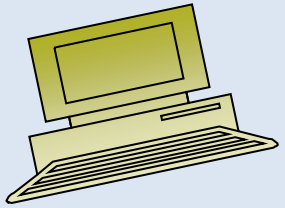




Average Run Length (ARL)

- Expected number of samples taken before shift is detected is called the Average Run Length (ARL)

$$ARL = \sum_{r=1}^{\infty} r\beta^{r-1}(1-\beta) = \frac{1}{(1-\beta)}$$

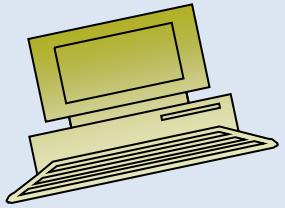


Performance of Any Shewhart Control Chart

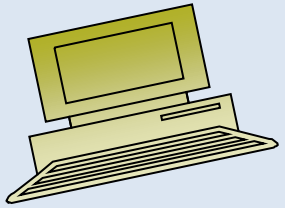
- In-Control ARL:
  - Average number of points plotted on control chart before a false alarm occurs (ideally, should be large)

- Out-of-Control ARL: $ARL_0 = \frac{1}{\alpha}$
  - Average number of points, after the process goes out-of-control, before the control chart detects it (ideally, should be small)

$$ARL_1 = \frac{1}{1-\beta}$$



Practice



Lecture 6

Introduction to Linear Regression and Correlation Analysis

Goals

After this, you should be able to:

- Calculate and interpret the simple correlation between two variables
- Determine whether the correlation is significant
- Calculate and interpret the simple linear regression equation for a set of data
- Understand the assumptions behind regression analysis
- Determine whether a regression model is significant

Goals

(continued)

After this, you should be able to:

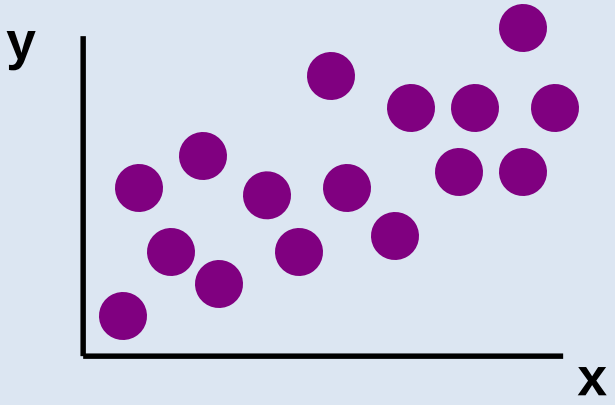
- Calculate and interpret confidence intervals for the regression coefficients
- Recognize regression analysis applications for purposes of prediction and description
- Recognize some potential problems if regression analysis is used incorrectly
- Recognize nonlinear relationships between two variables

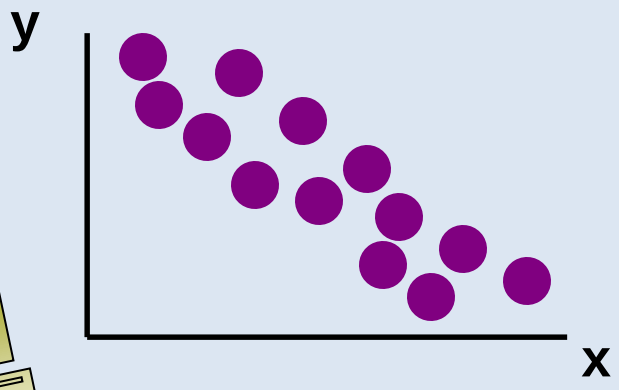
Scatter Plots and Correlation

- A scatter plot (or scatter diagram) is used to show the relationship between two variables
- Correlation analysis is used to measure strength of the association (linear relationship) between two variables
  - Only concerned with strength of the relationship
  - No causal effect is implied

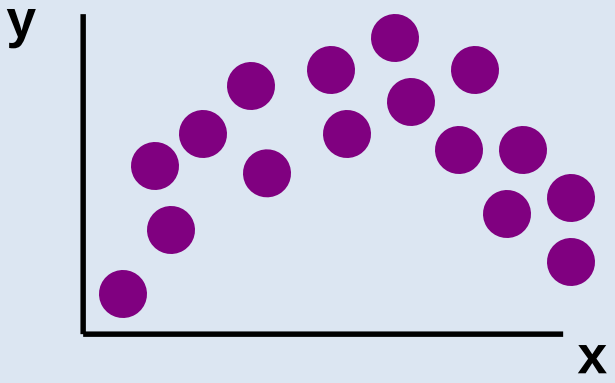
Scatter Plot Examples

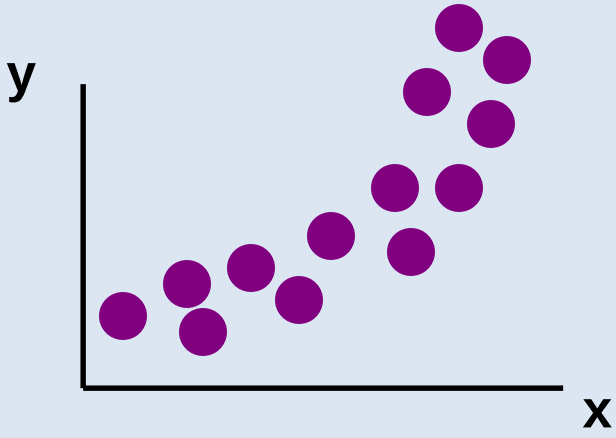
Linear relationships





Curvilinear relationships

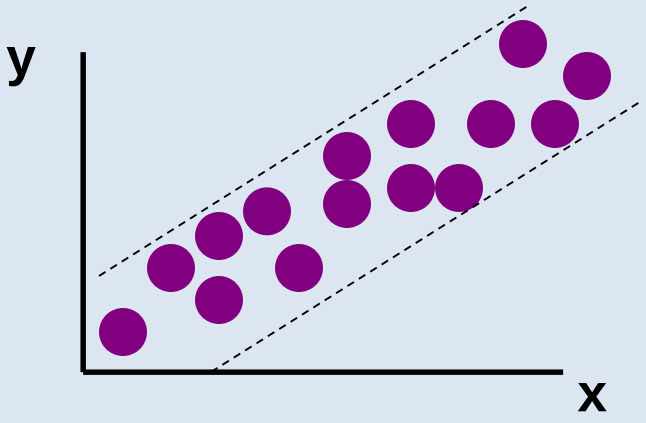


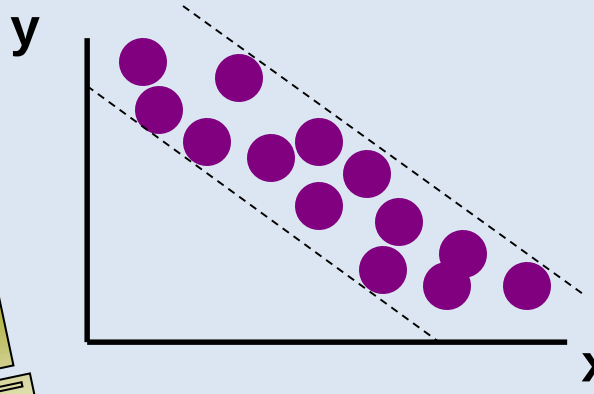


Scatter Plot Examples

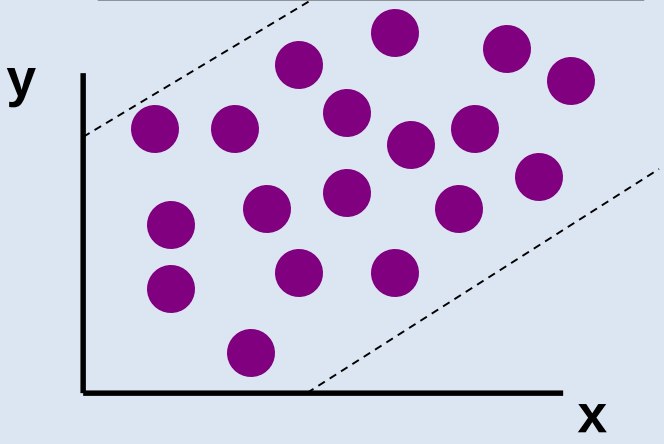
(continued)

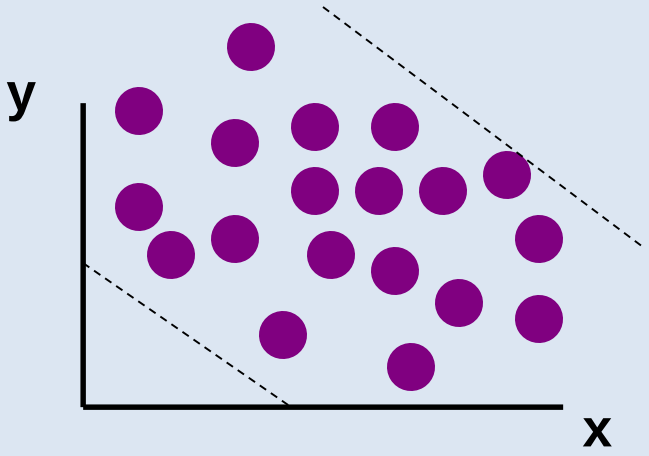
Strong relationships





Weak relationships

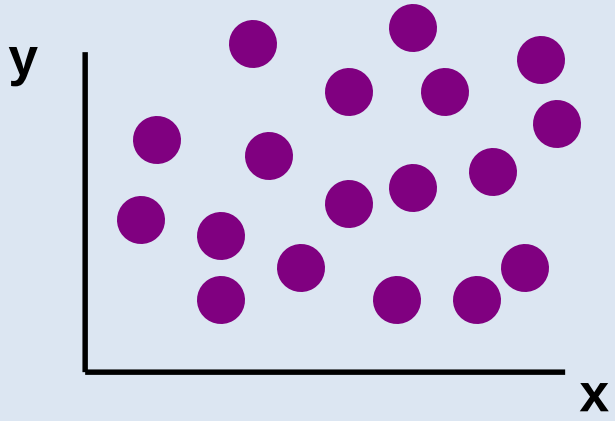


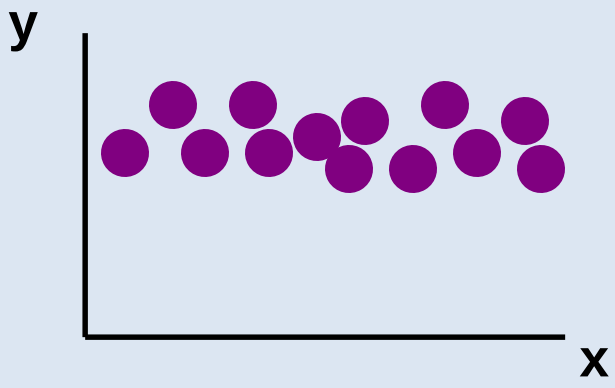


Scatter Plot Examples

(continued)

No relationship





Correlation Coefficient

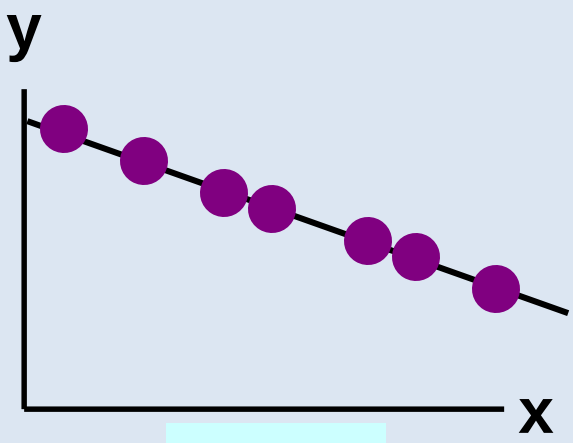
(continued)

- The population correlation coefficient ρ (rho) measures the strength of the association between the variables
- The sample correlation coefficient r is an estimate of ρ and is used to measure the strength of the linear relationship in the sample observations

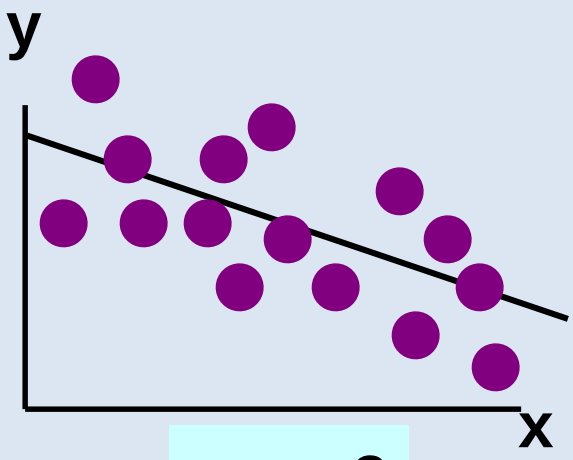
Features of ρ and r

- Unit free
- Range between -1 and 1
- The closer to -1, the stronger the negative linear relationship
- The closer to 1, the stronger the positive linear relationship
- The closer to 0, the weaker the linear relationship

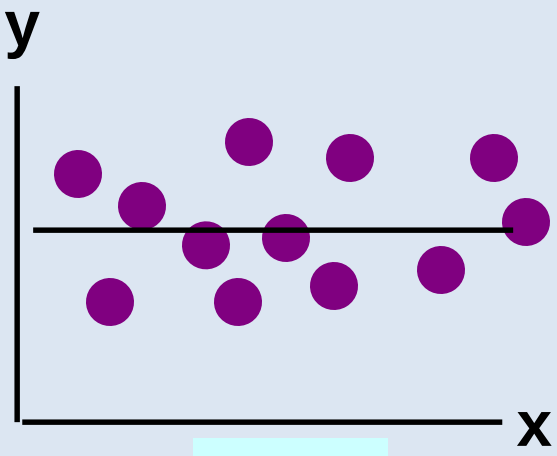
Examples of Approximate r Values



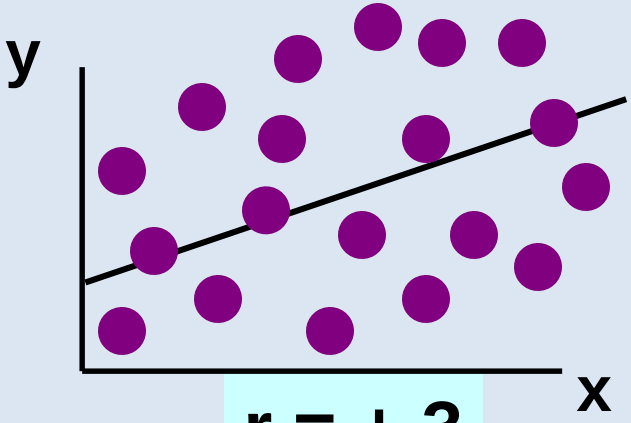
r = -1



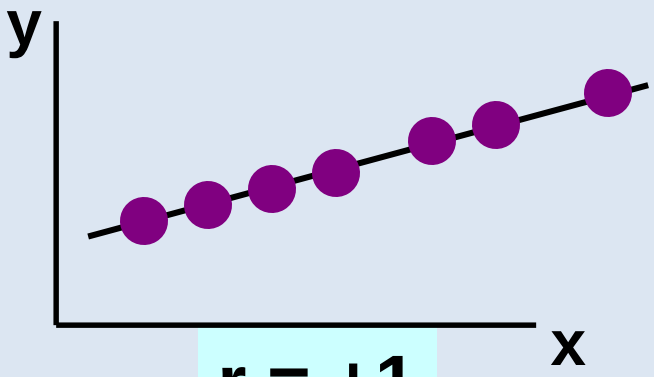
r = -.6



r = 0



r = +.3



r = +1

Calculating the Correlation Coefficient

Sample correlation coefficient:

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{[\sum (x - \bar{x})^2][\sum (y - \bar{y})^2]}}$$

or the algebraic equivalent:

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

where:

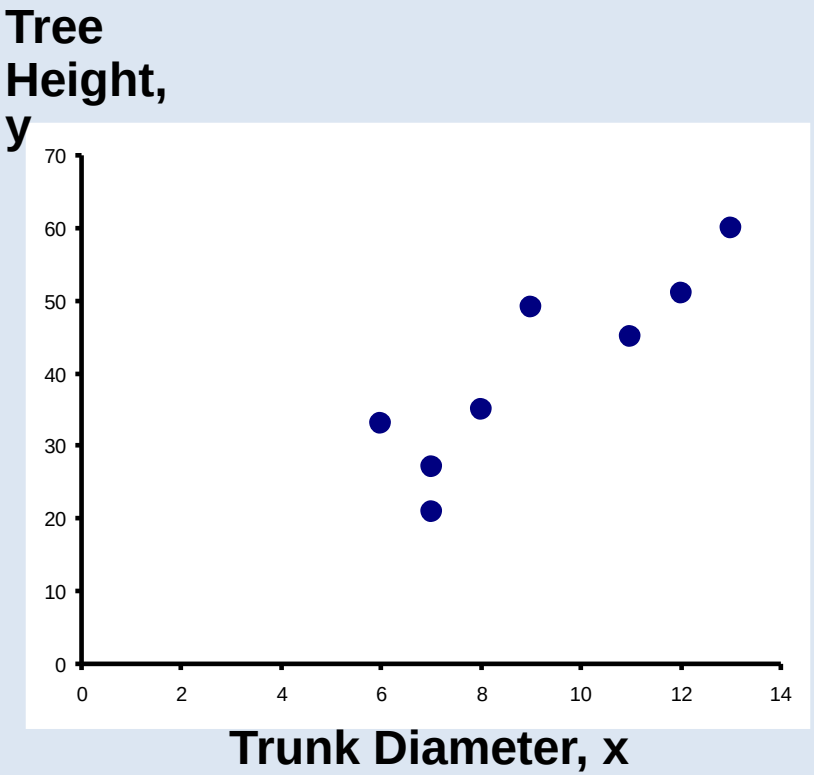
r = Sample correlation coefficient
n = Sample size
x = Value of the independent variable
y = Value of the dependent variable

Calculation Example

Tree Height	Trunk Diameter			
y	**x**	**xy**	**y^2**	**x^2**
35	8	280	1225	64
49	9	441	2401	81
27	7	189	729	49
33	6	198	1089	36
60	13	780	3600	169
21	7	147	441	49
45	11	495	2025	121
51	12	612	2601	144
Σ=321	**Σ=73**	**Σ=3142**	**Σ=14111**	**Σ=713**

Calculation Example

(continued)



$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$= \frac{8(3142) - (73)(321)}{\sqrt{[8(713) - (73)^2][8(14111) - (321)^2]}}$$

$$= 0.886$$

r = 0.886 → relatively strong positive linear association between x and y

Excel Output

Excel Correlation Output

Tools / data analysis / correlation…

	Tree Height	Trunk Diameter
Tree Height	1	
Trunk Diameter	0.886231	1

Correlation between

Tree Height and Trunk Diameter

Significance Test for Correlation

- Hypotheses

H_0: $\rho = 0$ (no correlation)

H_A: $\rho \neq 0$ (correlation exists)

- Test statistic

– $$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}}$$

(with n – 2 degrees of freedom)

Example: Produce Stores

Is there evidence of a linear relationship between tree height and trunk diameter at the .05 level of significance?

H_0: ρ = 0 (No correlation)

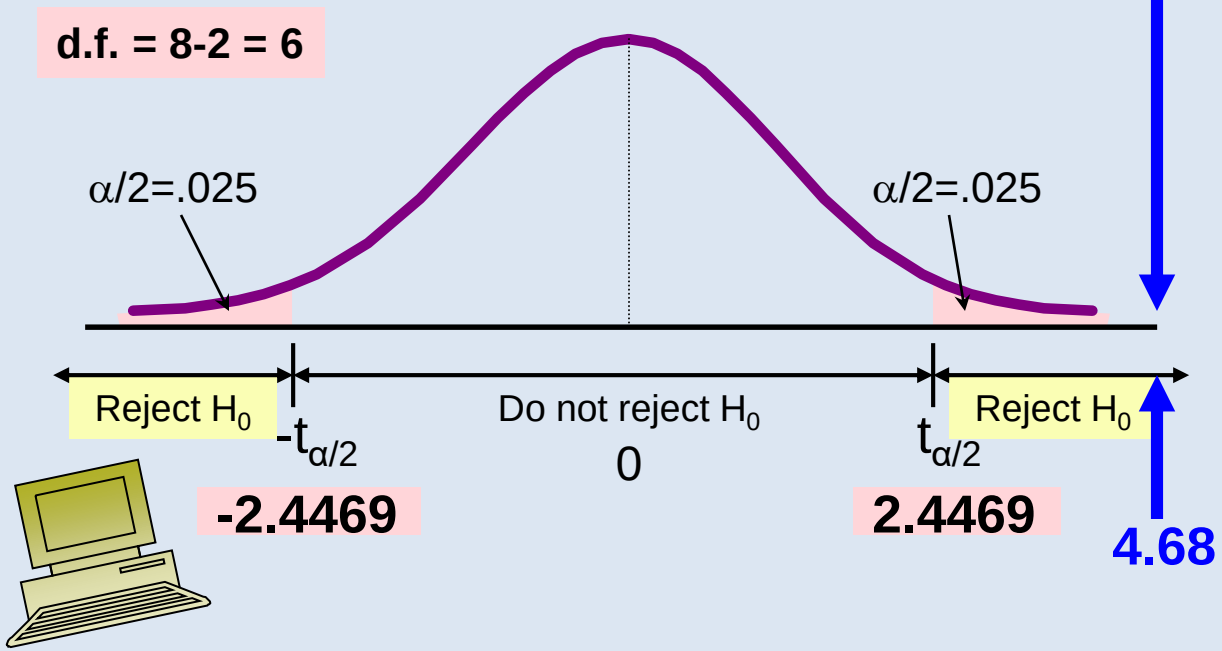
H_1: ρ ≠ 0 (correlation exists)

α =.05 , df = 8 - 2 = 6

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}} = \frac{.886}{\sqrt{\frac{1-.886^2}{8-2}}} = 4.68$$

Example: Test Solution

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}} = \frac{.886}{\sqrt{\frac{1-.886^2}{8-2}}} = 4.68$$



Decision:
Reject H_0

Conclusion:
There **is evidence** of a linear relationship at the 5% level of significance

Introduction to Regression Analysis

- Regression analysis is used to:
  - Predict the value of a dependent variable based on the value of at least one independent variable
  - Explain the impact of changes in an independent variable on the dependent variable

Dependent variable: the variable we wish to explain

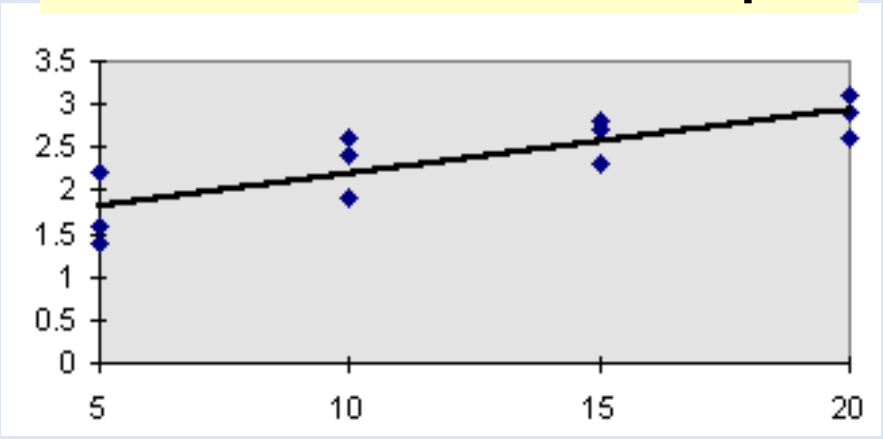
Independent variable: the variable used to explain the dependent variable

Simple Linear Regression Model

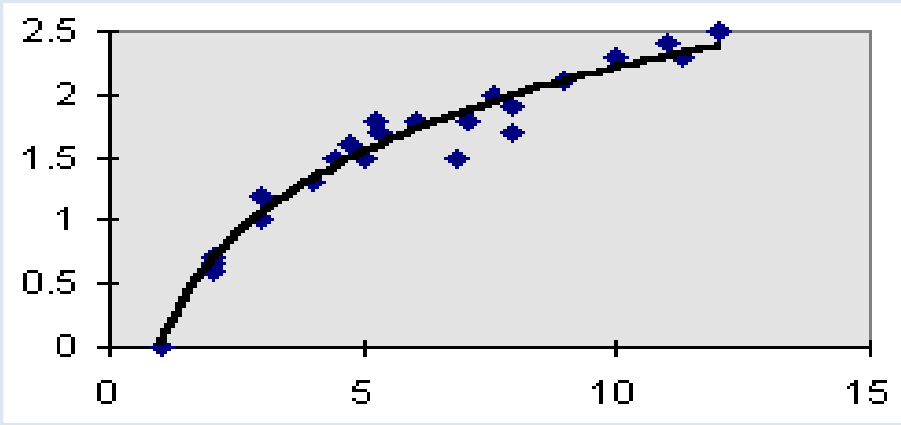
- Only **one** independent variable, x
- Relationship between x and y is described by a linear function
- Changes in y are assumed to be caused by changes in x

Types of Regression Models

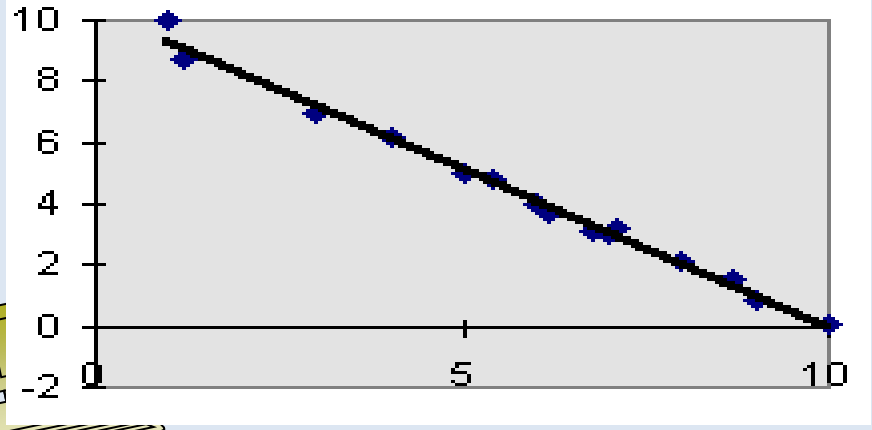
Positive Linear Relationship



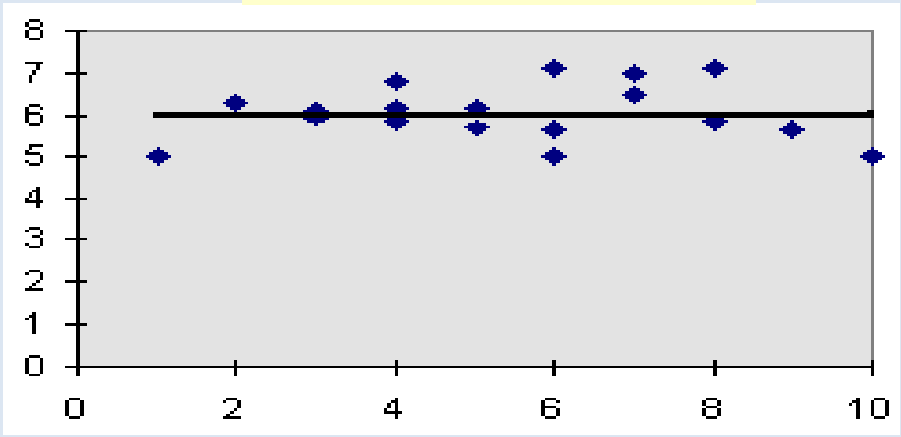
Relationship NOT Linear



Negative Linear Relationship

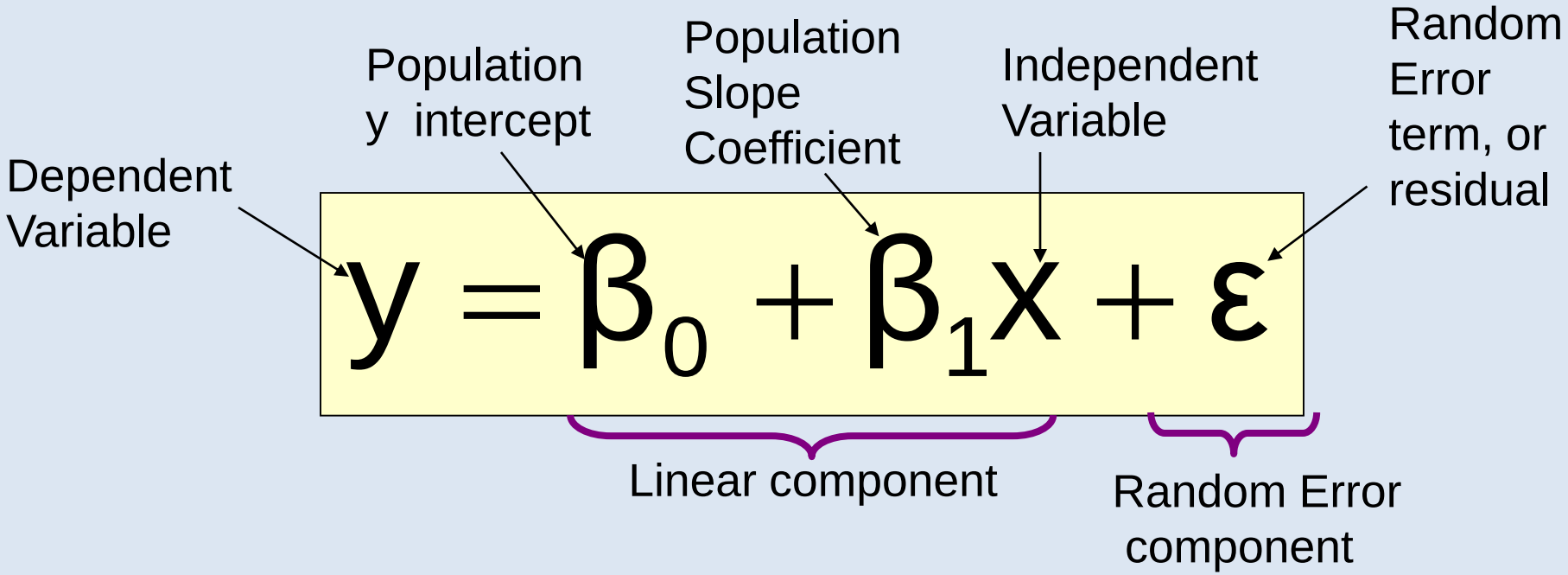


No Relationship



Population Linear Regression

The population regression model:



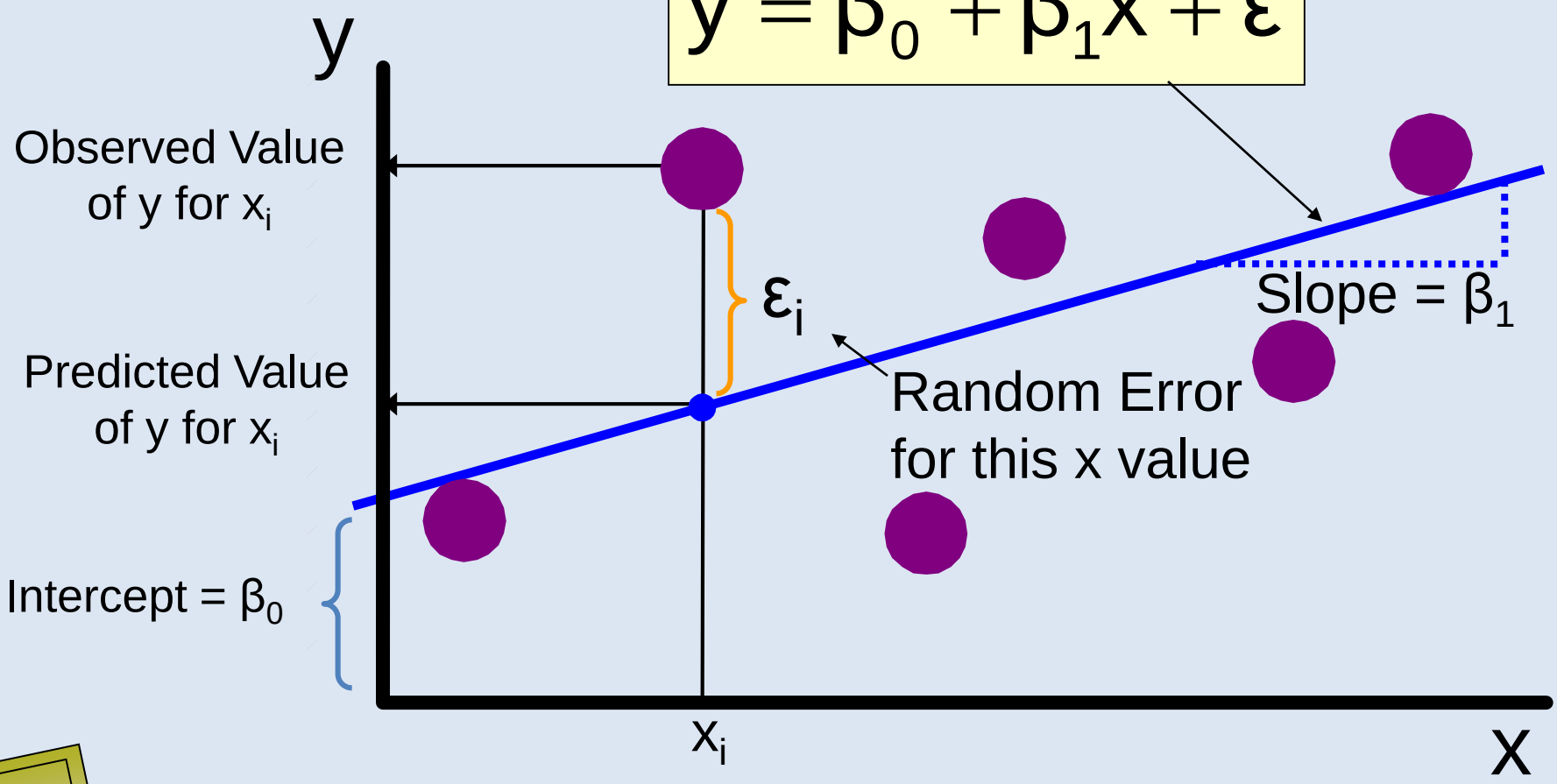
Linear Regression Assumptions

- Error values (ε) are statistically independent
- Error values are normally distributed for any given value of x
- The probability distribution of the errors is normal
- The probability distribution of the errors has constant variance
- The underlying relationship between the x variable and the y variable is linear

Population Linear Regression

(continued)

$$y = \beta_0 + \beta_1 x + \varepsilon$$



Estimated Regression Model

The sample regression line provides an estimate of the population regression line

Estimated (or predicted) y value

Estimate of the regression intercept

Estimate of the regression slope

Independent variable

$$\hat{y}_i = b_0 + b_1 x$$

The individual random error terms e_i have a mean of zero

Least Squares Criterion

- b_0 and b_1 are obtained by finding the values of b_0 and b_1 that minimize the sum of the squared residuals

$$\sum e^2 = \sum (y - \hat{y})^2$$

$$= \sum (y - (b_0 + b_1 x))^2$$

The Least Squares Equation

- The formulas for b_1 and b_0 are:

$$b_1 = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

algebraic equi

$$b_1 = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}$$

and

$$b_0 = \bar{y} - b_1 \bar{x}$$

Interpretation of the Slope and the Intercept

- b_0 is the estimated average value of y when the value of x is zero

- b_1 is the estimated change in the average value of y as a result of a one-unit change in x

Finding the Least Squares Equation

- The coefficients b_0 and b_1 will usually be found using computer software, such as Excel
- Other regression measures will also be computed as part of computer-based regression analysis

Simple Linear Regression Example

- A real estate agent wishes to examine the relationship between the selling price of a home and its size (measured in square feet)

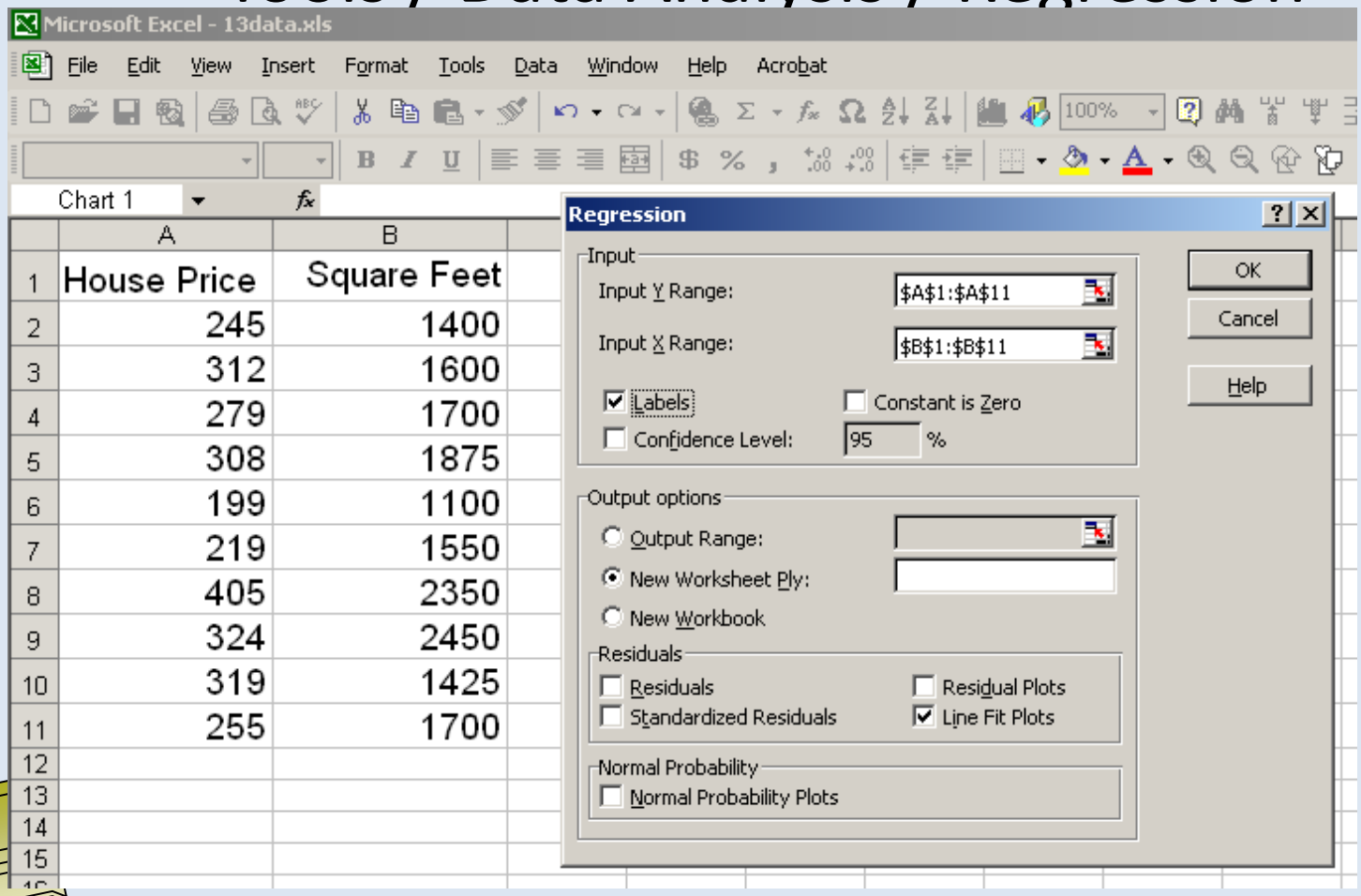
- A random sample of 10 houses is selected
  - Dependent variable (y) = house price in $1000s
  - Independent variable (x) = square feet

Sample Data for House Price Model

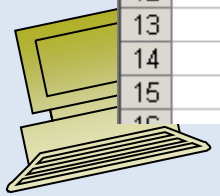
House Price in $1000s (y)	Square Feet (x)
245	1400
312	1600
279	1700
308	1875
199	1100
219	1550
405	2350
324	2450
319	1425
255	1700

Regression Using Excel

- Tools / Data Analysis / Regression



	A	B
1	House Price	Square Feet
2	245	1400
3	312	1600
4	279	1700
5	308	1875
6	199	1100
7	219	1550
8	405	2350
9	324	2450
10	319	1425
11	255	1700
12		
13		
14		
15		



Excel Output

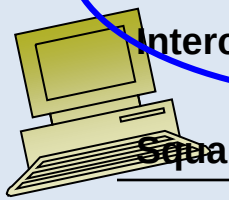
Regression Statistics	
Multiple R	0.76211
R Square	0.58082
Adjusted R Square	0.52842
Standard Error	41.33032
Observations	10

The regression equation is:

$$\widehat{\text{house price}} = 98.24833 + 0.10977\ (\text{square feet})$$

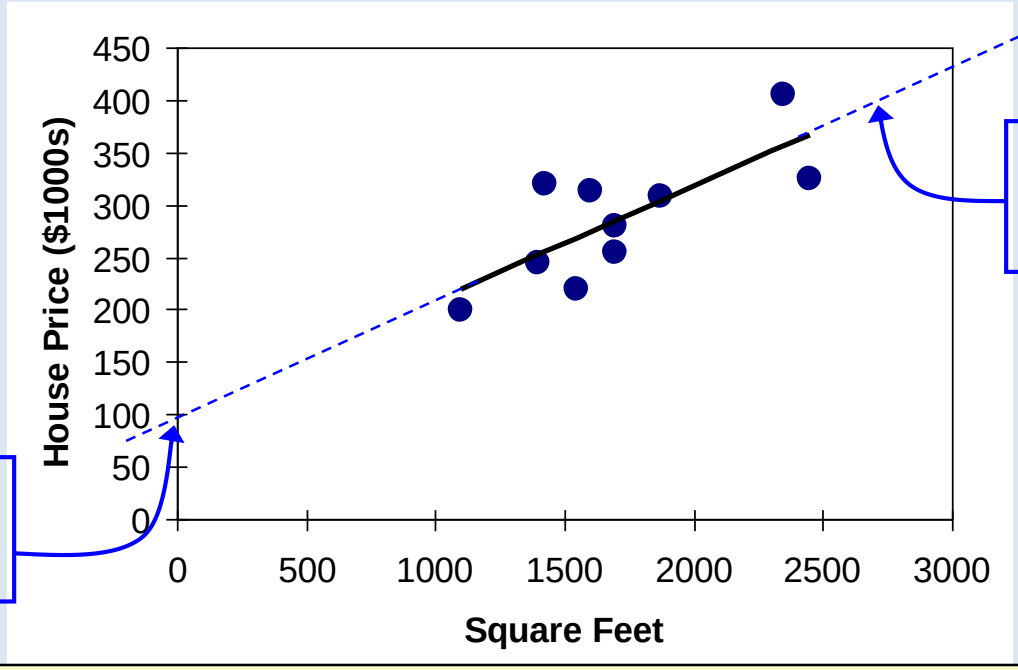
ANOVA	*df*	*SS*	*MS*	*F*	*Significance F*
Regression	1	18934.9348	18934.9348	11.0848	0.01039
Residual	8	13665.5652	1708.1957		
Total	9	32600.5000			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	98.24833	58.03348	1.69296	0.12892	-35.57720	232.07386
Square Feet	0.10977	0.03297	3.32938	0.01039	0.03374	0.18580



Graphical Presentation

- House price model: scatter plot and regression line



$$\widehat{\text{house price}} = 98.24833 + 0.10977\ (\text{square feet})$$

Interpretation of the Intercept, b_0

$$\widehat{\text{house price}} = 98.24833 + 0.10977\ (\text{square feet})$$

- b_0 is the estimated average value of Y when the value of X is zero (if x = 0 is in the range of observed x values)
  - Here, no houses had 0 square feet, so b_0 = 98.24833 just indicates that, for houses within the range of sizes observed, $98,248.33 is t portion of the house price not explained by square feet

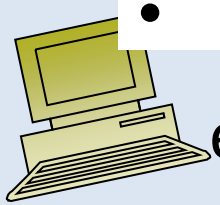
Interpretation of the Slope Coefficient, b_1

$$\widehat{\text{house price}} = 98.24833 + \boxed{0.10977}\,(\text{square feet})$$

- b_1 measures the estimated change in the average value of Y as a result of a one-unit change in X
  - Here, b_1 = .10977 tells us that the average value of a house increases by .10977($1000) = $109.77, on average, for each additional one square foot of size

Least Squares Regression Properties

- The sum of the residuals from the least squares regression line is 0 ( ) $\sum(y-\hat{y})=0$
- The sum of the squared residuals is a minimum (minimized) $\sum(y-\hat{y})^2$
- The simple regression line always passes through the mean of the y variable and the mean of the x variable
- The least squares coefficients are unbiased estimates of β_0 and β_1



Explained and Unexplained Variation

- Total variation is made up of two parts:

$$SST = SSE + SSR$$

Total sum of Squares	Sum of Squares Error	Sum of Squares Regression
$SST = \sum (y - \bar{y})^2$	$SSE = \sum (y - \hat{y})^2$	$SSR = \sum (\hat{y} - \bar{y})^2$

where:

$\bar{y}$ = Average value of the dependent variable

y = Observed values of the dependent variable

$\hat{y}$ = Estimated value of y for the given x value

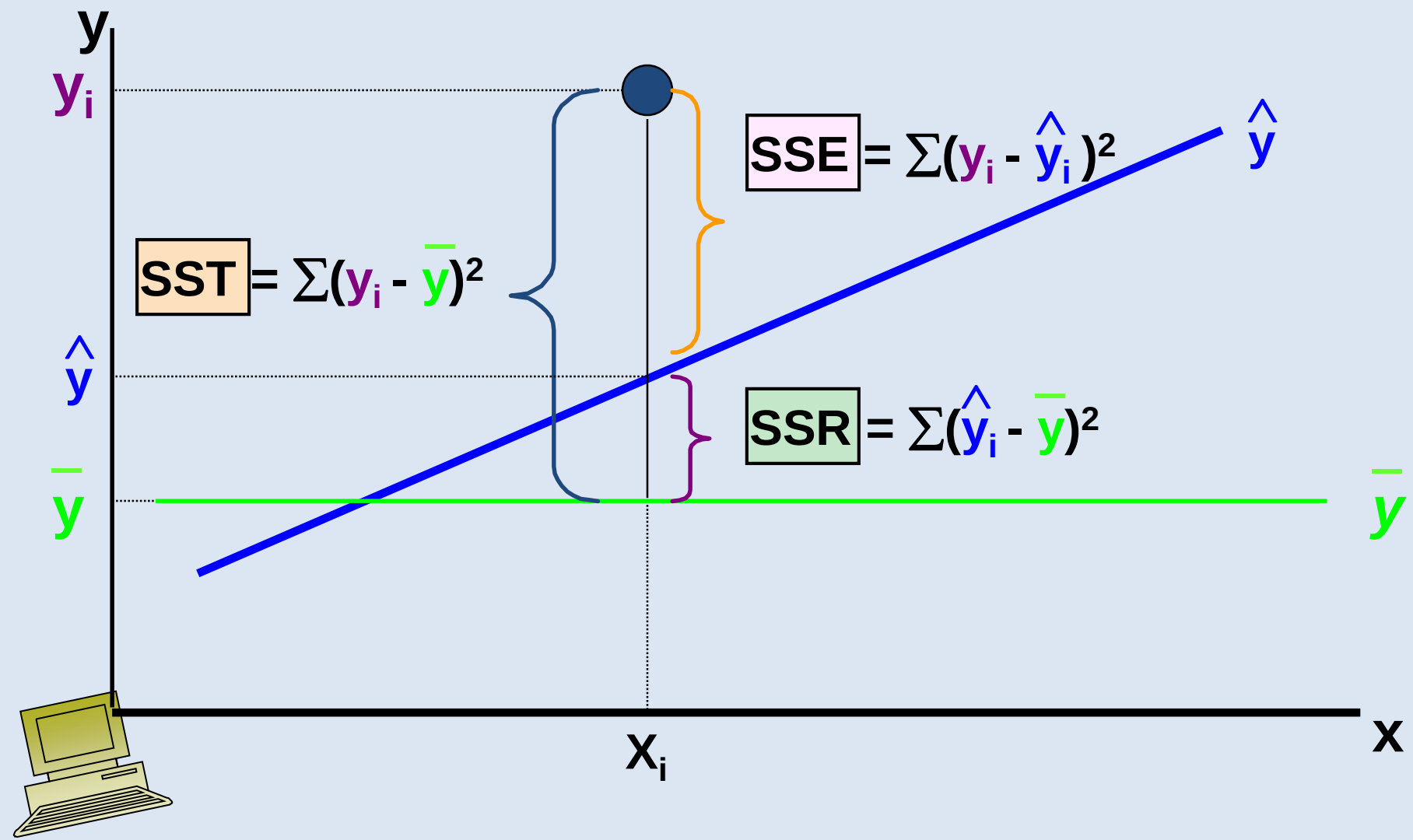
Explained and Unexplained Variation

(continued)

- SST = total sum of squares
  - Measures the variation of the y_i values around their mean y
  - SSE = error sum of squares
  - Variation attributable to factors other than the relationship between x and y
- SSR = regression sum of squares
  - Explained variation attributable to the relationship between x and y

Explained and Unexplained Variation

(continued)



Coefficient of Determination, R^2

- The coefficient of determination is the portion of the total variation in the dependent variable that is explained by variation in the independent variable

- The coefficient of determination is also called R-squared and is denoted as R^2

$$R^2 = \frac{SSR}{SST}$$

where $0 \le R^2 \le 1$

Coefficient of Determination, R^2

(continued)

Coefficient of determination

$$R^2 = \frac{SSR}{SST} = \frac{\text{sum of squares explained by regression}}{\text{total sum of squares}}$$

Note: In the single independent variable case, the coefficient of determination is

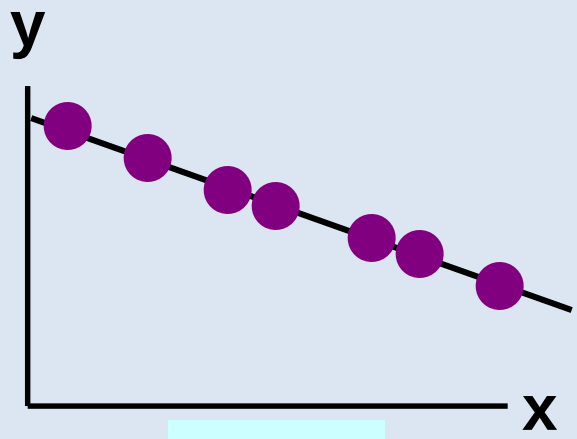
$$R^2 = r^2$$

where:

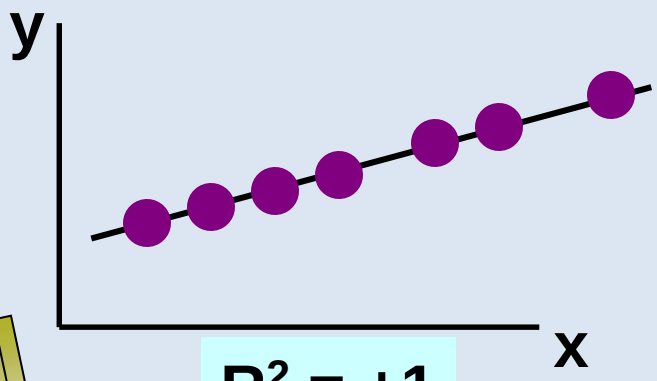
R^2 = Coefficient of determination

r = Simple correlation coefficient

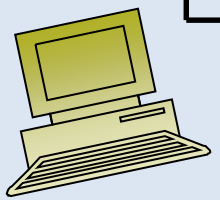
Examples of Approximate R^2 Values



$R^2 = 1$



$R^2 = +1$

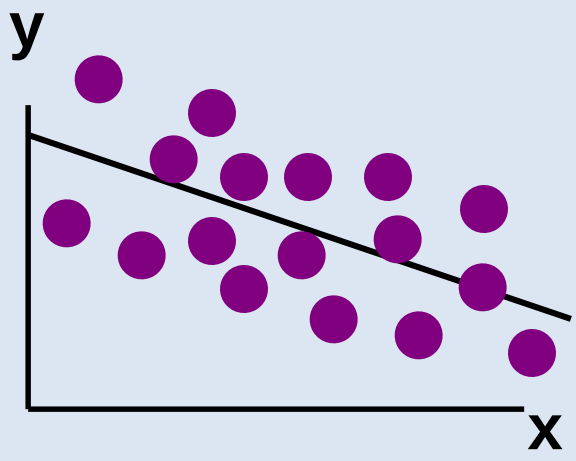


$R^2 = 1$

Perfect linear relationship between x and y:

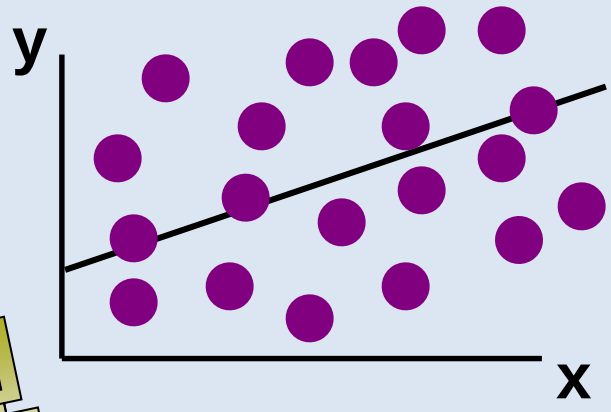
100% of the variation in y is explained by variation in x

Examples of Approximate R^2 Values

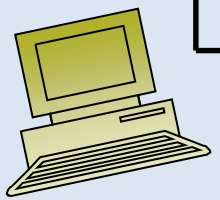


$0 < R^2 < 1$

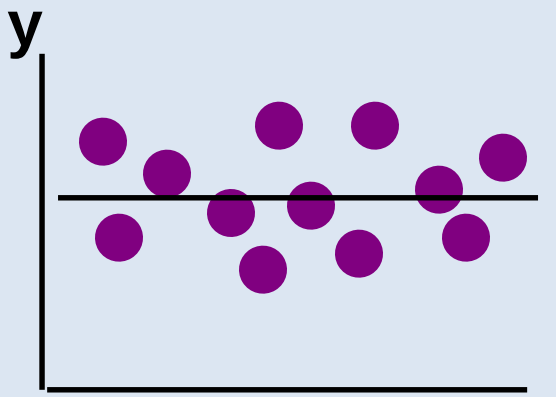
Weaker linear relationship between x and y:



Some but not all of the variation in y is explained by variation in x



Examples of Approximate R^2 Values



$R^2 = 0$

$R^2 = 0$

No linear relationship between x and y:

The value of Y does not depend on x. (None of the variation in y is explained by variation in x)

Standard Error of Estimate

- The standard deviation of the variation of observations around the regression line is estimated by

$$s_{\varepsilon} = \sqrt{\frac{SSE}{n-k-1}}$$

Where

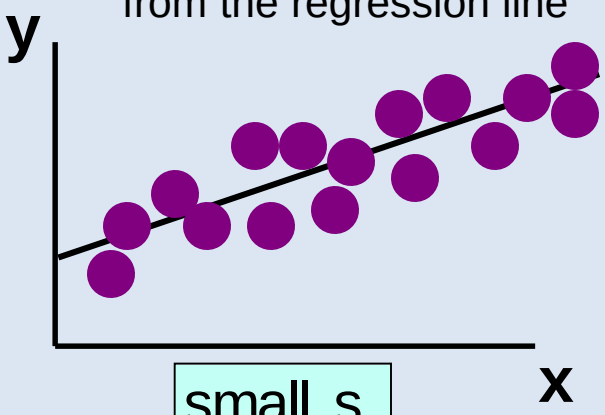
SSE = Sum of squares error

n = Sample size

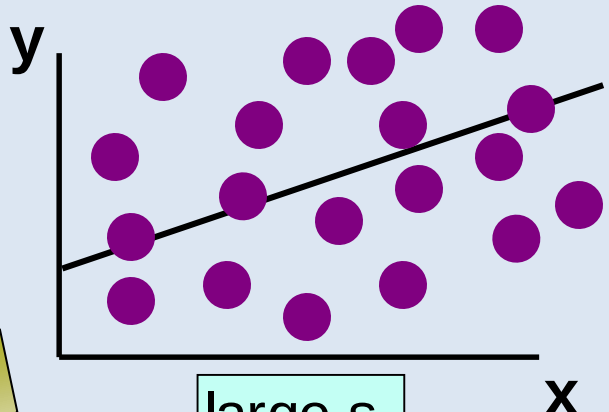
k = number of independent variables in the model

Comparing Standard Errors

Variation of observed y values from the regression line

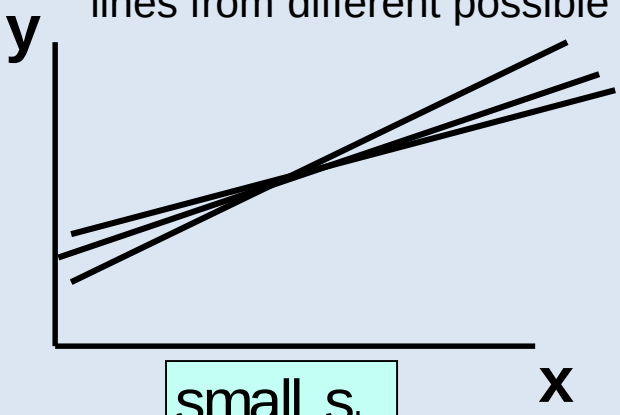


small s_ε

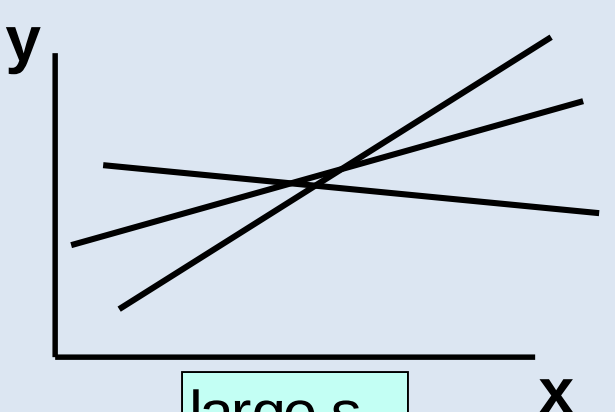


large s_ε

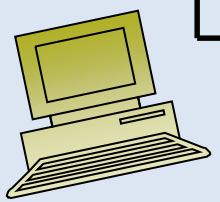
Variation in the slope of regression lines from different possible samples



small s_{b_1}



large s_{b_1}



Inference about the Slope: t Test

- t test for a population slope
  - Is there a linear relationship between x and y?
- Null and alternative hypotheses
  - H_0: $\beta_1 = 0$ (no linear relationship)
  - H_1: $\beta_1 \neq 0$ (linear relationship does exist)
- Test statistic
  -

$$t = \frac{b_1 - \beta_1}{s_{b_1}}$$

where:

b_1 = Sample regression slope coefficient

β_1 = Hypothesized slope

s_{b1} = Estimator of the standard error of the slope

$$d.f. = n - 2$$

Inference about the Slope: t Test

(continued)

House Price in $1000s (y)	Square Feet (x)
245	1400
312	1600
279	1700
308	1875
199	1100
219	1550
405	2350
324	2450
319	1425
255	1700

Estimated Regression Equation:

$$\widehat{\text{house price}} = 98.25 + 0.1098\ (\text{sq.ft.})$$

The slope of this model is 0.1098

Does square footage of the house affect its sales price?

Inferences about the Slope: t Test Example

Test Statistic: **t = 3.329**

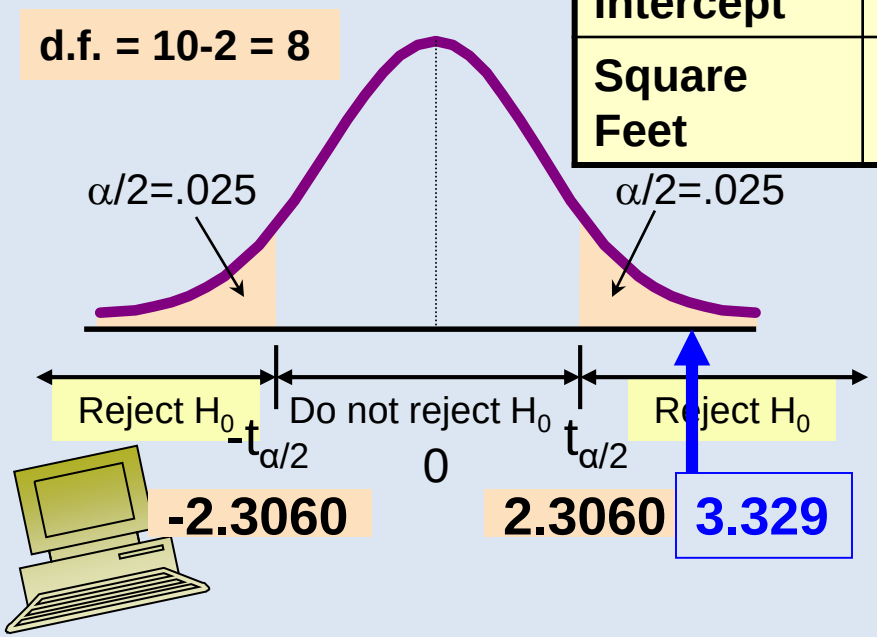
$H_0: \beta_1 = 0$

$H_A: \beta_1 \neq 0$

From Excel output:

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	**98.24833**	**58.03348**	**1.69296**	**0.12892**
Square Feet	**0.10977**	**0.03297**	**3.32938**	**0.01039**

b_1 S_{b_1} t



Reject H_0

There is sufficient evidence that square footage affects house price

Regression Analysis for Description

Confidence Interval Estimate of the Slope:

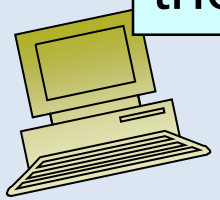
$$b_1 \pm t_{\alpha/2} s_{b_1}$$

d.f. = n - 2

Excel Printout for House Prices:

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	**98.24833**	**58.03348**	**1.69296**	**0.12892**	**-35.57720**	**232.07386**
Square Feet	**0.10977**	**0.03297**	**3.32938**	**0.01039**	**0.03374**	**0.18580**

At 95% level of confidence, the confidence interval for the slope is (0.0337, 0.1858)



Regression Analysis for Description

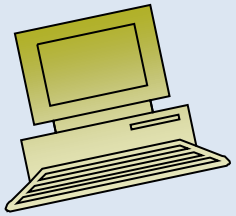
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	**98.24833**	**58.03348**	**1.69296**	**0.12892**	**-35.57720**	**232.07386**
Square Feet	**0.10977**	**0.03297**	**3.32938**	**0.01039**	**0.03374**	**0.18580**

Since the units of the house price variable is $1000, we are 95% confident that the average impact on sales price is between $33.70 and $185.80 per square foot of house size

This 95% confidence interval does not include 0.

Conclusion: There is a significant relationship between house price and square feet at the .05 level of significance

Lecture 7



- Random Numbers
- Checking Normality
- Hypothesis testing

HYPOTHESIS TESTING

HYPOTHESIS

What do you mean by a Hypothesis?

A supposition or explanation that is provisionally accepted in order to interpret certain events or phenomena, and to provide guidance for further investigation.

OR

A hypothesis is a tentative statement about the relationship between two or more variables.

HYPOTHESIS TESTING

Statistically, an assumption about certain characteristics of a population.

The statistical procedure for testing a hypothesis requires some understanding of the null hypothesis and alternative hypothesis.

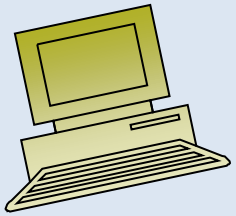
TYPES OF HYPOTHESIS

Null Hypothesis

A null hypothesis is "the hypothesis that there is no relationship between two or more variables. In a mathematical formulation of the null hypothesis there will typically be an equal sign. This hypothesis is denoted by H_0.

Alternate Hypothesis

The alternative hypothesis proposes a relationship between two or more variables. In a mathematical formulation of the alternative hypothesis there will typically be an inequality, or not equal to symbol. This hypothesis is denoted by either H_a or by H_1.



EXAMPLE OF HYPOTHESES

For example, you might have come up with a measurable hypothesis that children have a higher IQ if they eat oily fish for a period of time.

Null hypothesis: Children who eat oily fish for six months do not show a higher IQ.

Alternative hypothesis: Children who eat oily fish for six months will show a higher IQ

A common statistical method is to compare a population to the mean.

H_0 : The children will show no increase in mean intelligence.

i.e.

$H_0 : \ \mu = 100$

H_1 : The children will show an increase in mean intelligence.

i.e.

$H_1 : \ \mu > 100$

TYPES OF ERRORS

What are errors in Hypothesis Testing?

The purpose of Hypothesis Testing is to reject or not reject the Null Hypothesis based on statistical evidence

Hypothesis Testing is said to have resulted in an error when the decision regarding treatment of the Null Hypothesis is wrong

There are basically **two types of errors** we can make:

TYPE I Error

TYPE II Error

TYPES OF ERRORS

Type-I Error (Ho right but rejected)

When Null Hypothesis is rejected despite the test on data showing that the outcome was true

Type-II Error (Ho wrong but not rejected)

When Null Hypothesis is not rejected despite the test on data showing that the outcome was false

Type I Error

- A **type I error**, also known as an **error of the first kind**, occurs when the null hypothesis (H_0) is true, but is rejected.
- A type I error may be compared with a so called ***false positive.***
- The rate of the type I error is called the *size* of the test and denoted by **the Greek letter α (alpha).**
- It usually equals the **significance level of a test**.
- If type I error is fixed at 5 %, it means that there are about 5 chances in 100 that we will reject H_0 when H_0 is true.

Type II Error

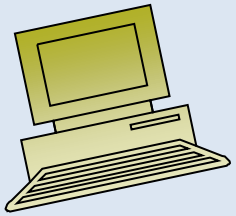
- **Type II error**, also known as an **error of the second kind**, occurs when the null hypothesis is false, but erroneously fails to be rejected.
- Type II error means accepting the hypothesis **which should have been rejected**.
- A type II error may be compared with a so-called ***False Negative.***
- The rate of the type II error is denoted by the **Greek letter β (beta)** and related to the power of a test **(which equals 1-β)**.

In the tabular form two error can be presented as follows:

	Null hypothesis (H_0) is true	Null hypothesis (H_0) is false
Reject null hypothesis	Type I error False decision	Correct True decision
Fail to reject null hypothesis	Correct True decision	Type II error False decision

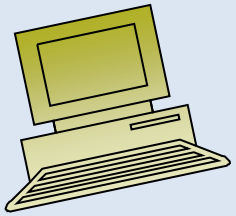
Reducing Type I Errors

The chances of making a Type I error is reduced by increasing the level of confidence.



Reducing Type II Errors

Test condition and acceptance criteria are in turn reduces Type II errors. This increases the number of times we reject the Null hypothesis – **with a resulting increase *in the number of Type I errors*.**



Type II Error and Power

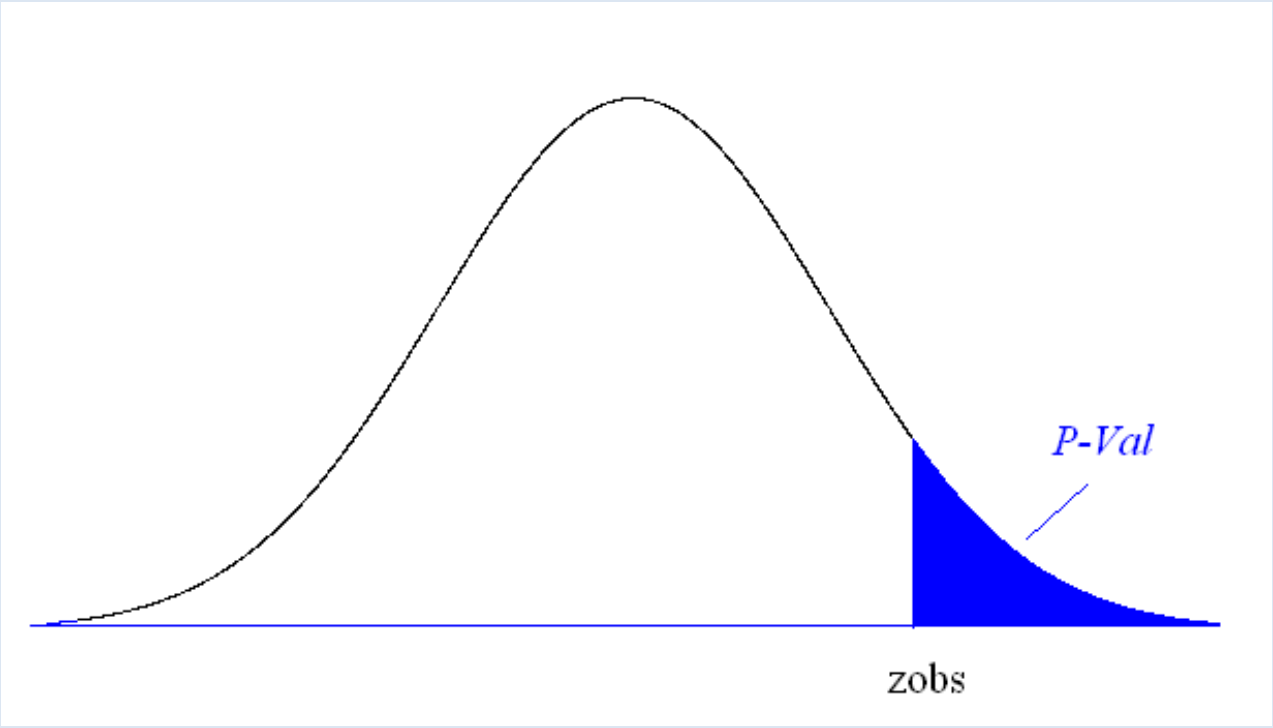
- "**Power**" of a test is the probability of rejecting null when alternative is true.
- **"Power" = 1 - P(Type II error)**
- To minimize the P(Type II error), we equivalently want to maximize power.
- But power depends on the value under the alternative hypothesis .

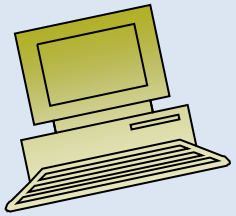
P-value

***P*-value -** Measure of the strength of evidence the sample data provides against the null hypothesis:

P(Evidence This strong or stronger against H_0 | H_0 is true)

$$P-val: p = P(Z \geq z_{obs})$$





INTERPRETING RESULTS

Interpreting the weight of evidence against the Null Hypothesis for rejecting / not rejecting Ho

If the p-value for testing Ho is less than –

- **< 0.10**, we have some evidence that Ho is false
- **< 0.05**, we have strong evidence that Ho is false
- **< 0.01**, we have very strong evidence that Ho is false
- **< 0.001**, we have extremely strong evidence that Ho is false

Simple and Composite Hypothesis

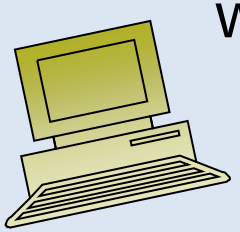
A **simple hypothesis** is one in which all parameters of the distribution are specified. For example, if the heights of college students are normally distributed with , $\sigma^2 = 4$ the hypothesis that its mean μ is, say, $62"$, that is $H : \mu = 62$, we have stated a simple hypothesis, as the mean and variance together specify a normal distribution completely.

Simple and Composite Hypothesis

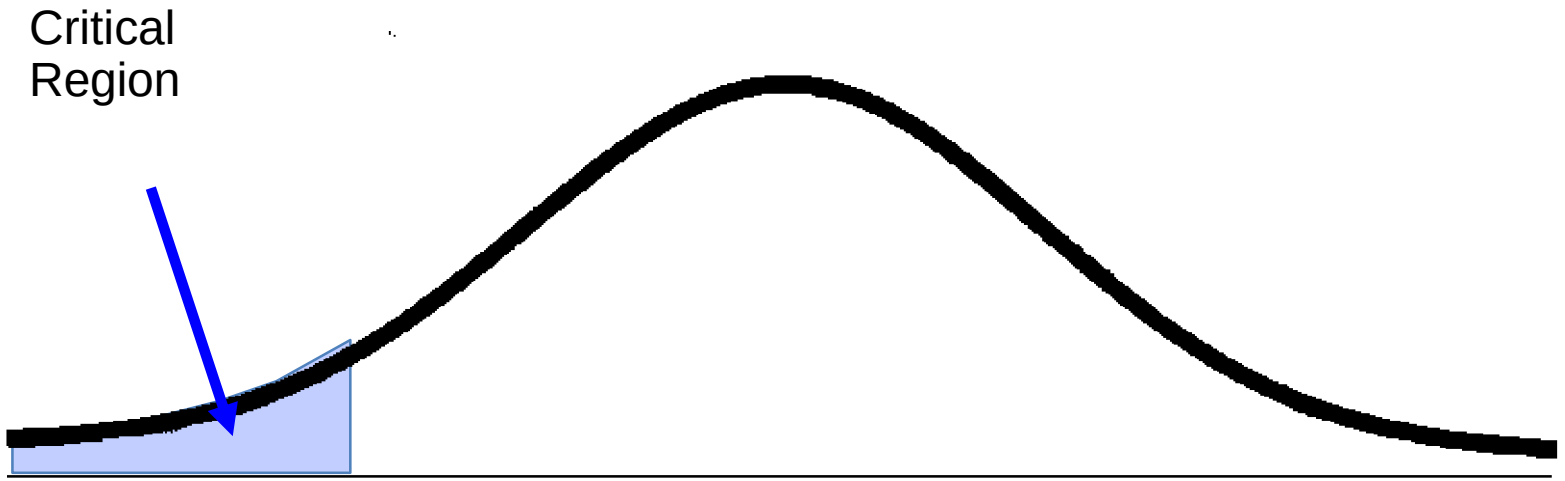
A hypothesis which is not simple (i.e. in which not all of the parameters are specified) is called a **composite hypothesis**. For instance, if we hypothesize that $H : \mu > 62$(and $\sigma^2 = 4$) or $H : \mu = 62$ and $\sigma^2 < 4$, the hypothesis becomes a composite hypothesis because we cannot know the exact distribution of the population in either case. Obviously, the parameters $\mu > 62"$ and $\sigma^2 < 4$ have more than one value and no specified values are being assigned.

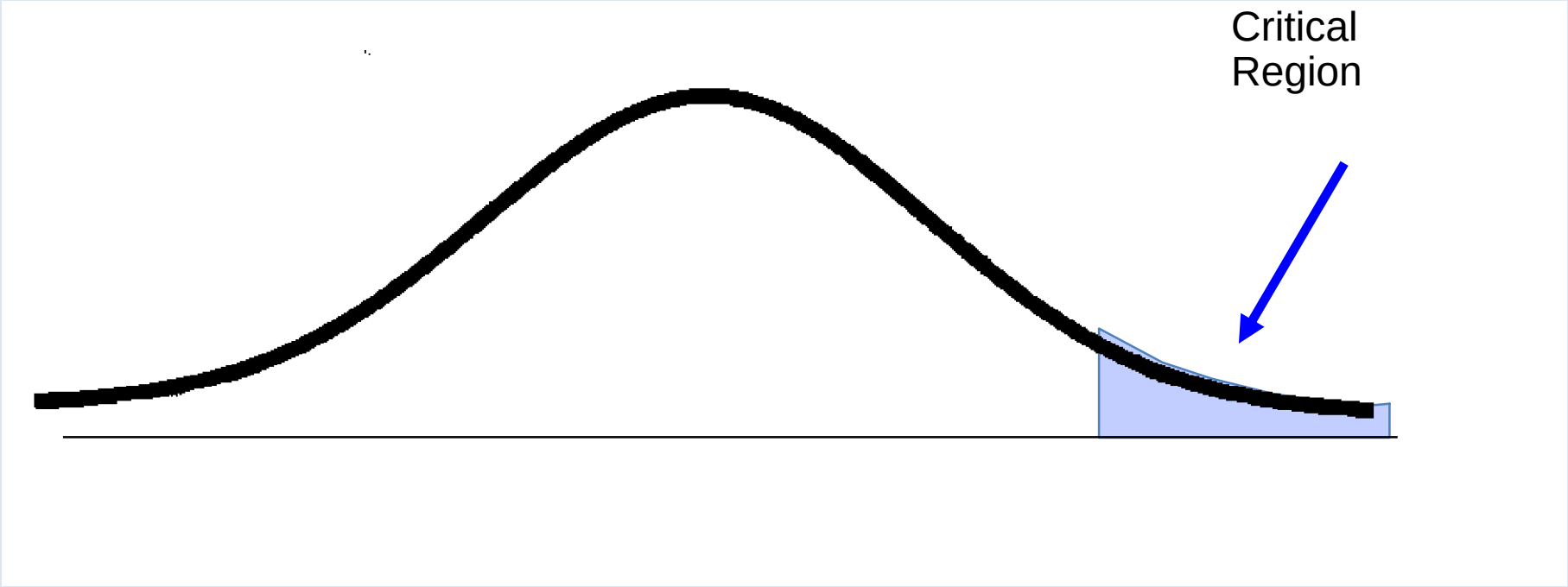
Critical Region (or Rejection Region)

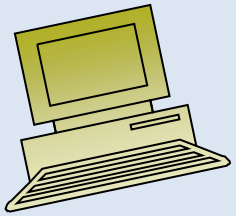
The critical region CR, or rejection region RR, is a set of values of the test statistic for which the null hypothesis is rejected in a hypothesis test. That is, the sample space for the test statistic is partitioned into two regions; one region (the critical region) will lead us to reject the null hypothesis H0, the other will not. So, if the observed value of the test statistic is a member of the critical region, we conclude "Reject H0"; if it is not a member of the critical region then we conclude "Do not reject H0".

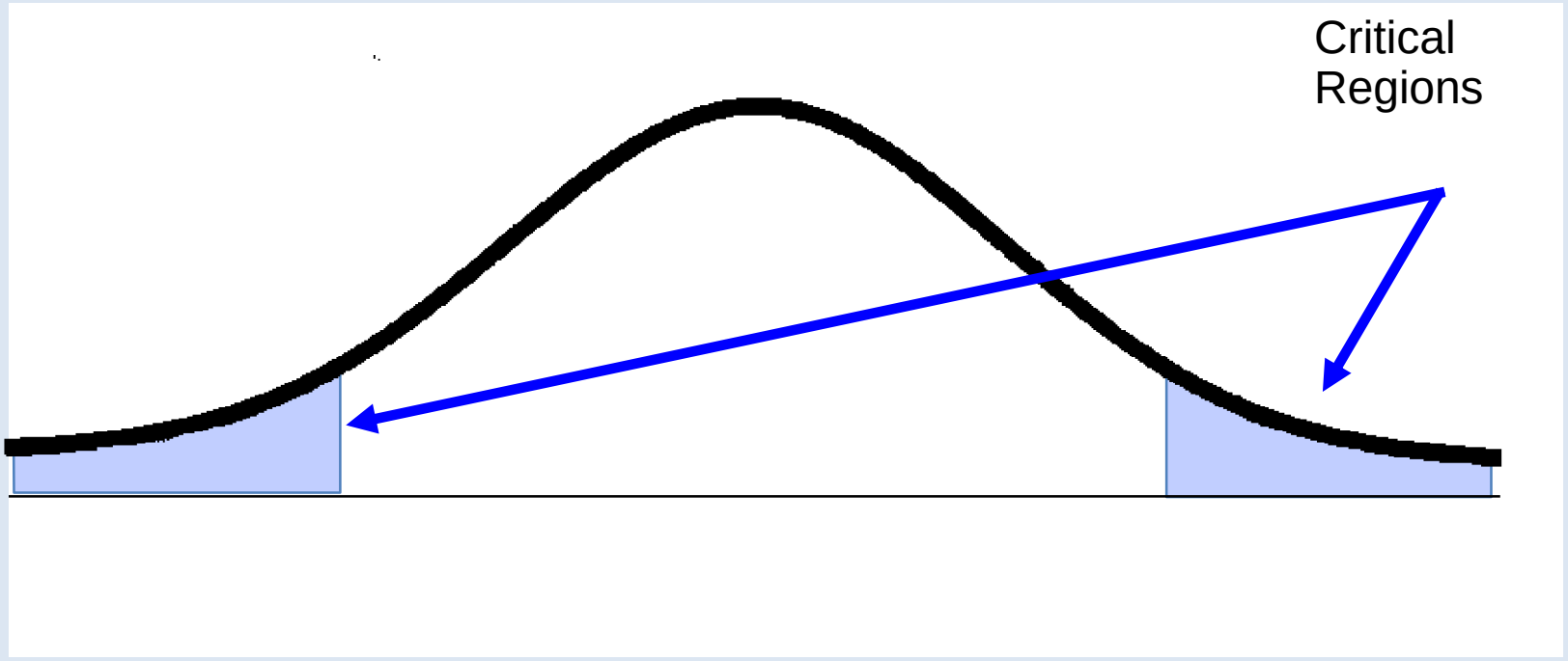


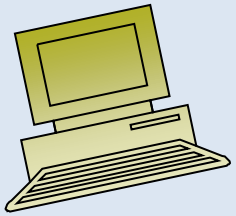
Critical Region




Critical
Region

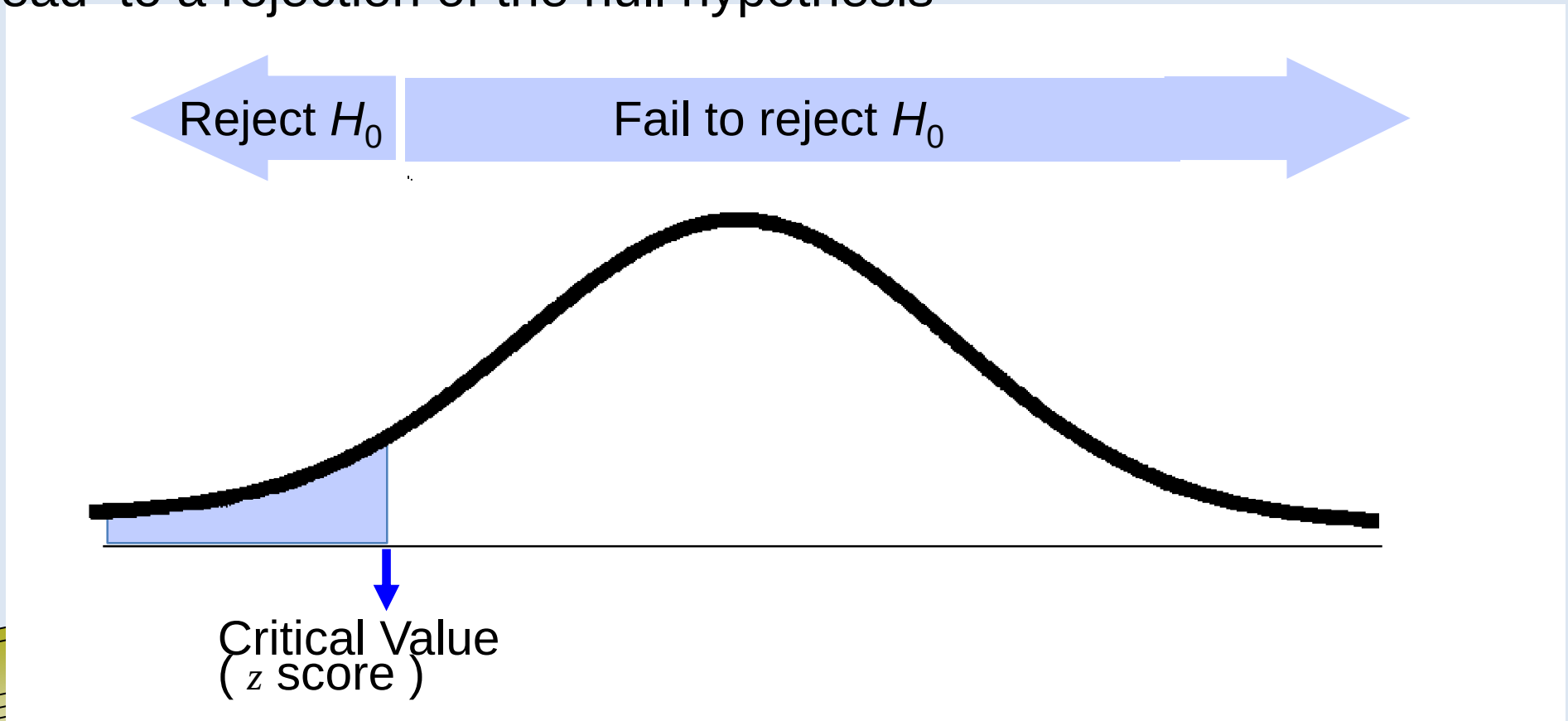



Critical
Regions



Critical Value

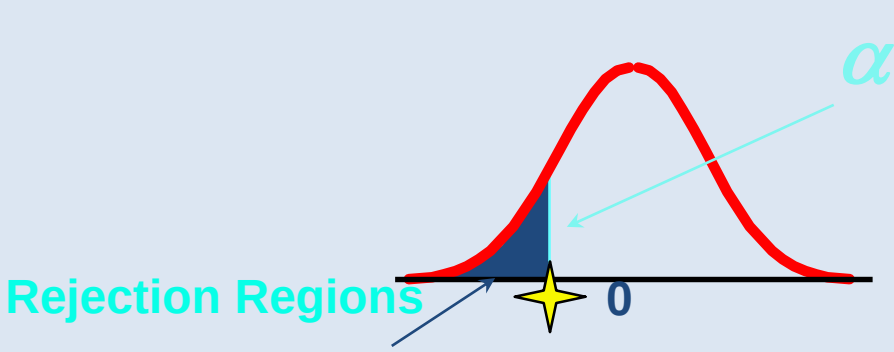
Any value that separates the critical region (where we reject the null hypothesis) from the values of the test statistic that do not lead to a rejection of the null hypothesis



Level of Significance, α and the Rejection Region

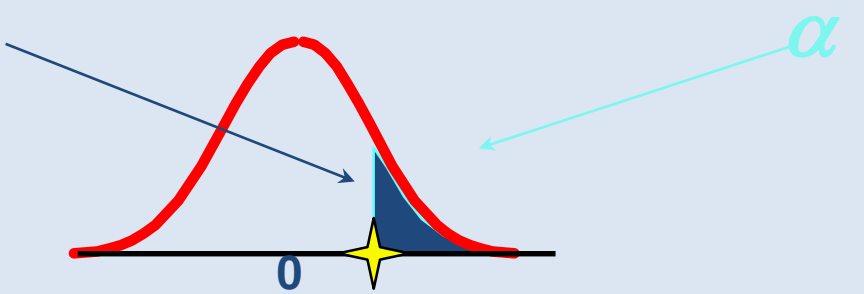
$H_0: \mu \geq 3$

$H_1: \mu < 3$



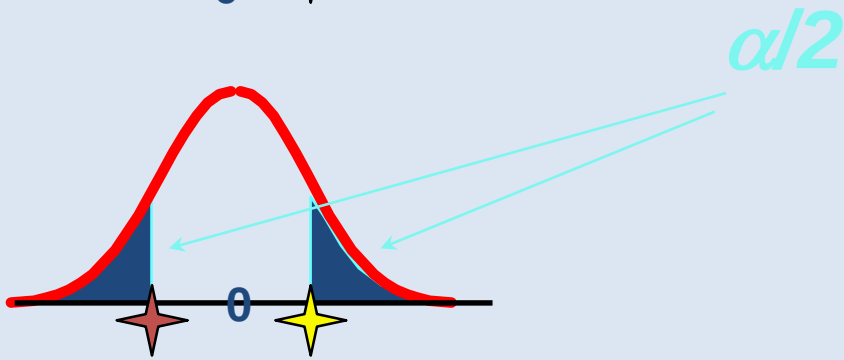
$H_0: \mu \leq 3$

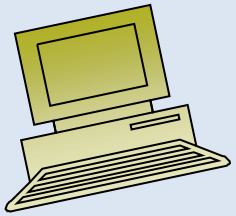
$H_1: \mu > 3$



$H_0: \mu = 3$

$H_1: \mu \neq 3$





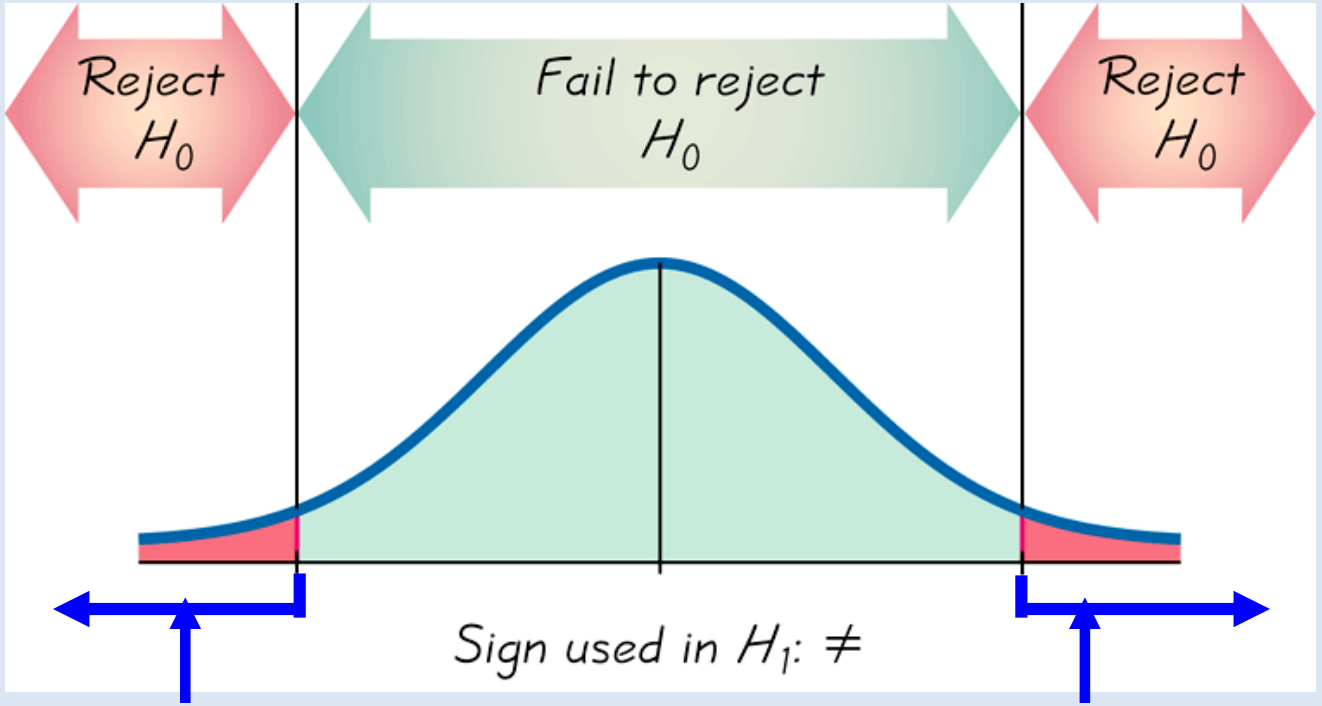
Two-tailed Test

H_0: =

H_1: ≠

Means less than or greater than

α is divided equally between the two tails of the critical region



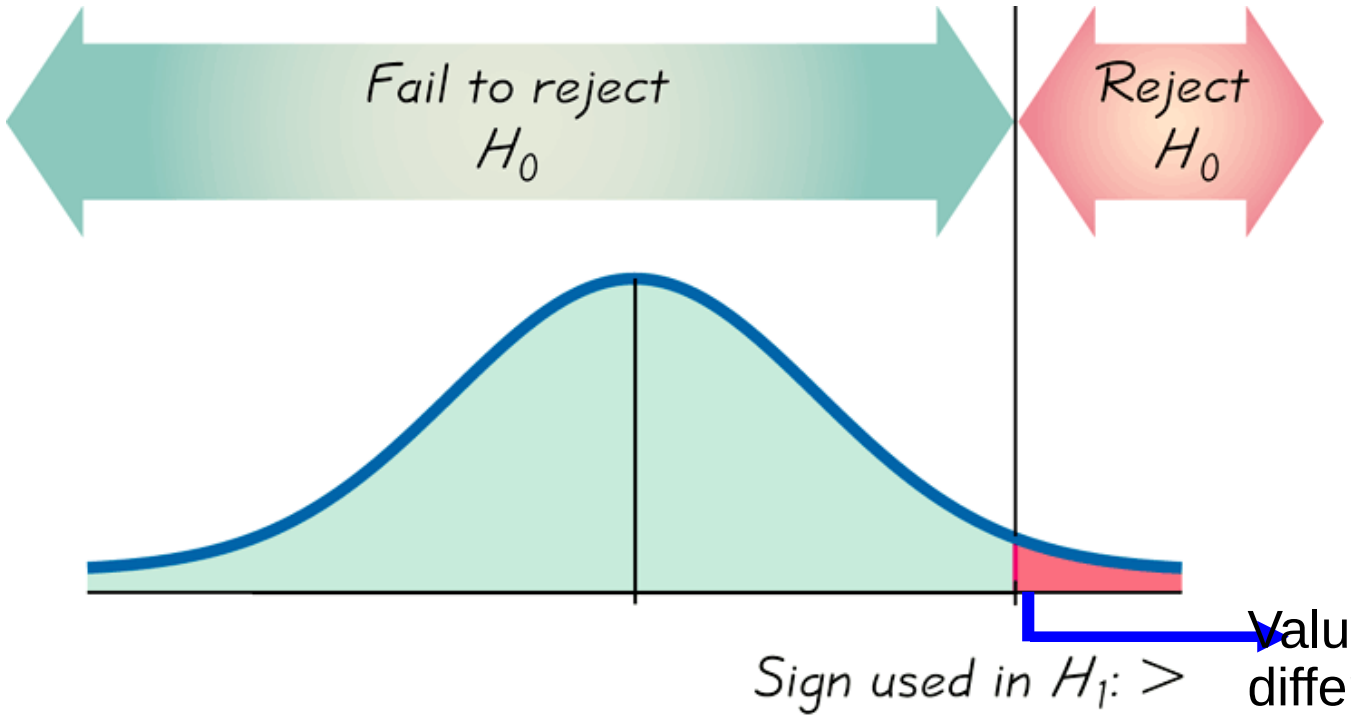
Values that differ significantly from H_0

Right-tailed Test

H_0: =

H_1: >

Points Right

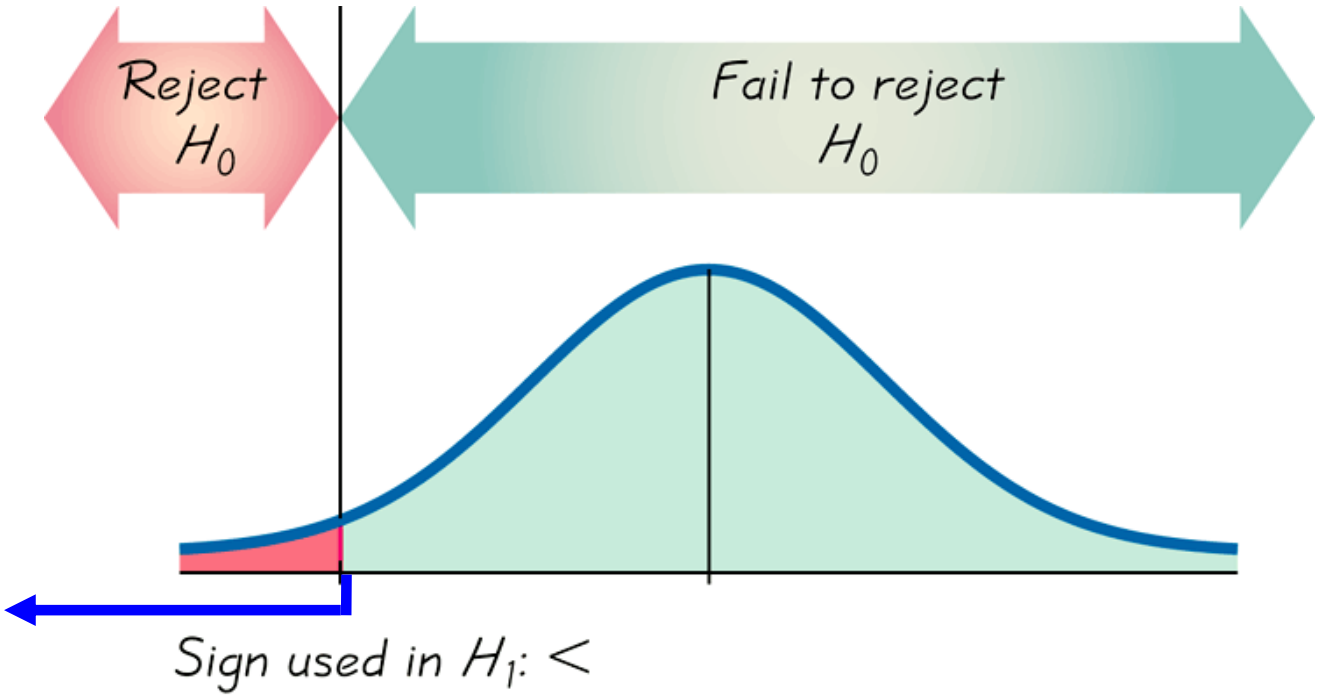


Left-tailed Test

H_0: =

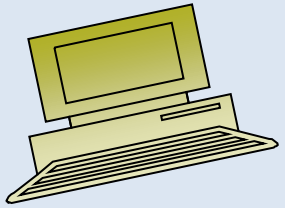
H_1: <

Points Left



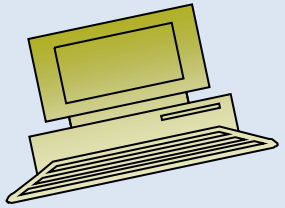
Values that differ significantly from H_o

Lecture 9

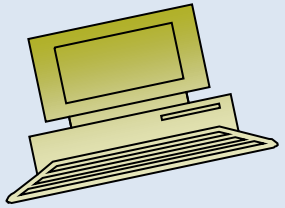


Contents

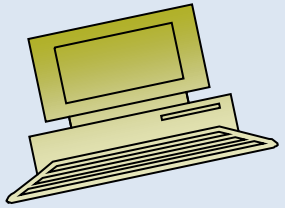
- Non parametric Test
  - ✓ Wilcoxon Signed-Rank Test for a Single Sample
  - ✓ Mann-Whitney Test for Two Independent Samples
  - ✓ Wilcoxon Signed-Rank Test for Paired Samples
- Matrix
  - ✓ Transpose
  - ✓ Determinant
  - ✓ Inverse
  - ✓ Eigen Values



How to Use Excel to Apply Non-Parametric Tests



How to Use Excel to Solve a Matrix



An Introduction to IBM SPSS

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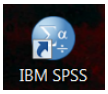
Beginning an SPSS Session	3
Basic Information	4
Entering New Data	7
Example	9
Saving Data	11
Opening Existing Data	12

Beginning An SPSS Session

Begin by opening SPSS 10 for Windows.

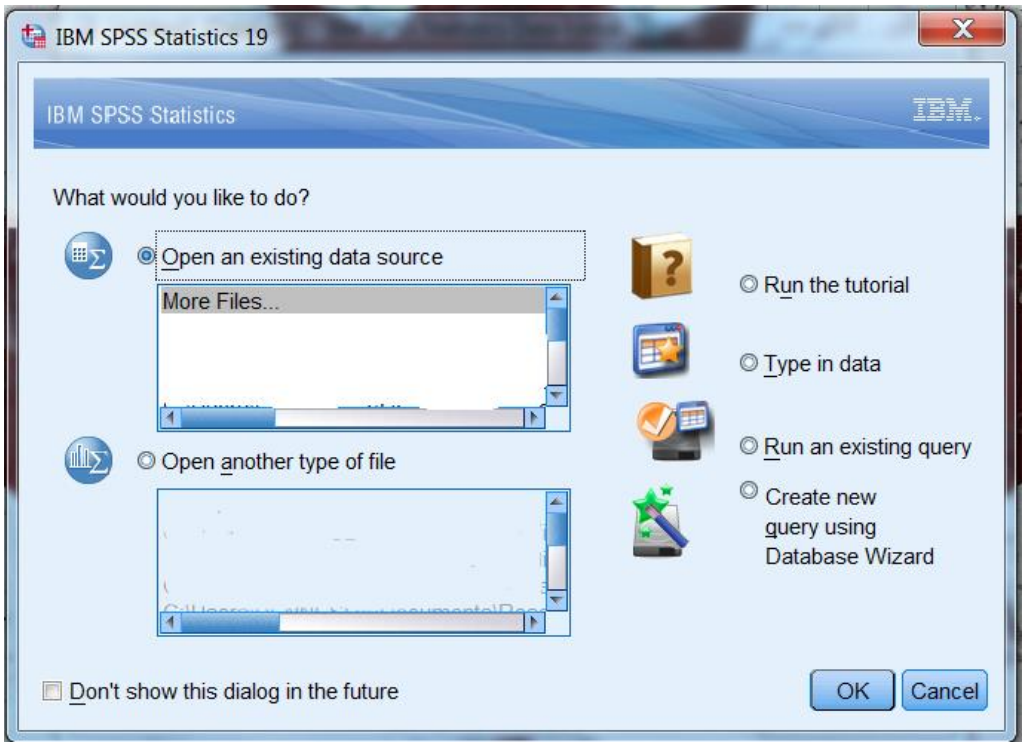


1. Click on the *IBM SPSS* shortcut button on your desktop.

OR

2. Go to *START*, click on *PROGRAMS*, and click on *IBM SPSS* .

Below is an illustration of how the first page should appear.



The three most common options at this point are:

a. Run a tutorial – this will take you through a tour of how to use SPSS 10

b. Type in data – this is used to begin entering a new data set

c. Open an existing data source – this is used to work with an existing data set

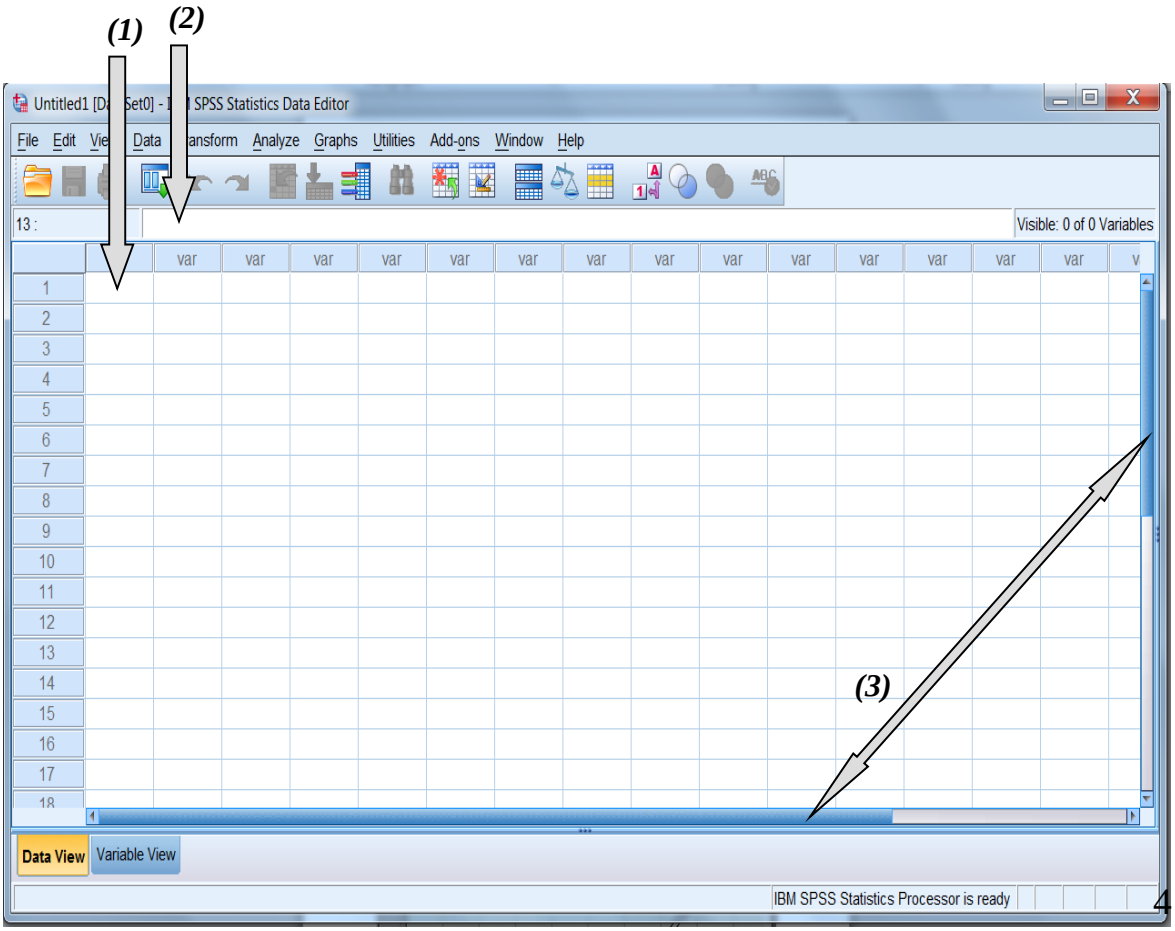
Basic Information

Each little box is called a "cell" ***(1)***. To access a cell, simply move the mouse pointer to the cell and click the left mouse button.

Once you have entered data in a cell, you can either hit ENTER, or one of the direction arrows.

To modify information in a cell, you can also use the data view "window" ***(2)***. Click on the cell you want to modify and then click the mouse pointer in the data view window (the data presently in the cell should appear once you click on the window). Now you move around in the data by using the direction arrows to delete all or some of the information (by using the DELETE button) or add new information.

Along the side and bottom of SPSS, you will find "scroll bars" ***(3)*** that allow you to move the view up/down or left/right (by clicking the appropriate arrows at the ends of the scroll bars).



Basic Information

Above the data cells, you will find the toolbar ***(1)*** and the menu bar ***(2)***. Each provides the functions you will need to open/save data, perform transformations, and conduct your statistical operations.

Toolbar:

The toolbar contains icons to facilitate easy point and click operations. Because these can be customized, they may vary slightly between systems. Below is a description of the main icons:

Open files

Save data

Print file

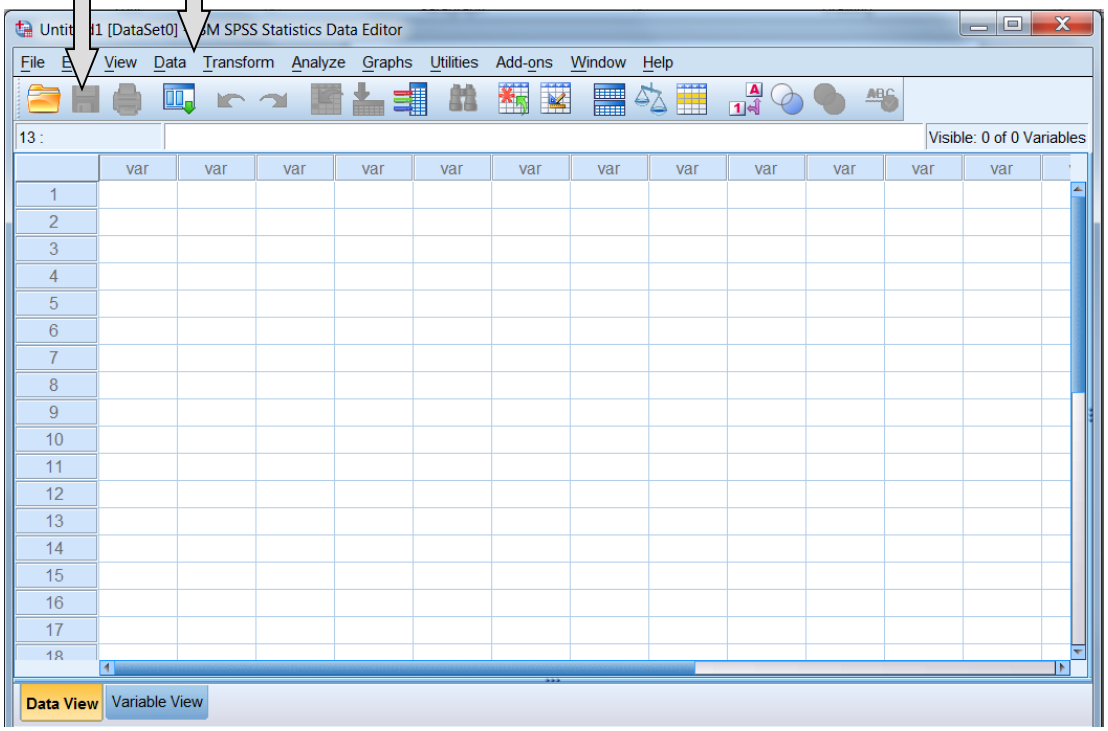
Find data

Undo last operation

Redo last operation

Insert case

Insert variable



Basic Information

Menu Bar:

The menu bar provides a series of "drop down" commands to perform most essential SPSS functions. By clicking on a menu command, a further series of menu options will appear. Many of these submenu commands will be discussed in further detail in this manual.

File: These are the basic file management operations.

e.g. opening, saving, and printing files

Edit: This allows you to perform editing functions on the current data set.

e.g. cut, copy, clear, undo changes and redo changes

View: Allows you to change the current view of data, as well as toolbar options.

e.g. grid lines, value labels

Data: These functions deal with the configuration, defining, and management of data.

e.g. insert variables/cases, sort data, merge files

Transform: This allows you to transform the data set you've entered.

e.g. calculating new variables, recoding, missing values

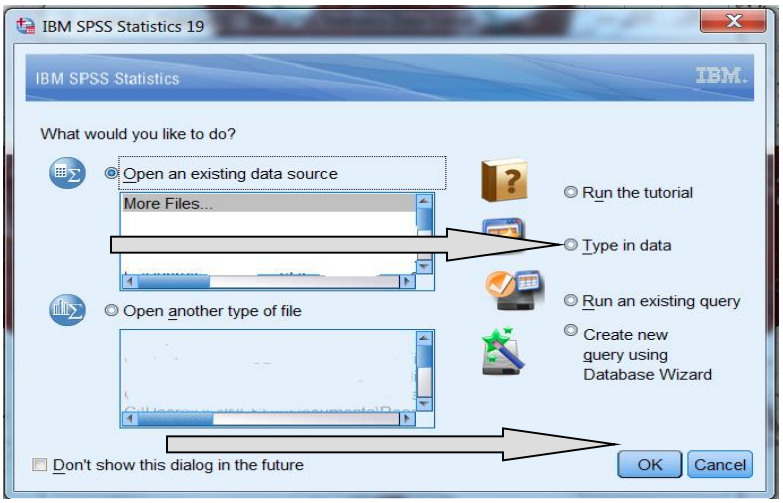
Analyze: Includes the main data analysis functions.

e.g. descriptive statistics, t-Tests, ANOVA, correlation, data reduction

Windows: Allows you to alter the appearance, format, position of the SPSS windows.

Entering New Data

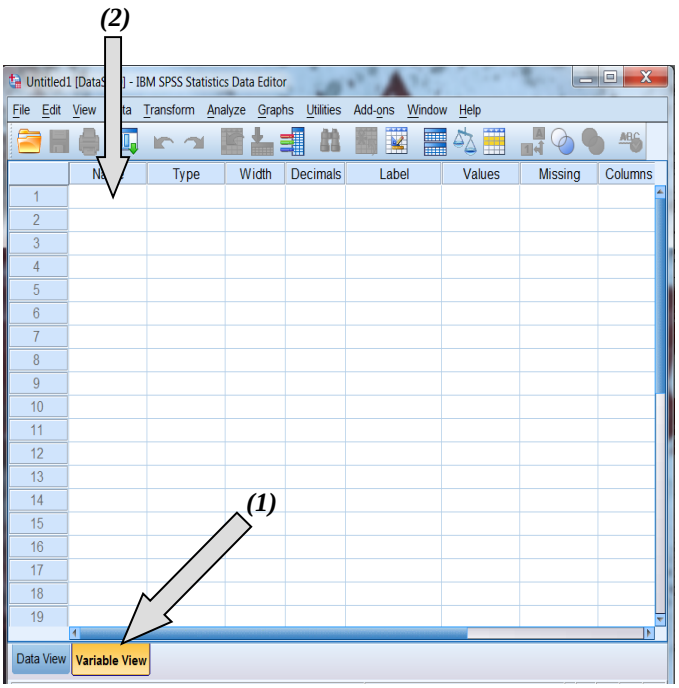
To begin, click on the *"Type in data"* option from the opening SPSS for Windows menu.



Then click *"OK"*.

Before you can enter any data, you have to ***define your variables***.

To do this, we click on the *VARIABLES VIEW* tab *(1)* from the bottom left corner.



Each row represents a different variable that you will use in your analysis.

At the very least you need to enter a name for each variable. It is always good to use a name you might remember.

However, SPSS doesn't let you use blanks in the name.

To enter a variable name, click on the "name" cell *(2)* and type in the name of the variable.

Repeat this for each variable you will have data for.

You can also change the options available for your data at this time (although you can come back at any time simply by clicking the *VARIABLE VIEW* tab.

At this point the options that you might consider are as follows:

TYPE:

- The default is numeric data.
- This will allow you to change from numeric data to other formats.
- You can change formats by clicking the cell, then clicking the three dots in the right corner of the cell.
- The most common format change is to "string" data, which will allow you to enter words rather than numbers.

WIDTH

- This allows you to set the maximum number of digits (or letters in a string format) you can enter in cell.
- The decimal and all decimal points count as digits in the width.

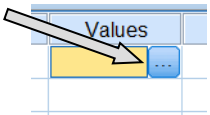
DECIMALS

- This indicates how many decimal points you can have in your cell.

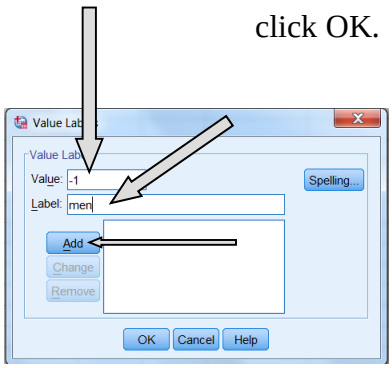
LABELS

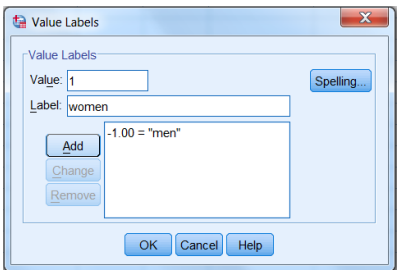
- You can enter a longer description of the variable here.

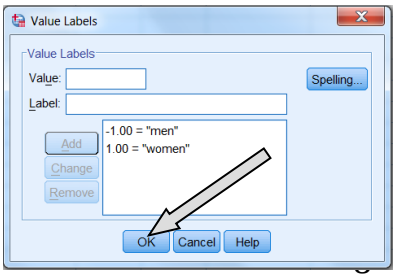
VALUES

- You can tell SPSS what the values (mainly for categorical data like gender) actually represent. For example if you have a code of "-1" for men and "1" for women you would **click the "values" box**, then the "dots": 

.Now in the new box (see below), you enter the value (-1) and label (men) then click "Add". You will now see the value and label in the big box. Repeat this for all values & labels & click OK.







EXAMPLE 1:

Suppose we have collected a list that contains both a person's weight and their height.

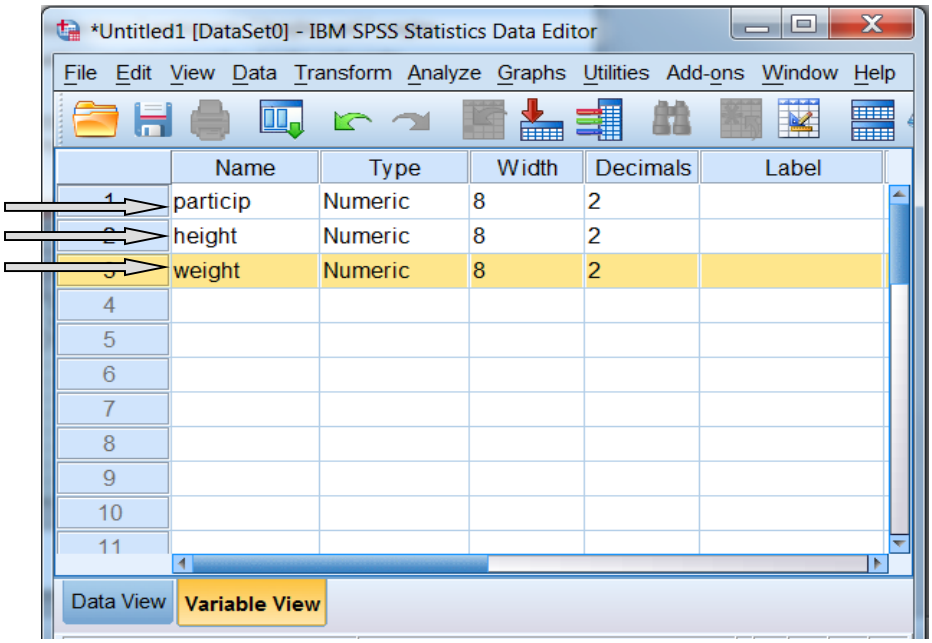
Each person has a different measurement of height and weight (see Table below).

	Height	Weight
Participant 1	79	200
Participant 2	79	185
Participant 3	51	111
Participant 4	49	85
Participant 5	55	117
Participant 6	64	125
Participant 7	66	129

From this set of data, we see that we have 7 participants, 7 heights, and 7 weights.

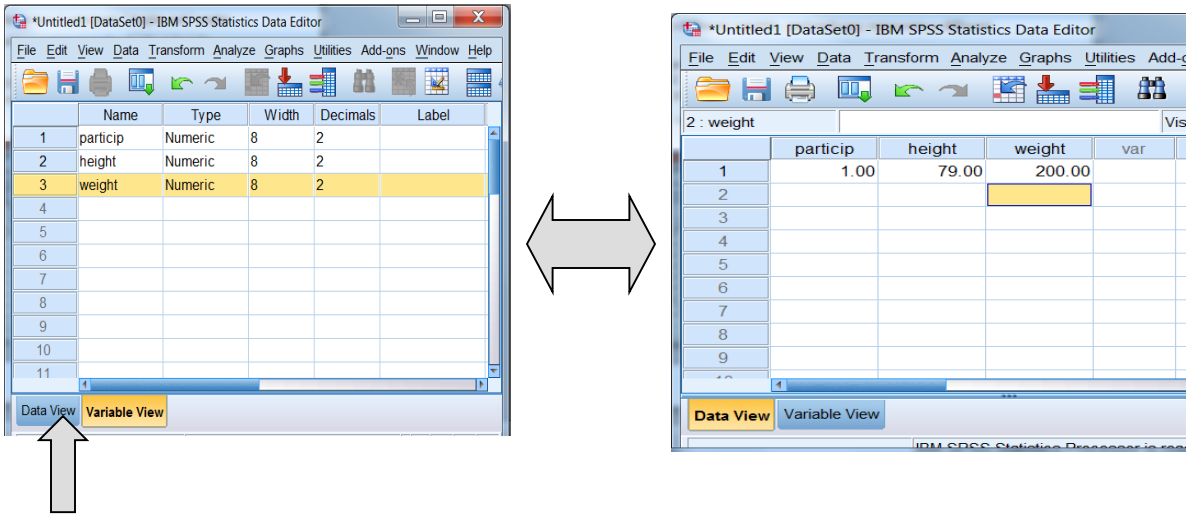
In other words, we have 3 variables – participant number, height, and weight.

In the variable box, type PARTIC (remember only 8 characters maximum) then arrow down to the next variable and type HEIGHT, then arrow down again and type WEIGHT.



You are now ready to begin entering the actual data.

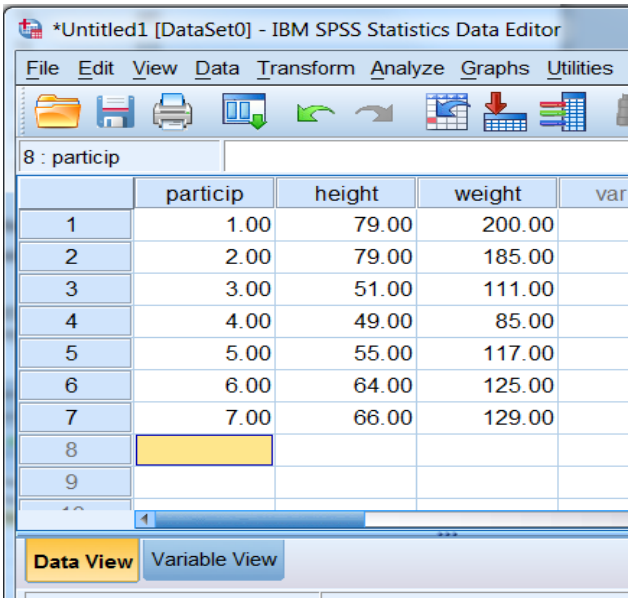
Click on the *DATA VIEW* tab (next to the *VARIABLE VIEW* tab) at the bottom at the bottom left corner.



Each <u>column</u> now represents a different variable, and each <u>row</u> represents a different participant.

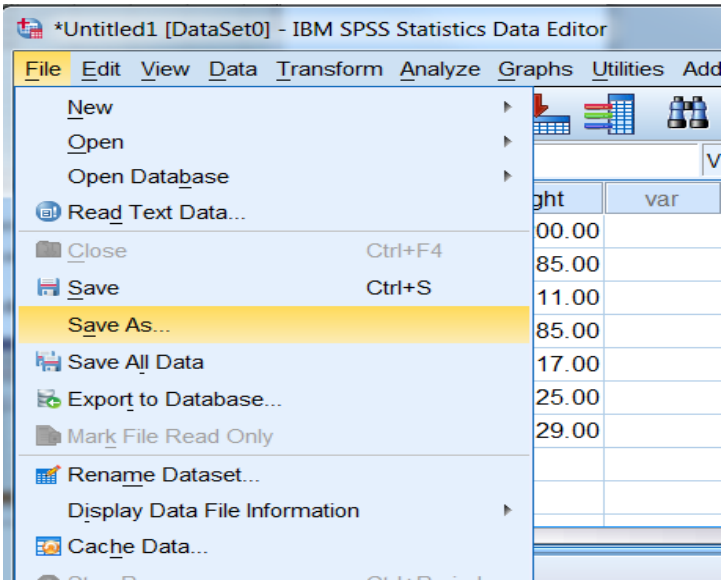
With the present example, begin by entering the values for participant 1 as shown above. Type in "1" for PARTIC value, then arrow to the right, type in 79 for HEIGHT, arrow to the right and type in 200 for WEIGHT.

Then move on to Participant 2 and continue. The final data set should look as follows:



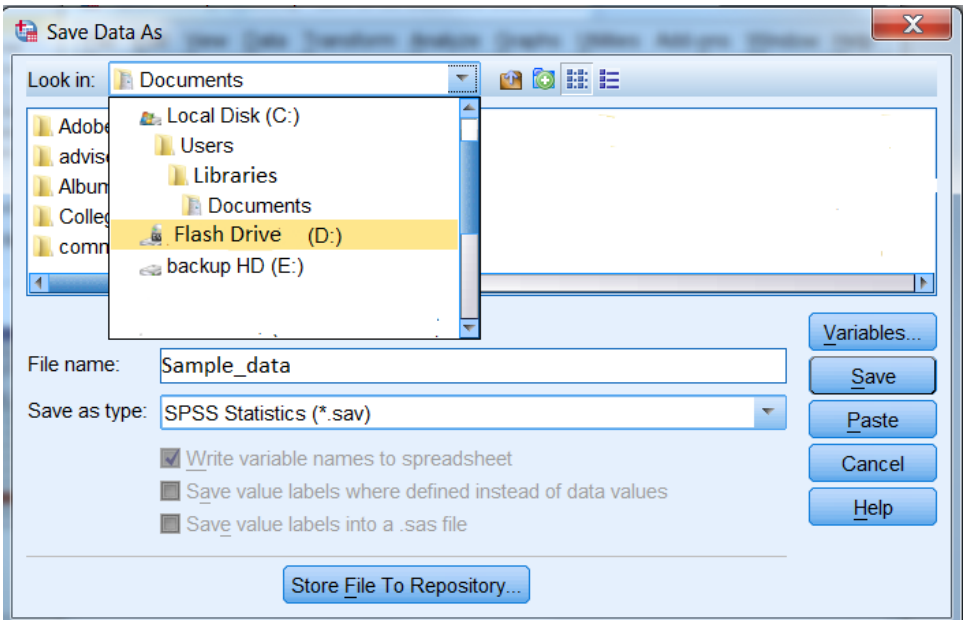
The final step in entering new data is to save your work.

Click on *FILE*, then *SAVE AS*...



Save to flash drive so choose whatever drive represents your flash (D: in this case).

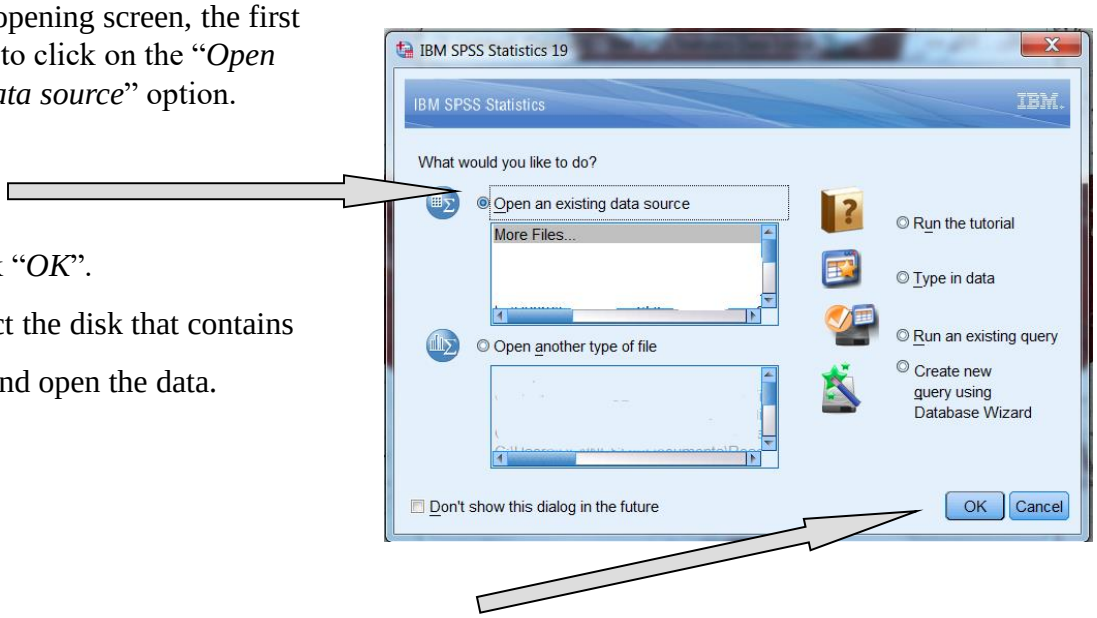
Finally, enter a name (sample_data) for your file and hit save .



Opening Existing Data

There are two ways of accessing existing data from a saved file.

From the opening screen, the first method is to click on the "*Open existing data source*" option.

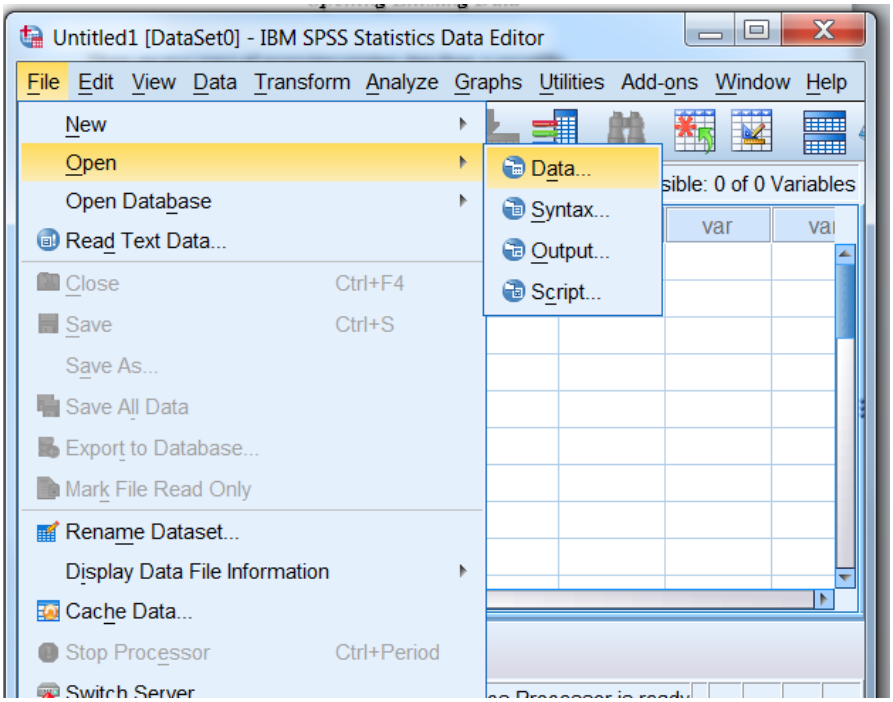


Then click "*OK*".

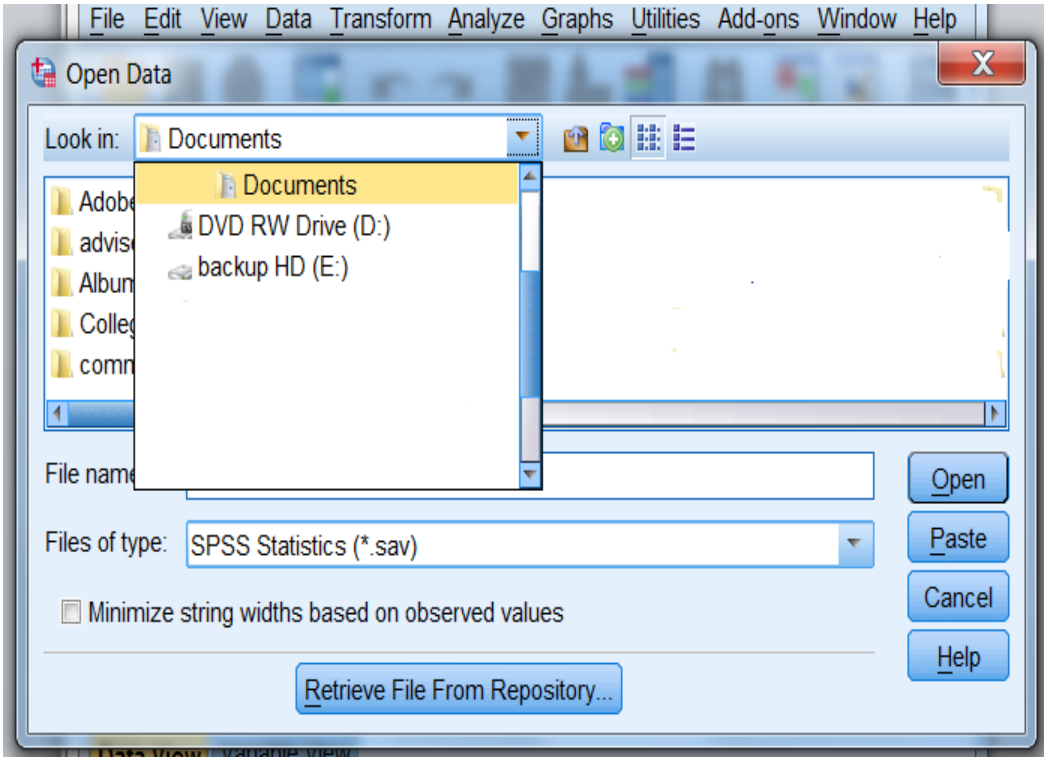
Then select the disk that contains the data, and open the data.

You can also access saved data when you are already in the data screen.

Click on *FILE*, *OPEN*, then *DATA*.



Now simply go to the drive that has your data, highlight the data file and click Open.



At this point, you are free to add new variables, remove old variables, change existing data, or add new cases.

When you are finished, you can save your data as a new file (i.e. new name) by using Save As, or simply overwrite your existing file by using the Save function.

***A word of warning regarding the Save function. If you use Save, you automatically overwrite anything you had in the file before the changes. Ask yourself first if this is what you really want. There are times when people have accidentally hit the save button on the toolbar and overwritten information they did not want to lose. Its always a safe bet to go to File and use Save As.*

Moving Existing Data

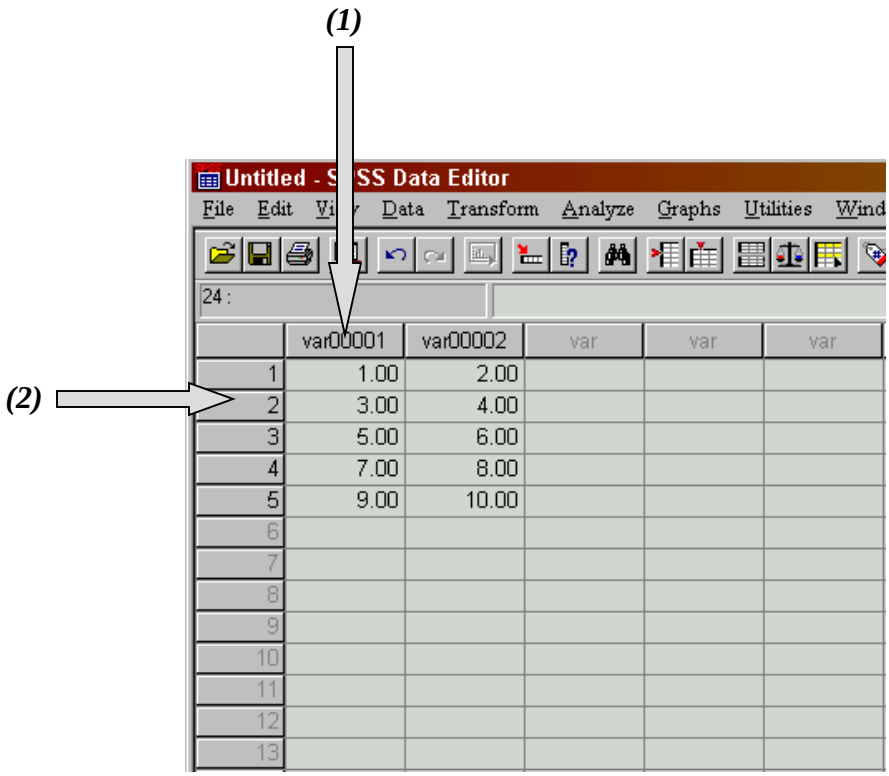
Cutting and Pasting:

You can move a variable, a case, or a data cell.

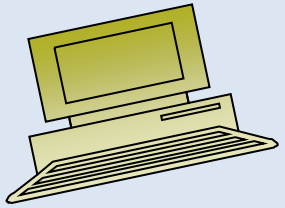
To move a variable to the end of your current data, click on the variable name ***(1)*** across the top of the data set. The column should turn dark. Now click on cut (to completely remove) or copy (to make a copy). Click on a new column at the end of your data set (it should turn dark) and click paste. The new data should appear in that column.

The same applies to moving a particular case to the end of your data set. Click on the row you want to move ***(2)*** (i.e. on the case number along the left edge of the data set), click cut or copy, click on a new row, then click paste.

When moving an individual cell, click on the cell, click cut or copy, click on the new cell, then click paste. Note, when you cut a cell, the data will be replaced with a "." which indicates missing data.

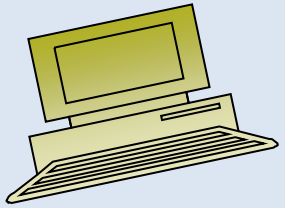


Lecture 11

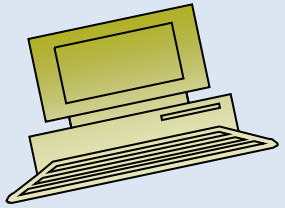


Contents

- Data Entry and importing data from excel
- Measure of Central Tendency
- Mean
- Median
- Mode
- Grouping Data
- Frequency Distribution
- Practical exercise in Data Preparation.



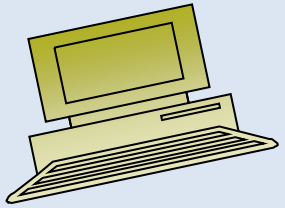
How to Use Excel to Find the Mean, Median & Mode



Objectives

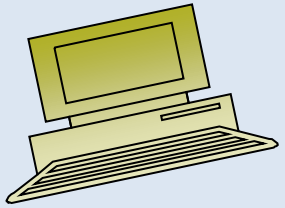
After completing this lecture, you should be able to:

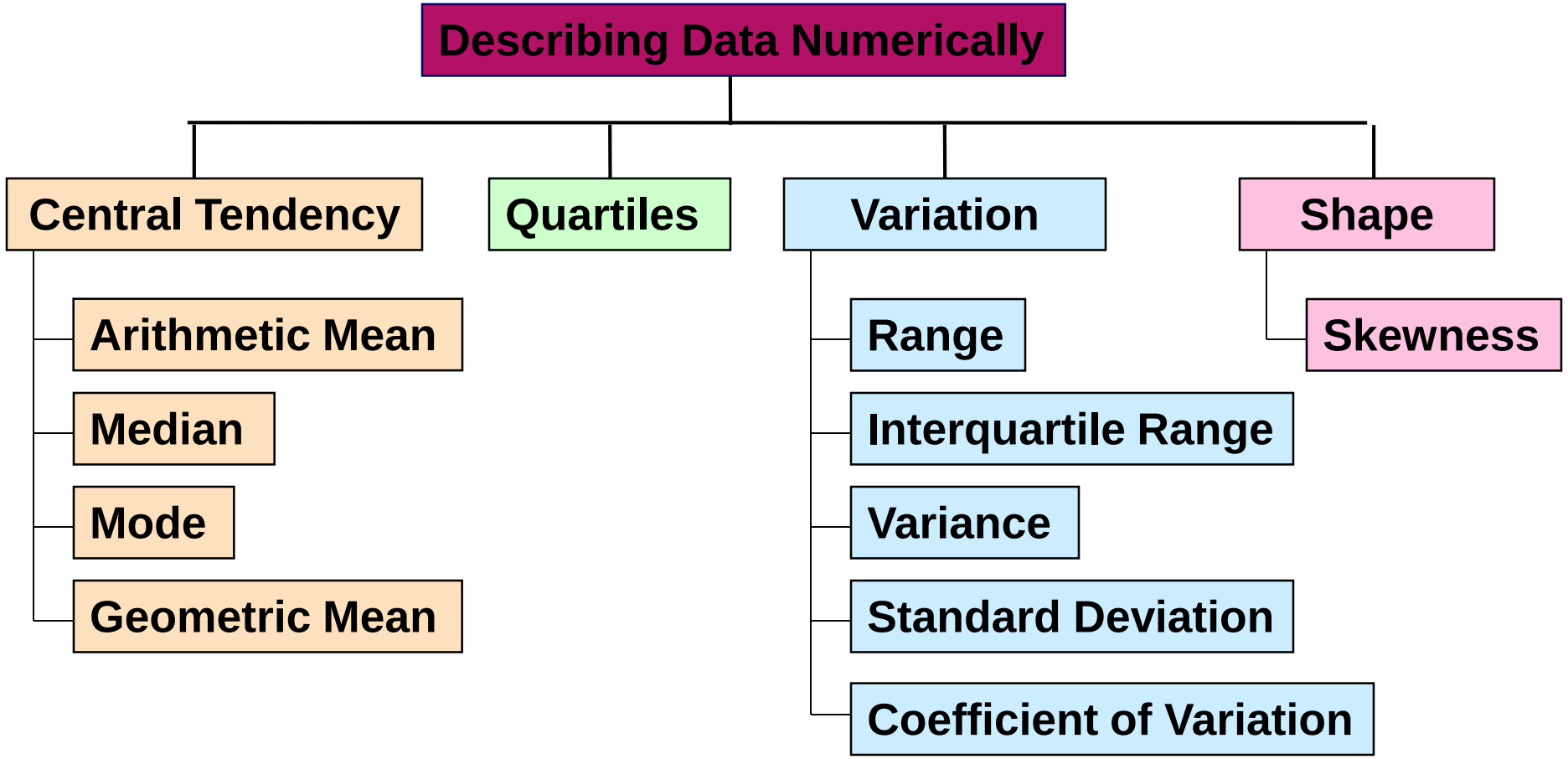
- Compute and interpret the mean, median, and mode for a set of data.
- Group data and frequency distribution.



Central Tendency:

- In statistics a central tendency (or, more commonly, a measure of central tendency) is a central or typical value for a probability distribution. It may also be called a center or location of the distribution.
- The most common measures of central tendency are the arithmetic mean, the median and the mode.
- A central tendency can be calculated for either a finite set of values




Describing Data Numerically
Central Tendency
Arithmetic Mean
Median
Mode
Geometric Mean
Quartiles
Variation
Range
Interquartile Range
Variance
Standard Deviation
Coefficient of Variation
Shape
Skewness

Measures of Central Tendency

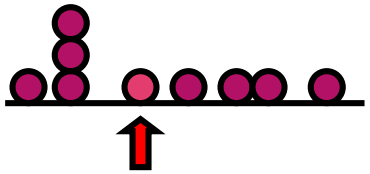
Overview

Central Tendency

Arithmetic Mean

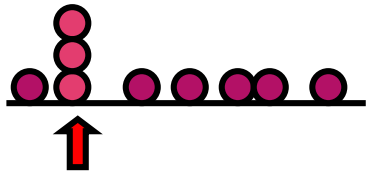
$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

Median



Midpoint of ranked values

Mode



Most frequently observed value

Geometric Mean

$$\bar{X}_G = (X_1 \times X_2 \times \cdots \times X_n)^{1/n}$$

Arithmetic Mean

- The arithmetic mean (mean) is the most common measure of central tendency
  - For a sample of size n:

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

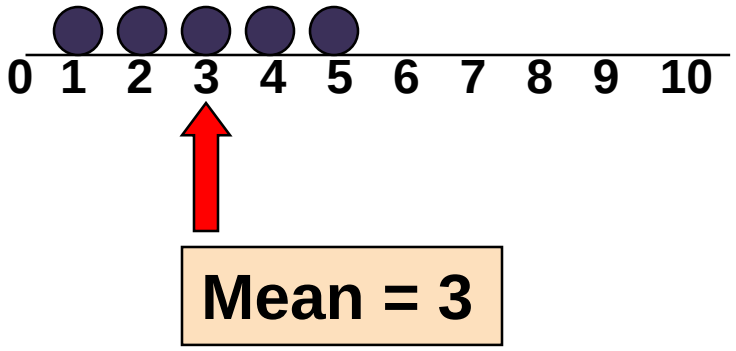
Sample size

Observed values

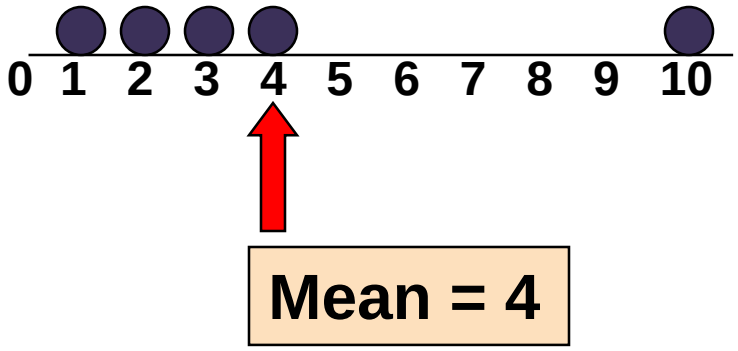
Arithmetic Mean

(continued)

- The most common measure of central tendency
- Mean = sum of values divided by the number of values
- Affected by extreme values (outliers)



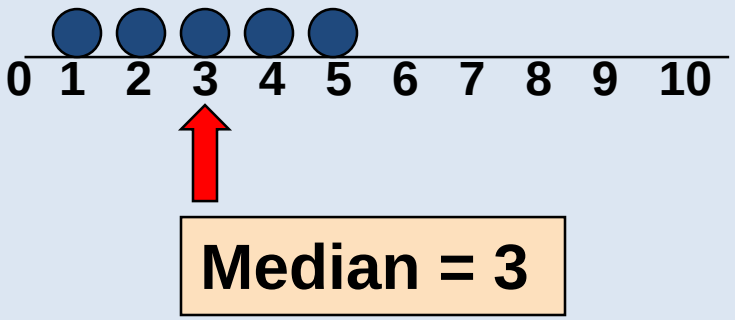
$$\frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$$

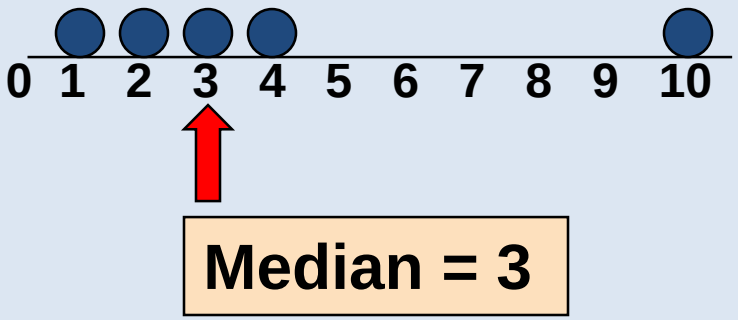


$$\frac{1+2+3+4+10}{5} = \frac{20}{5} = 4$$

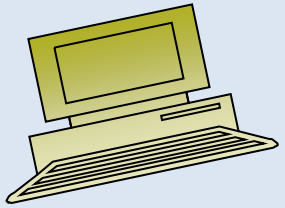
Median

- In an ordered array, the median is the “middle” number (50% above, 50% below)





- Not affected by extreme values



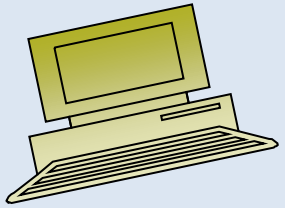
Finding the Median

- The location of the median:

$$\text{Median position} = \frac{n+1}{2} \text{ position in the ordered data}$$

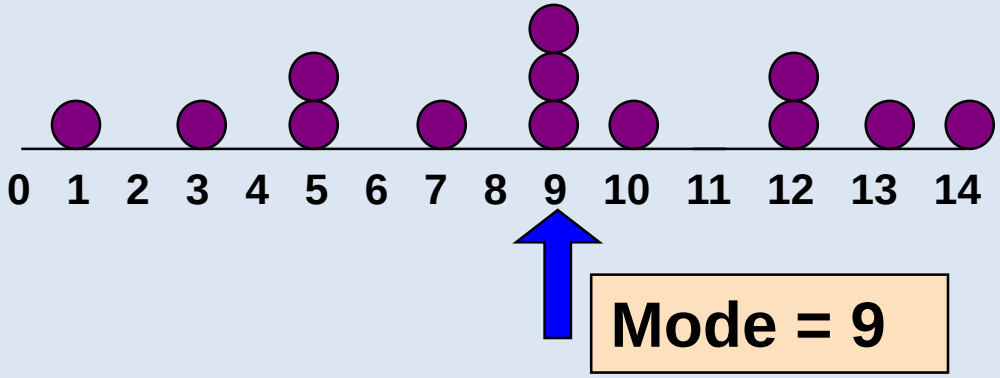
  - If the number of values is odd, the median is the middle number
  - If the number of values is even, the median is the average of the two middle numbers

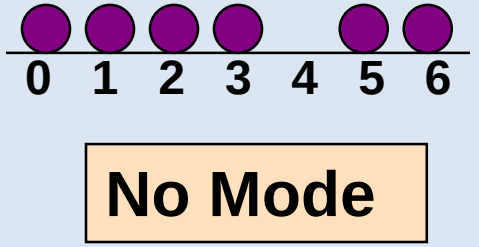
- Note that $\frac{n+1}{2}$ is not the value of the median, only the position of the median in the ranked data

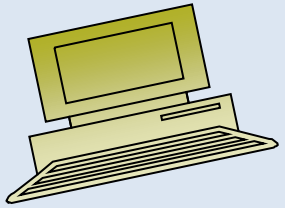


Mode

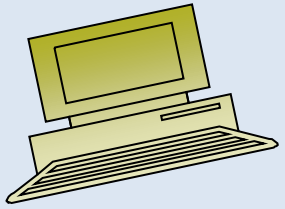
- A measure of central tendency
- Value that occurs most often
- Not affected by extreme values
- Used for either numerical or categorical data
- There may may be no mode
- There may be several modes



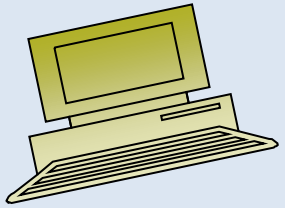




Practical Exercise in Data Preparation

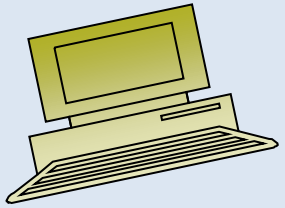


Lecture 12

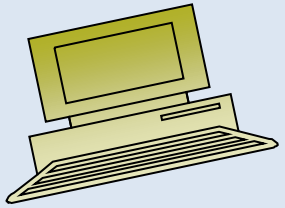


Contents

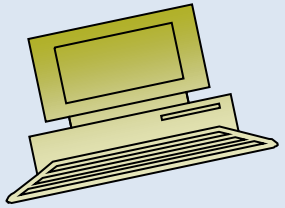
- Measure of Dispersion
- Skewness
- Kurtosis
- Graphs
- Customizing tables
- Practical exercise in Data Preparation.



Practical Exercise in Data Preparation

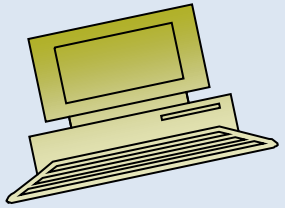


Lecture 13



Contents

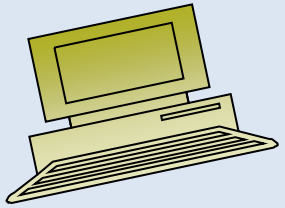
- Graphs
- Quality control charts
- Practical exercise in Data Preparation.



Control Charts

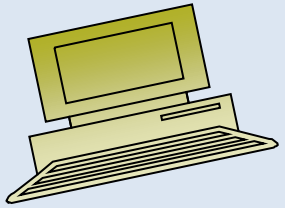
originally developed by

Walter A. Shewhart



History - beginnings

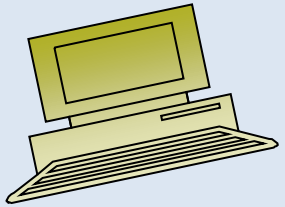
- The control chart - invented by Walter Shewhart
- How to improve the reliability of telephony transmission systems?
- Shewhart framed the problem in terms of common and special causes of variation
- In 1924, May 16, in an internal memo Shewhart introduced the control chart as a tool for distinguishing between the two causes of variation.
- Shewhart's boss, George Edwards: "Dr. Shewhart prepared a little memorandum only about a page in length… all of the essential principles and considerations which are involved in what we know today as process quality control."



Control Charts

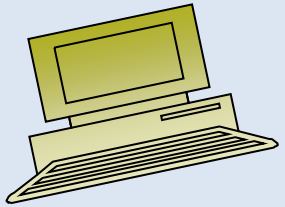
The *control chart* is a statistical quality control tool used in the monitoring variation in the characteristics of

a product or service



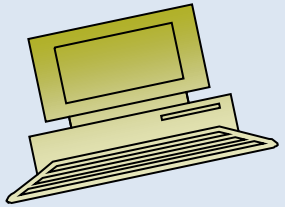
Control Charts

The control chart focuses on the time dimension and the nature of the variability in the system.



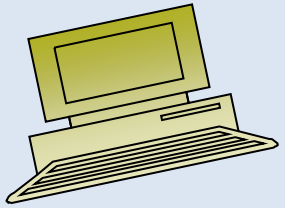
Control Charts

The control chart may be used to study past performance and/or to evaluate present conditions.

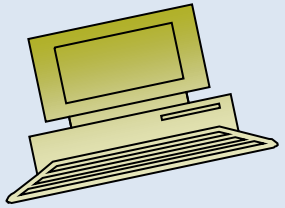


Control Charts

Data collected from a control chart may form the basis for process improvement.

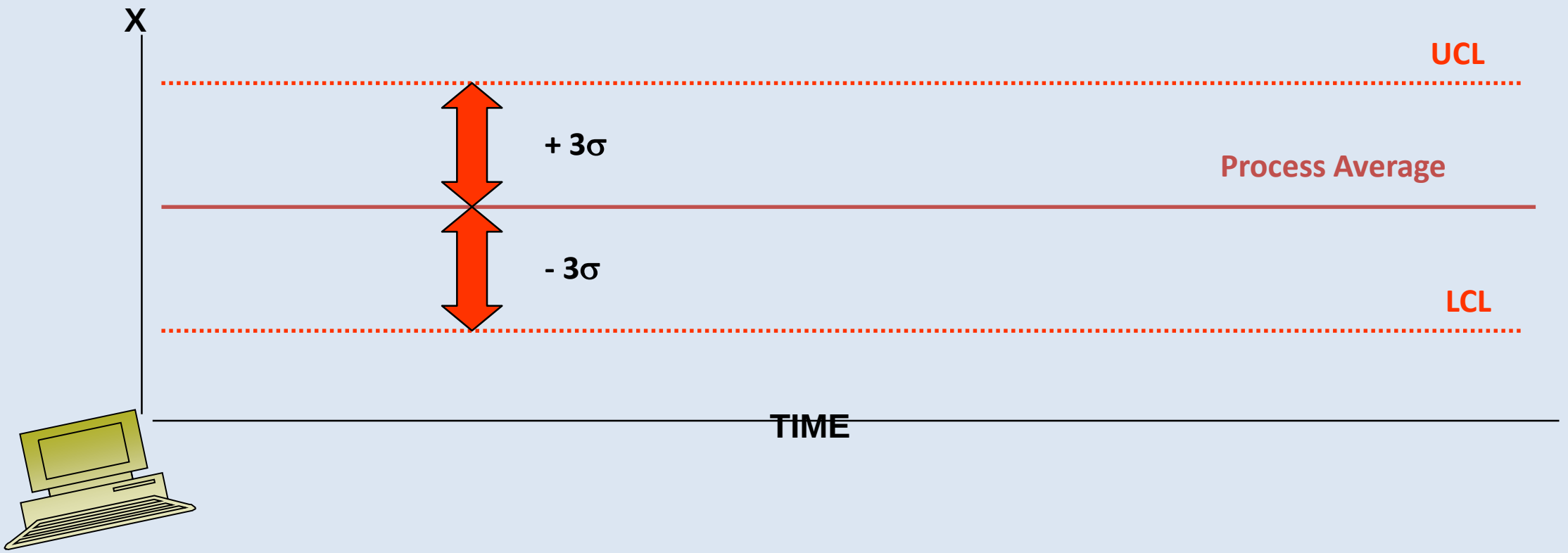


- Control chart - a graph used in SPC that shows whether a sample of data falls within the normal range of variation.
- Control chart: central line (CL), upper (UCL) and lower control limits (LCL).
  - Control limits separate common from assignable causes of variation
- Control charts for variables
  - monitor characteristics that can be measured
- Control charts for attributes
  - monitor characteristics that can be counted



Control Charts

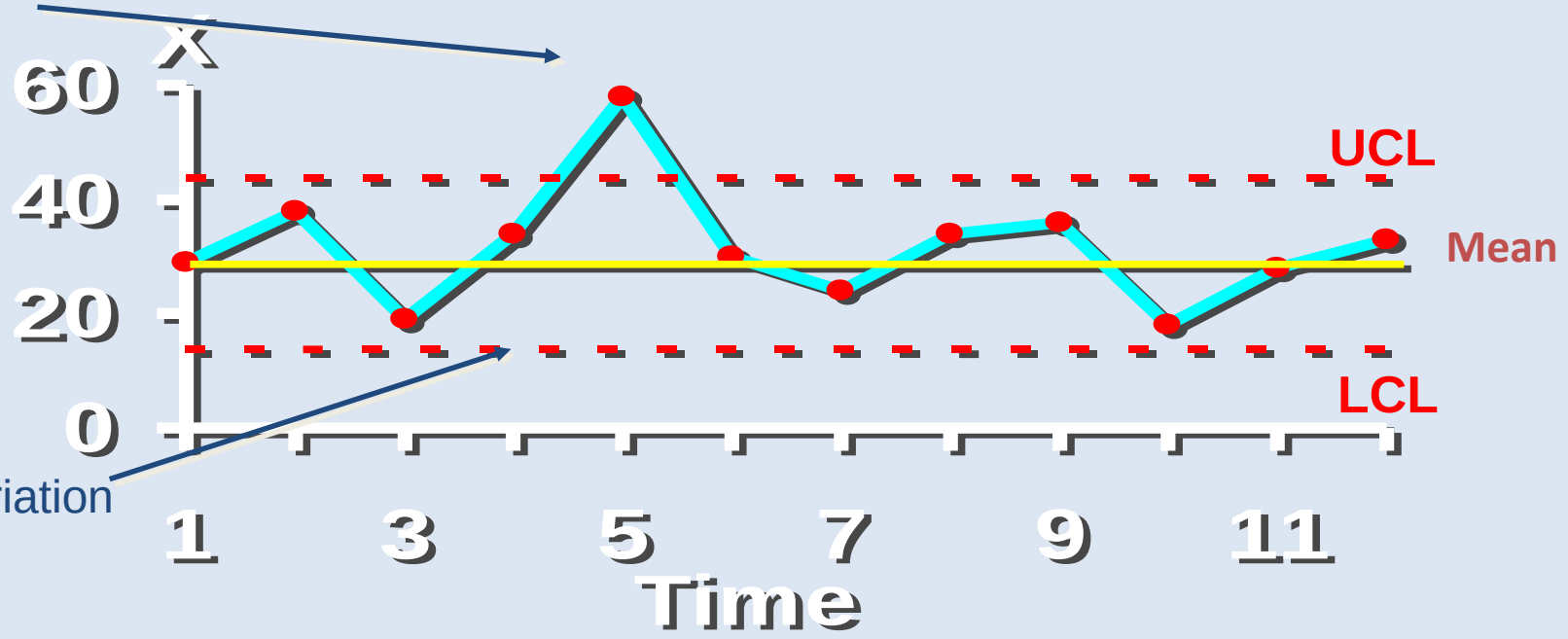
- UCL = Process Average + 3 Standard Deviations
- LCL = Process Average - 3 Standard Deviations



Control Charts

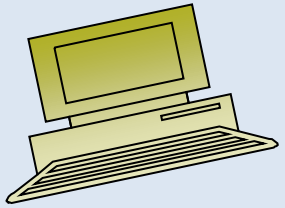
- Graph of sample data plotted over time

Assignable Cause Variation

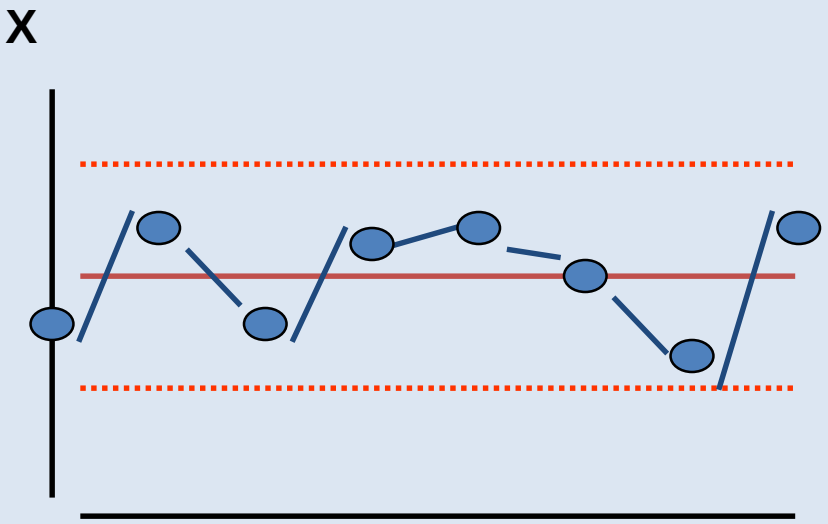


Process Average ±3σ

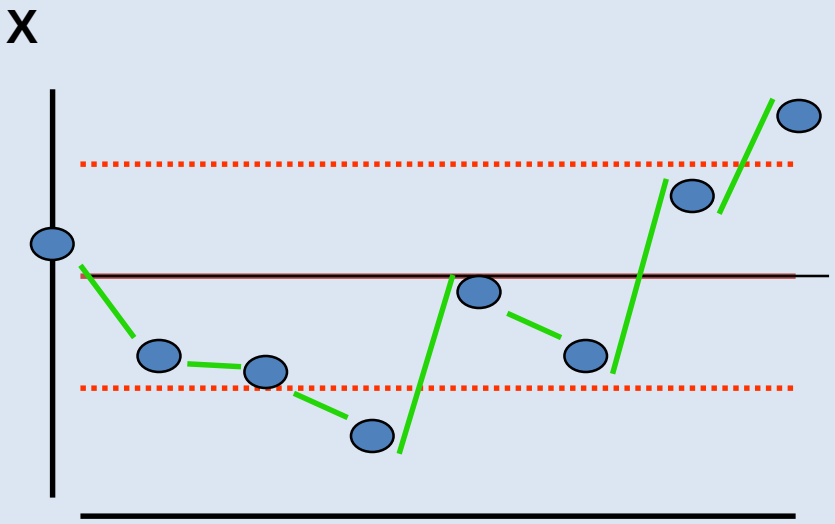
Random Variation



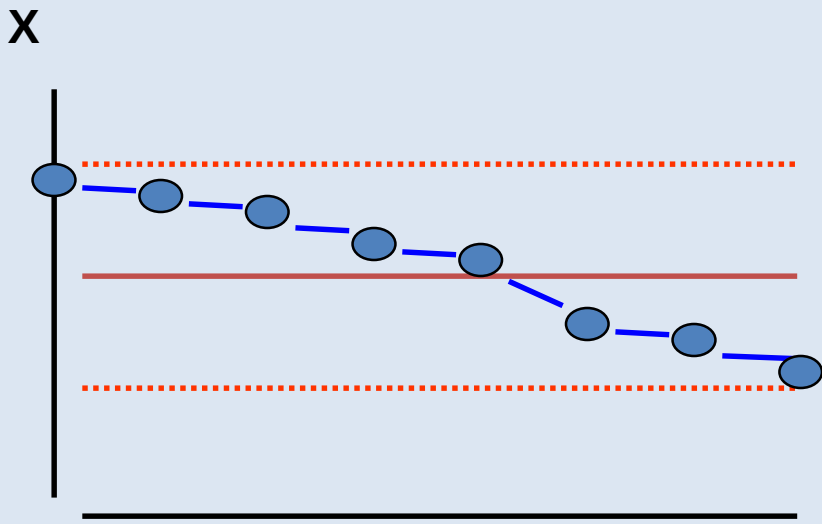
Control Charts



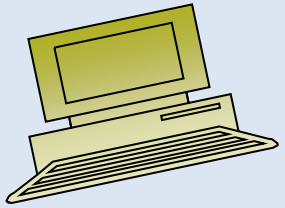
Common Cause Variation: no points outside control limit



Special Cause Variation: two points outside control limit

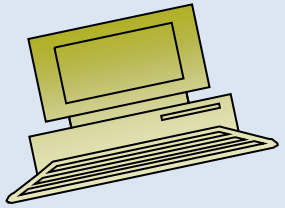


Downward Pattern: no points outside control limit; however, eight or more points in trend



Control Charts

- **Attribute charts**
- **Variables charts**



Control Charts

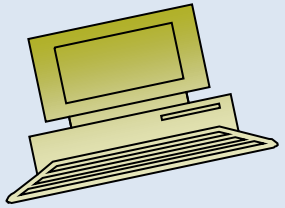
Charts may

be used for

categorical

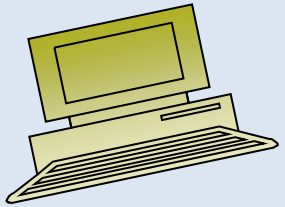
variables.

[i.e.: attributes]



Control Charts

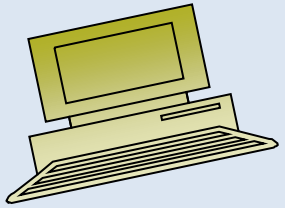
Whenever a character of interest is measured on a nominative or an interval, i.e.: categorical, scale…an *attributes* control chart is used to monitor a process.



Control Charts

- **Attributes Control Charts**

counts	[c-chart]
proportion	[p-charts]



Control Charts

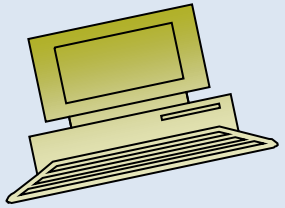
- **Attributes Control Charts**

when sample size are not constant

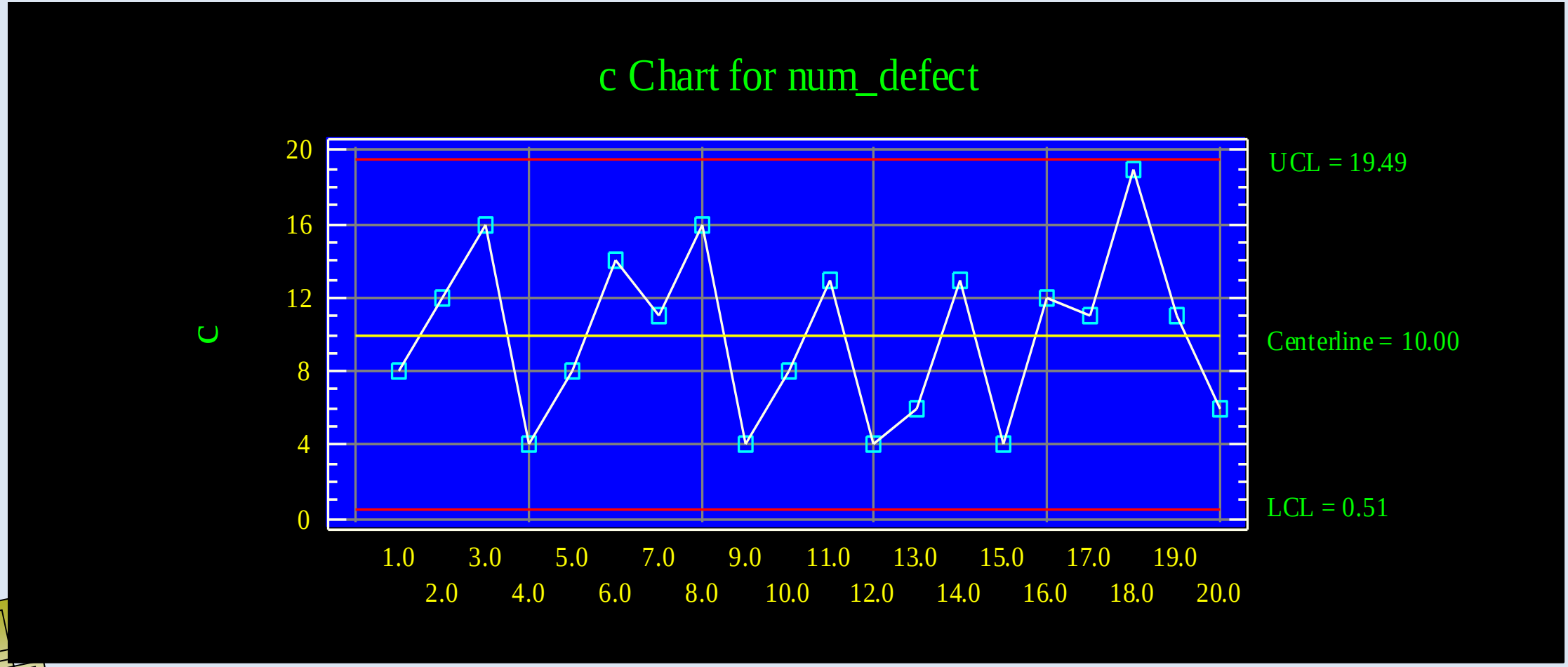
and/or are unknown

use counts charts

[c-charts]



Control Charts



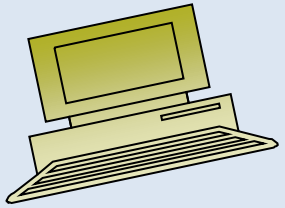
Control Charts

- **Attributes Control Charts**

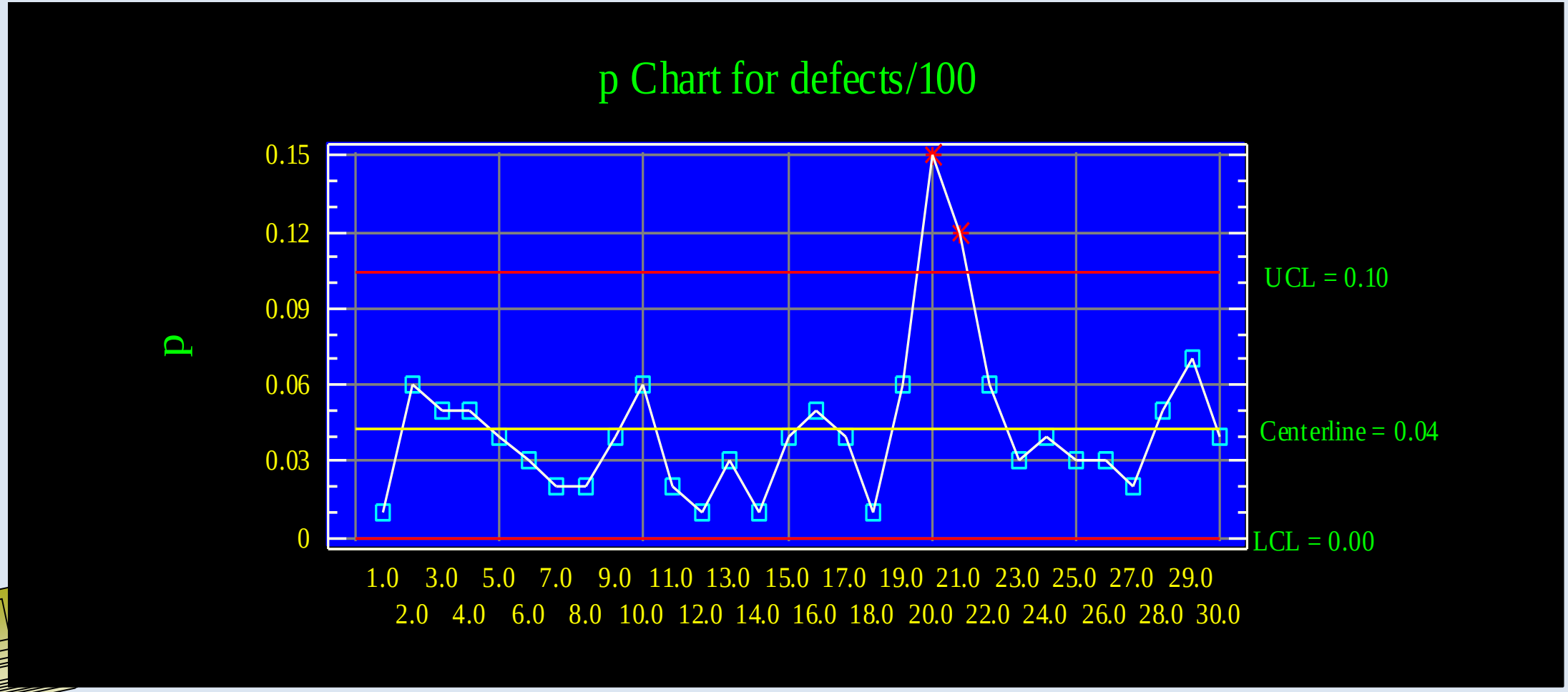
when sample size are constant and are known

use proportion charts

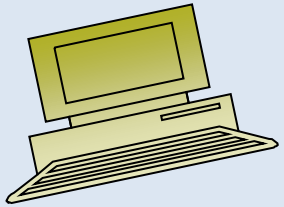
[p-charts]



Control Charts



Control Charts

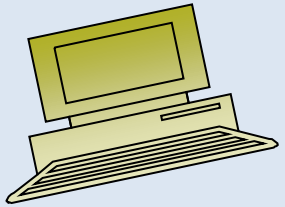


Charts may

be used for interval or ratio data [i.e.: *variables*]

Control Charts

Whenever a character of interest is measured on an interval or a ratio scale, a *variables* control chart is used to monitor a process.

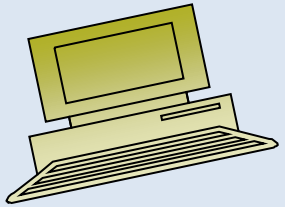


Control Charts

- **Variables Control Charts**

  Mean and Range charts

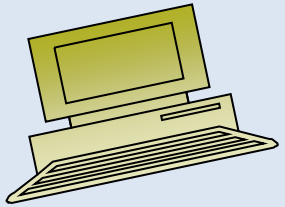
  [x-bar & R charts]



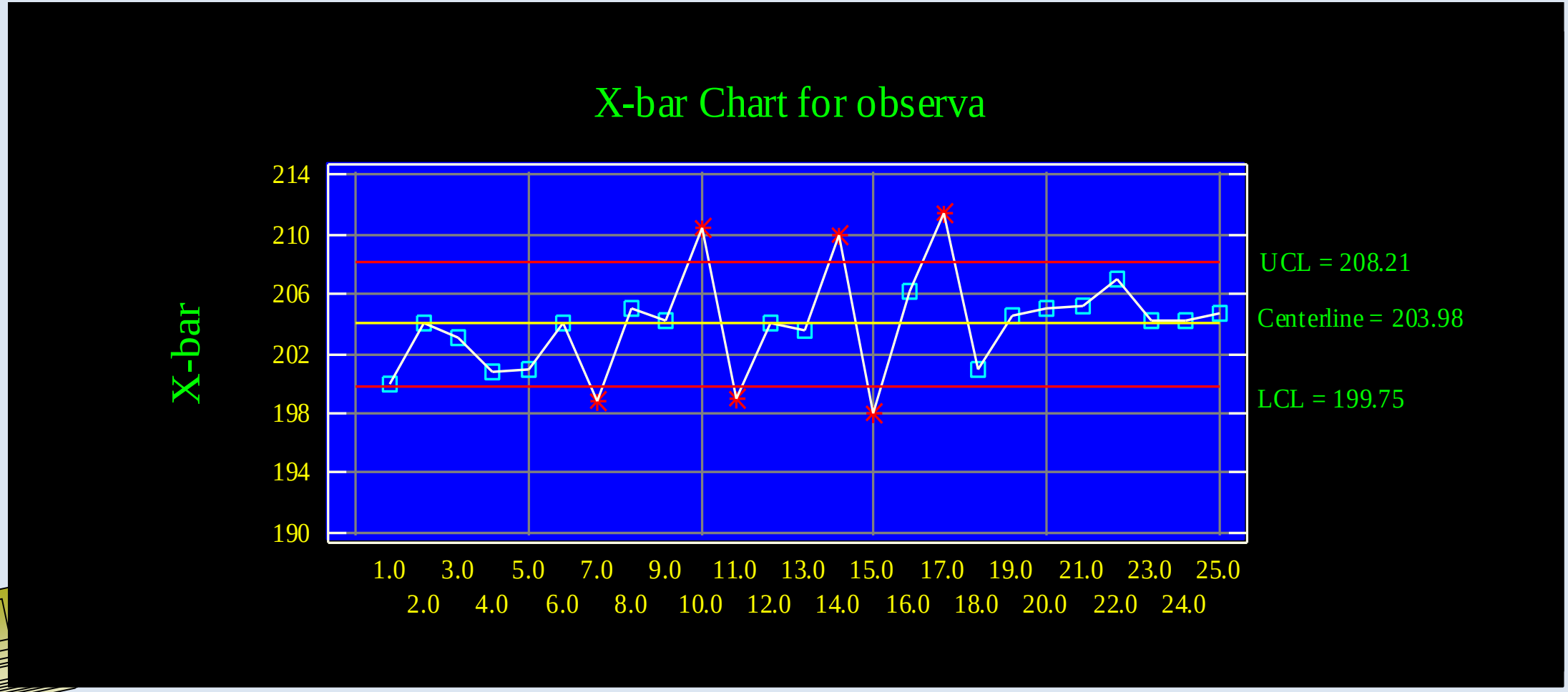
Control Charts

Variables **control charts are typically used in pairs.**

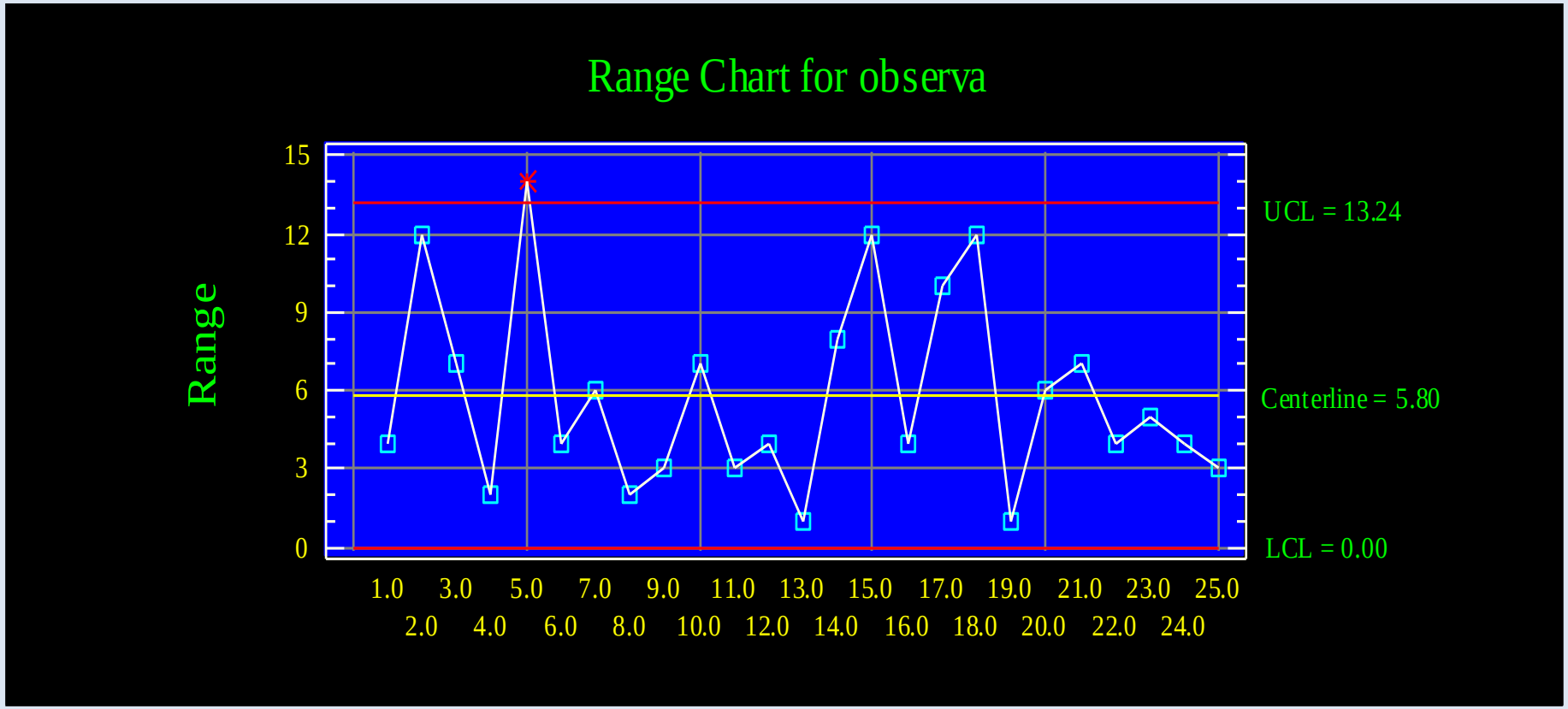
…… one chart monitors *process average* while the other monitors the *variation* in a process.

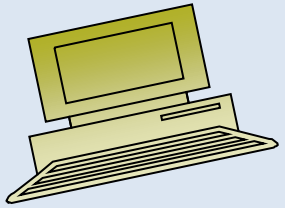


Control Charts



Control Charts

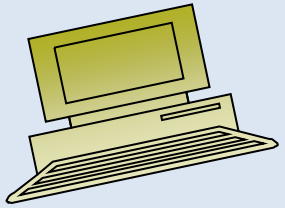




Average Run Length (ARL)

- Expected number of samples taken before shift is detected is called the Average Run Length (ARL)

$$ARL = \sum_{r=1}^{\infty} r\beta^{r-1}(1-\beta) = \frac{1}{(1-\beta)}$$

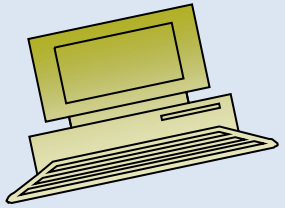


Performance of Any Shewhart Control Chart

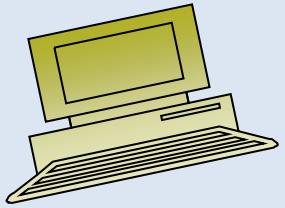
- In-Control ARL:
  - Average number of points plotted on control chart before a false alarm occurs (ideally, should be large)

- Out-of-Control ARL: $ARL_0 = \frac{1}{\alpha}$
  - Average number of points, after the process goes out-of-control, before the control chart detects it (ideally, should be small)

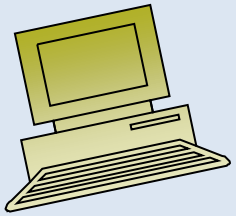
$$ARL_1 = \frac{1}{1-\beta}$$



Practice



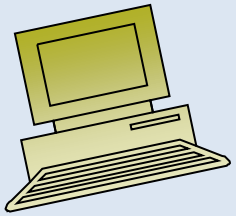
Lecture 14



Hypothesis testing

HYPOTHESIS TESTING

- T-test
- F-test



Lecture 15

Introduction to Linear Regression and Correlation Analysis in SPSS

Goals

After this, you should be able to:

- Calculate and interpret the simple correlation between two variables
- Determine whether the correlation is significant
- Calculate and interpret the simple linear regression equation for a set of data
- Understand the assumptions behind regression analysis
- Determine whether a regression model is significant

Goals

(continued)

After this, you should be able to:

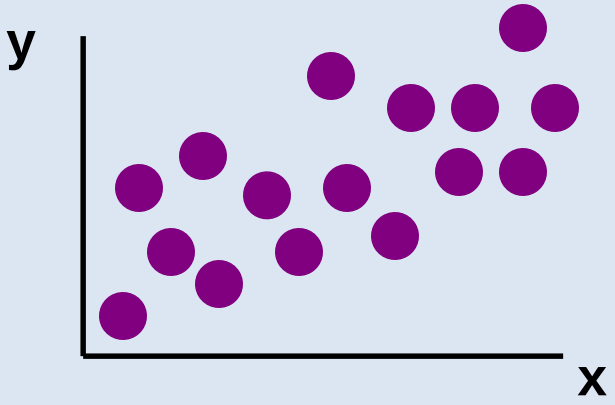
- Calculate and interpret confidence intervals for the regression coefficients
- Recognize regression analysis applications for purposes of prediction and description
- Recognize some potential problems if regression analysis is used incorrectly
- Recognize nonlinear relationships between two variables

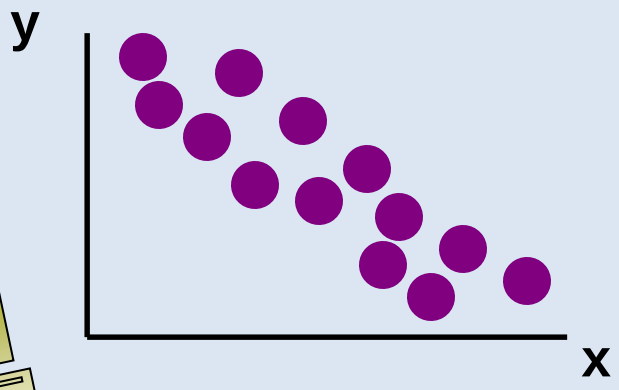
Scatter Plots and Correlation

- A scatter plot (or scatter diagram) is used to show the relationship between two variables
- Correlation analysis is used to measure strength of the association (linear relationship) between two variables
  - Only concerned with strength of the relationship
  - No causal effect is implied

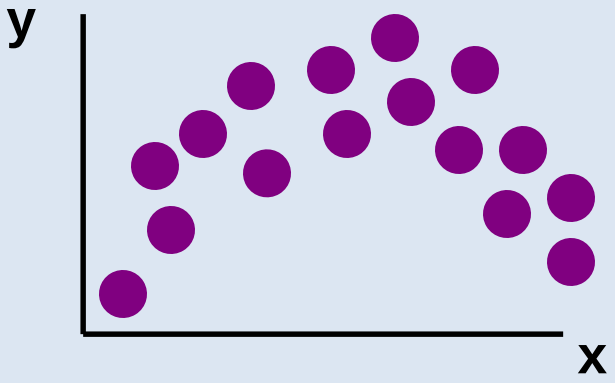
Scatter Plot Examples

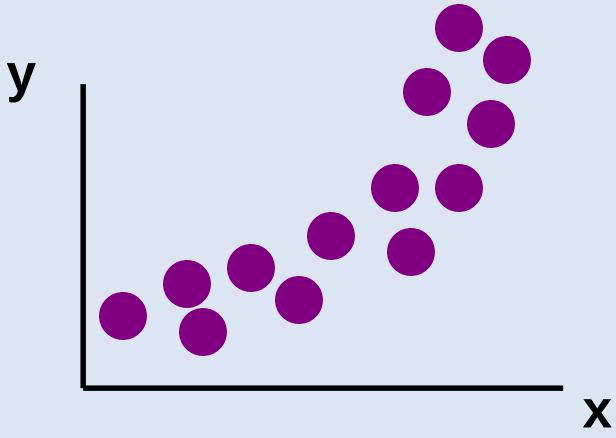
Linear relationships





Curvilinear relationships

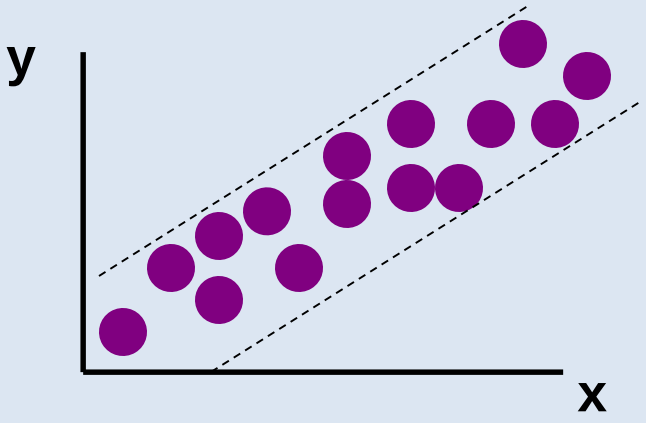


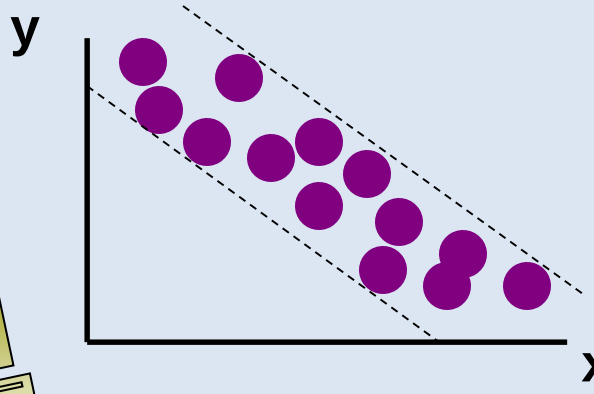


Scatter Plot Examples

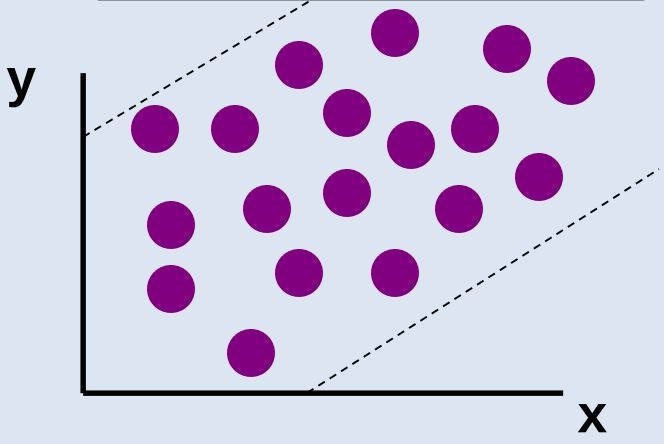
(continued)

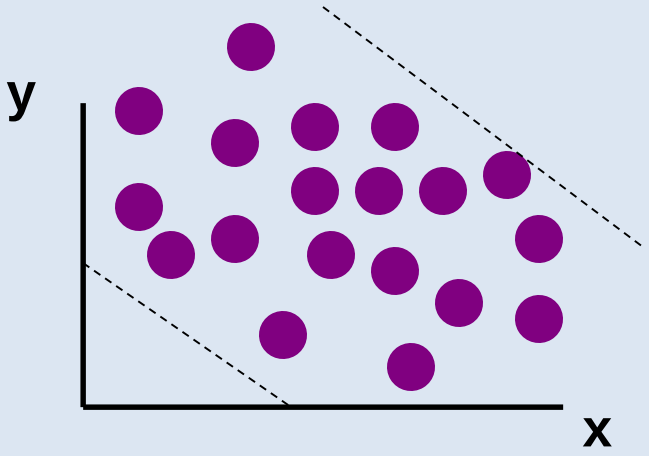
Strong relationships





Weak relationships

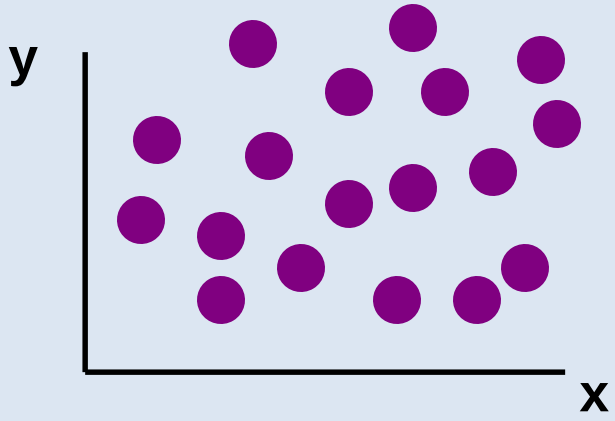


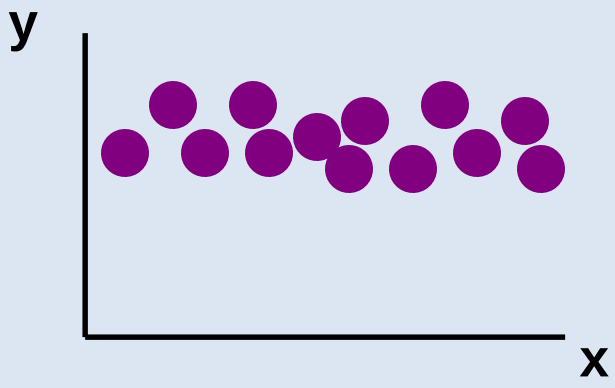


Scatter Plot Examples

(continued)

No relationship





Correlation Coefficient

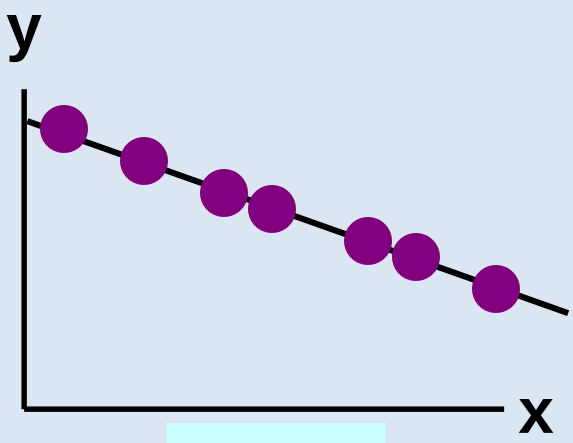
(continued)

- The population correlation coefficient ρ (rho) measures the strength of the association between the variables
- The sample correlation coefficient r is an estimate of ρ and is used to measure the strength of the linear relationship in the sample observations

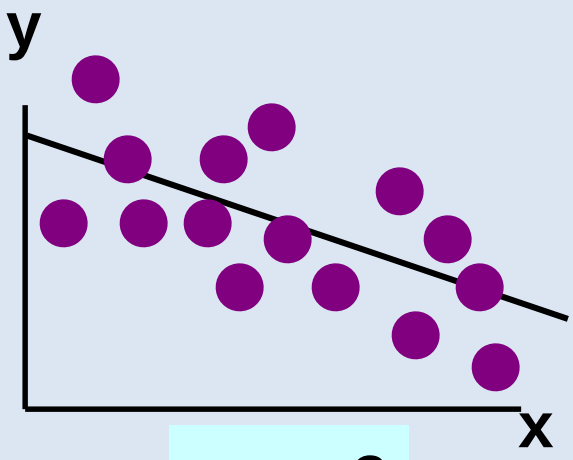
Features of ρ and r

- Unit free
- Range between -1 and 1
- The closer to -1, the stronger the negative linear relationship
- The closer to 1, the stronger the positive linear relationship
- The closer to 0, the weaker the linear relationship

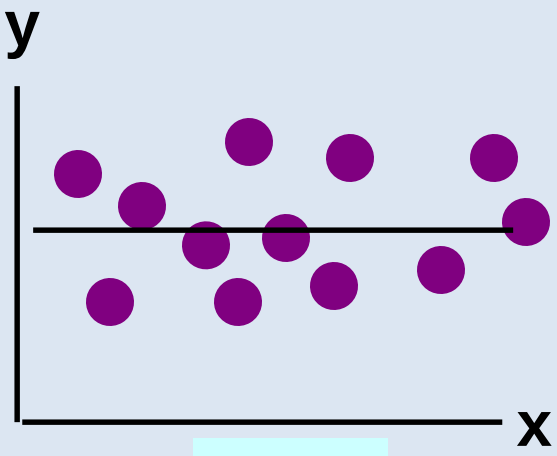
Examples of Approximate r Values



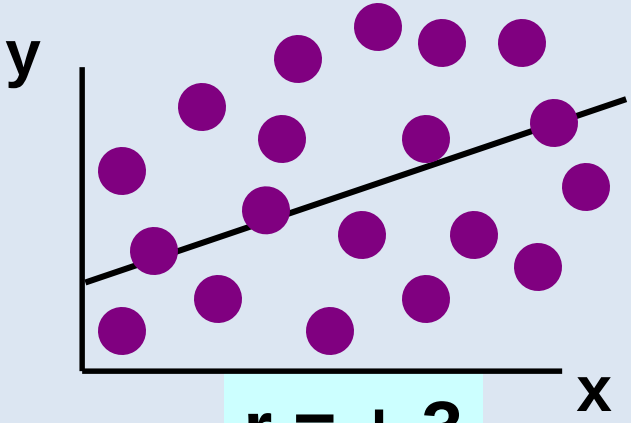
r = -1



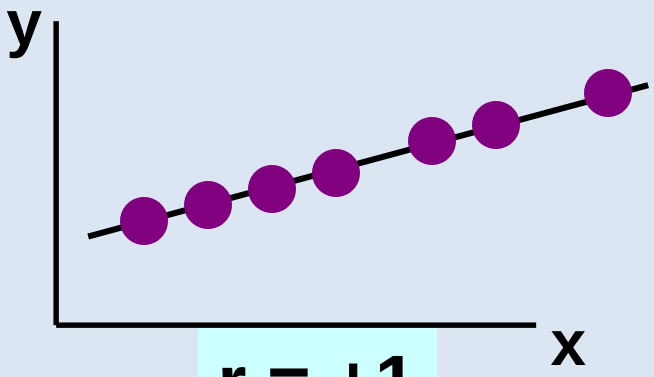
r = -.6



r = 0



r = +.3



r = +1

Calculating the Correlation Coefficient

Sample correlation coefficient:

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{[\sum (x - \bar{x})^2][\sum (y - \bar{y})^2]}}$$

or the algebraic equivalent:

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

where:

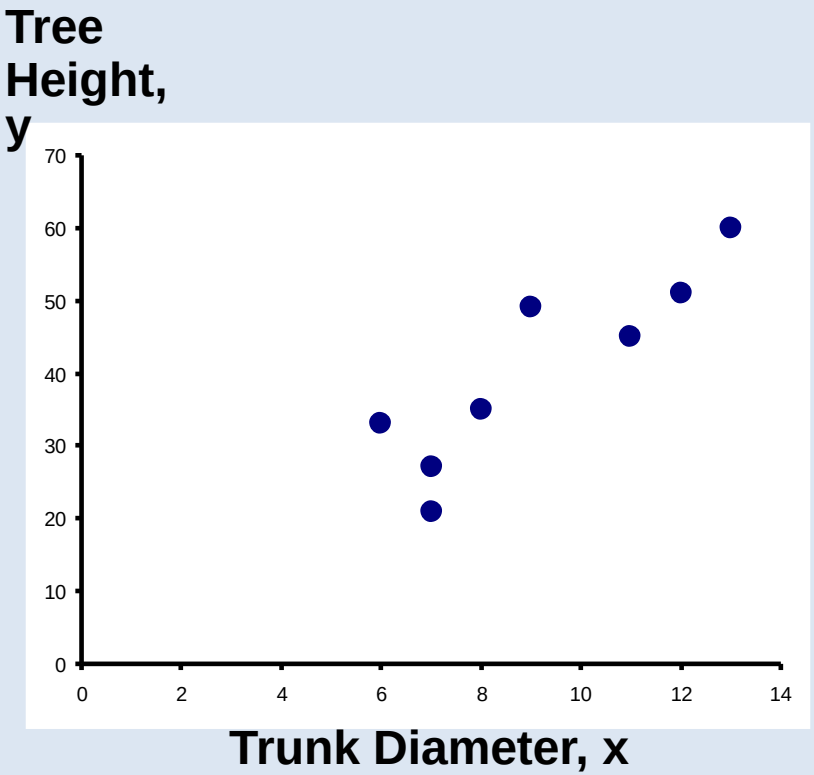
r = Sample correlation coefficient
n = Sample size
x = Value of the independent variable
y = Value of the dependent variable

Calculation Example

Tree Height	Trunk Diameter			
y	x	xy	y^2	x^2
35	8	280	1225	64
49	9	441	2401	81
27	7	189	729	49
33	6	198	1089	36
60	13	780	3600	169
21	7	147	441	49
45	11	495	2025	121
51	12	612	2601	144
Σ=321	**Σ=73**	**Σ=3142**	**Σ=14111**	**Σ=713**

Calculation Example

(continued)



$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$= \frac{8(3142) - (73)(321)}{\sqrt{[8(713) - (73)^2][8(14111) - (321)^2]}}$$

$$= 0.886$$

r = 0.886 → relatively strong positive linear association between x and y

Excel Output

Excel Correlation Output

Tools / data analysis / correlation…

	Tree Height	Trunk Diameter
Tree Height	1	
Trunk Diameter	0.886231	1

Correlation between

Tree Height and Trunk Diameter

Significance Test for Correlation

- Hypotheses

H_0: $\rho = 0$ (no correlation)

H_A: $\rho \neq 0$ (correlation exists)

- Test statistic

– $$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}}$$

(with n – 2 degrees of freedom)

Example: Produce Stores

Is there evidence of a linear relationship between tree height and trunk diameter at the .05 level of significance?

H_0: ρ = 0 (No correlation)

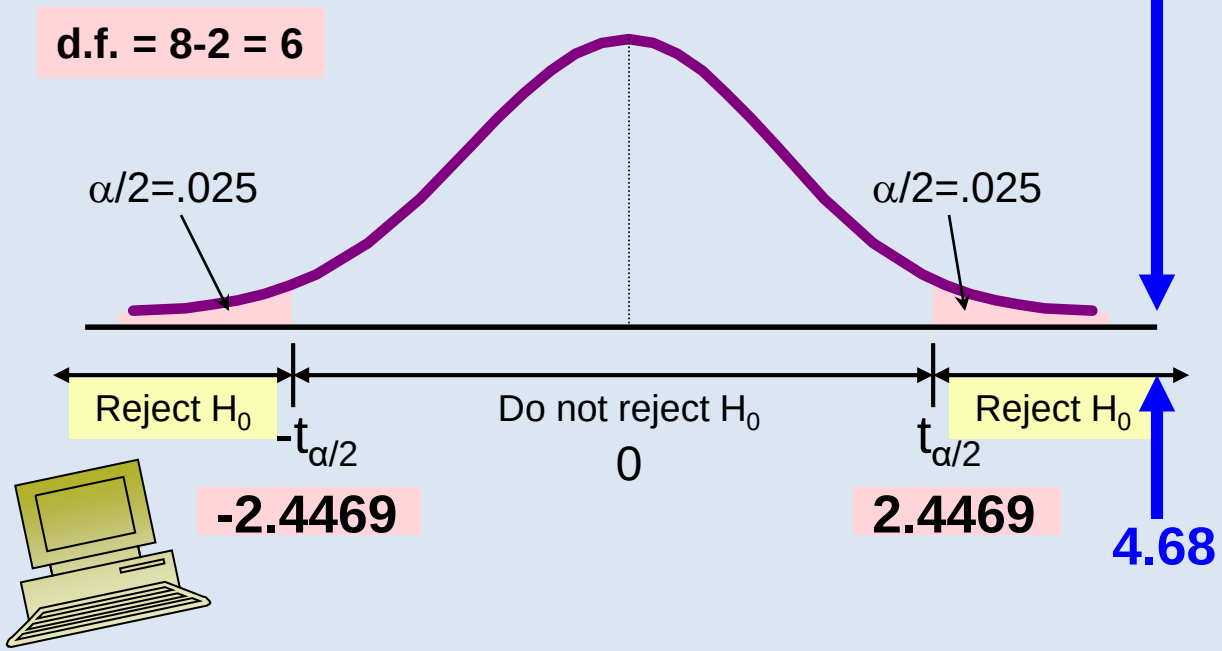
H_1: ρ ≠ 0 (correlation exists)

α =.05 , df = 8 - 2 = 6

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}} = \frac{.886}{\sqrt{\frac{1-.886^2}{8-2}}} = 4.68$$

Example: Test Solution

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}} = \frac{.886}{\sqrt{\frac{1-.886^2}{8-2}}} = 4.68$$



Decision:
Reject H_0

Conclusion:
There **is evidence** of a linear relationship at the 5% level of significance

Introduction to Regression Analysis

- Regression analysis is used to:
  - Predict the value of a dependent variable based on the value of at least one independent variable
  - Explain the impact of changes in an independent variable on the dependent variable

Dependent variable: the variable we wish to explain

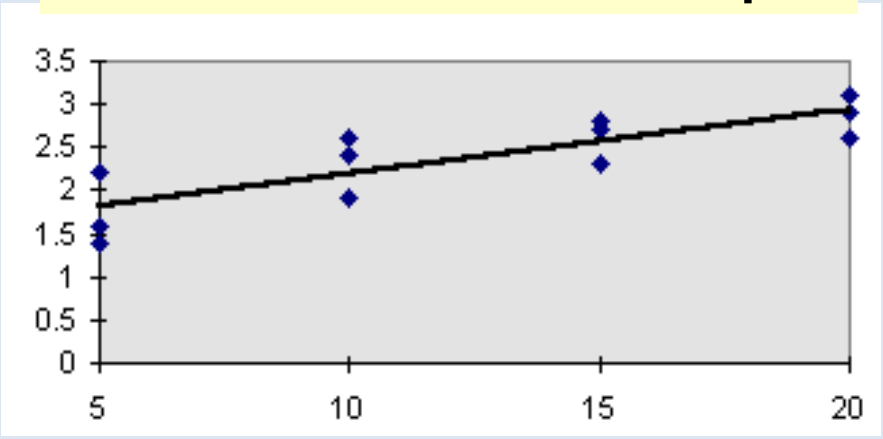
Independent variable: the variable used to explain the dependent variable

Simple Linear Regression Model

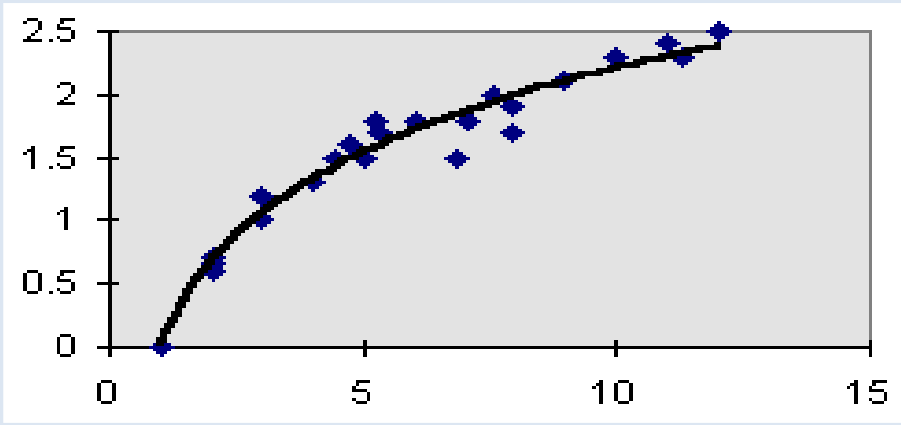
- Only **one** independent variable, x
- Relationship between x and y is described by a linear function
- Changes in y are assumed to be caused by changes in x

Types of Regression Models

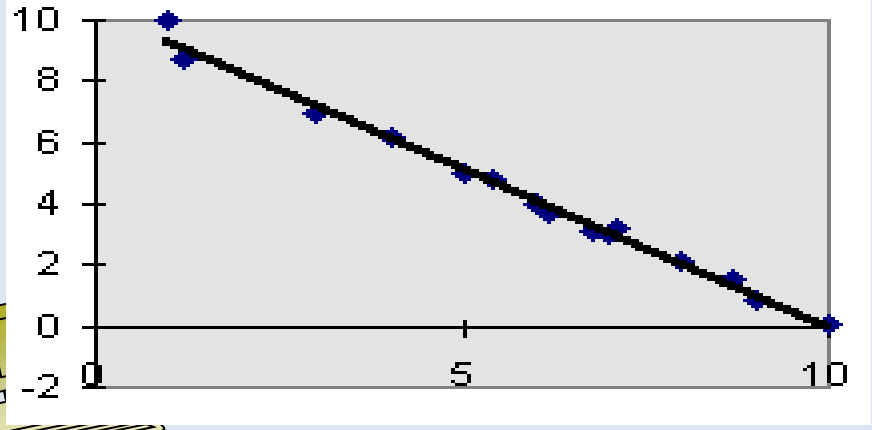
Positive Linear Relationship



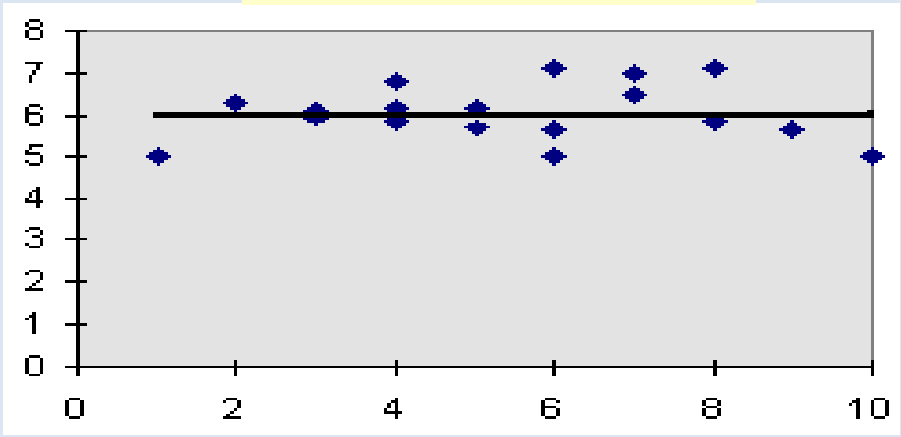
Relationship NOT Linear



Negative Linear Relationship

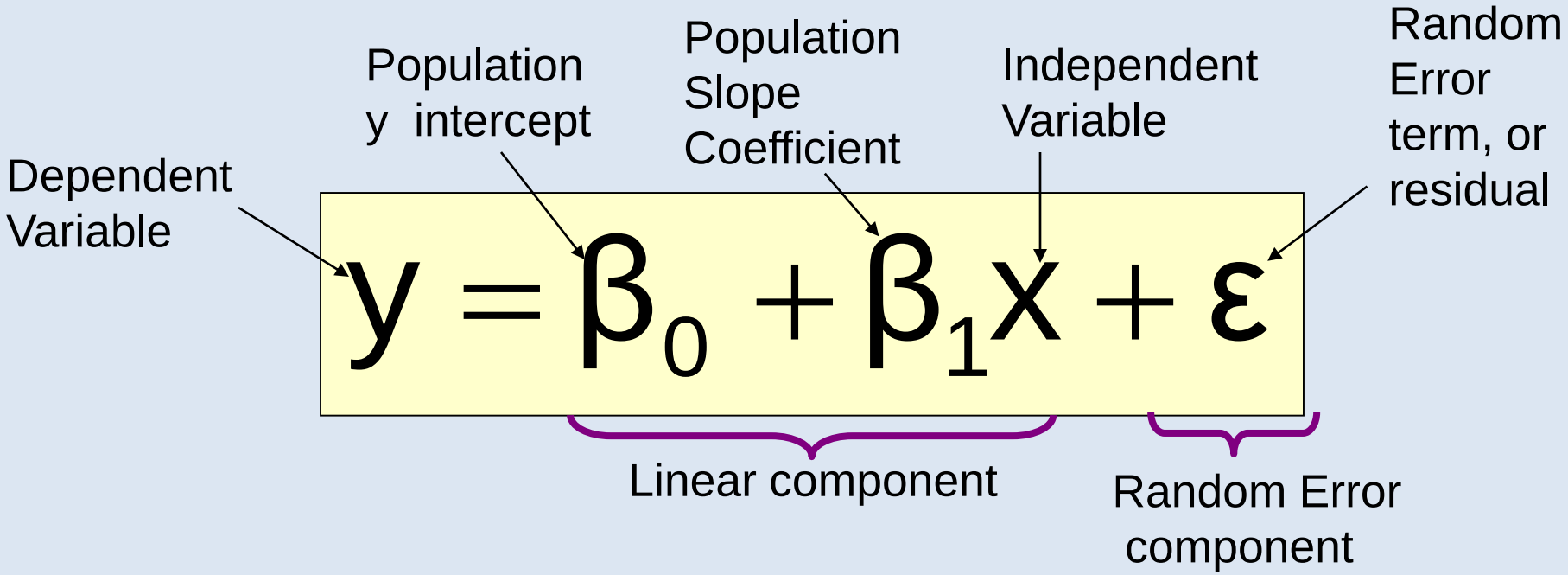


No Relationship



Population Linear Regression

The population regression model:



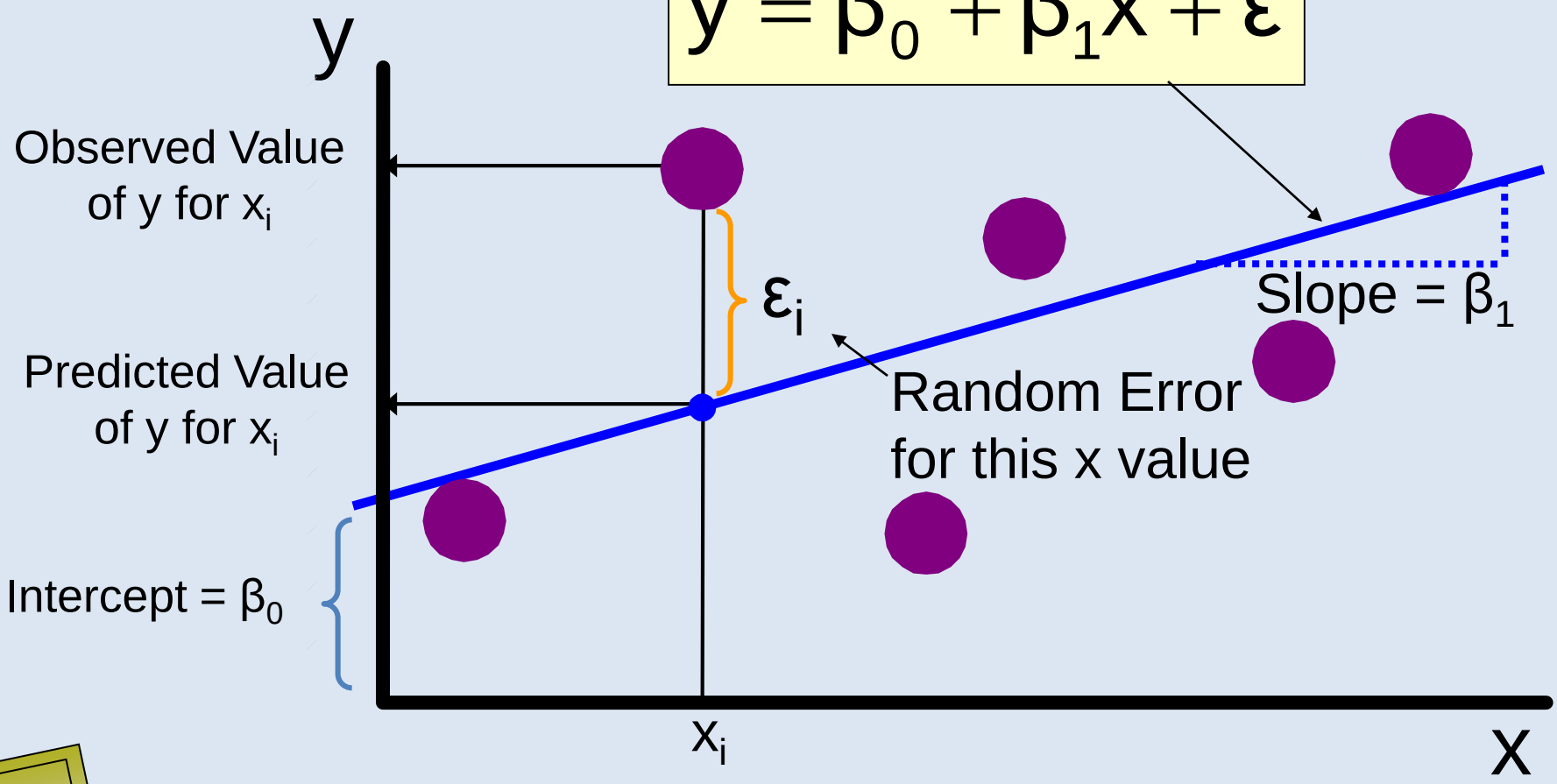
Linear Regression Assumptions

- Error values (ε) are statistically independent
- Error values are normally distributed for any given value of x
- The probability distribution of the errors is normal
- The probability distribution of the errors has constant variance
- The underlying relationship between the x variable and the y variable is linear

Population Linear Regression

(continued)

$$y = \beta_0 + \beta_1 x + \varepsilon$$



Estimated Regression Model

The sample regression line provides an estimate of the population regression line

Estimated (or predicted) y value

Estimate of the regression intercept

Estimate of the regression slope

Independent variable

$$\hat{y}_i = b_0 + b_1 x$$

The individual random error terms e_i have a mean of zero

Least Squares Criterion

- b_0 and b_1 are obtained by finding the values of b_0 and b_1 that minimize the sum of the squared residuals

$$\sum e^2 = \sum (y - \hat{y})^2$$

$$= \sum (y - (b_0 + b_1 x))^2$$

The Least Squares Equation

- The formulas for b_1 and b_0 are:

$$b_1 = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

algebraic equi

$$b_1 = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}$$

and

$$b_0 = \bar{y} - b_1 \bar{x}$$

Interpretation of the Slope and the Intercept

- b_0 is the estimated average value of y when the value of x is zero

- b_1 is the estimated change in the average value of y as a result of a one-unit change in x

Finding the Least Squares Equation

- The coefficients b_0 and b_1 will usually be found using computer software, such as Excel
- Other regression measures will also be computed as part of computer-based regression analysis

Simple Linear Regression Example

- A real estate agent wishes to examine the relationship between the selling price of a home and its size (measured in square feet)

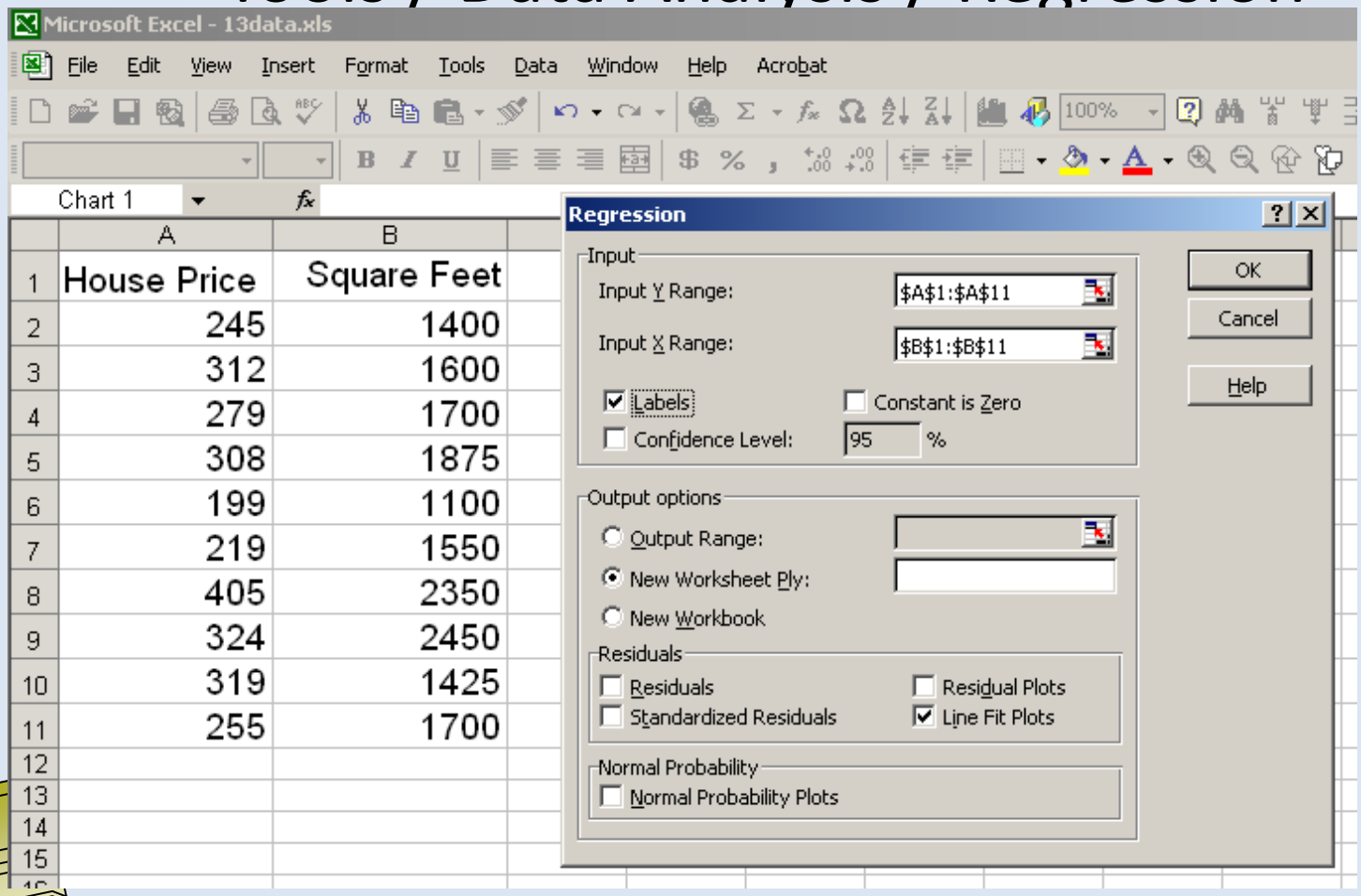
- A random sample of 10 houses is selected
  - Dependent variable (y) = house price in $1000s
  - Independent variable (x) = square feet

Sample Data for House Price Model

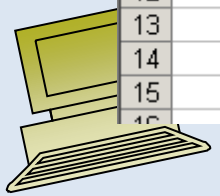
House Price in $1000s (y)	Square Feet (x)
245	1400
312	1600
279	1700
308	1875
199	1100
219	1550
405	2350
324	2450
319	1425
255	1700

Regression Using Excel

- Tools / Data Analysis / Regression



	A	B
1	House Price	Square Feet
2	245	1400
3	312	1600
4	279	1700
5	308	1875
6	199	1100
7	219	1550
8	405	2350
9	324	2450
10	319	1425
11	255	1700
12		
13		
14		
15		



Excel Output

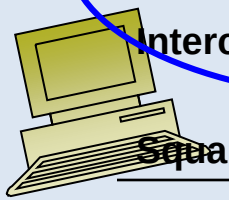
Regression Statistics	
Multiple R	0.76211
R Square	0.58082
Adjusted R Square	0.52842
Standard Error	41.33032
Observations	10

The regression equation is:

$$\widehat{\text{house price}} = 98.24833 + 0.10977\ (\text{square feet})$$

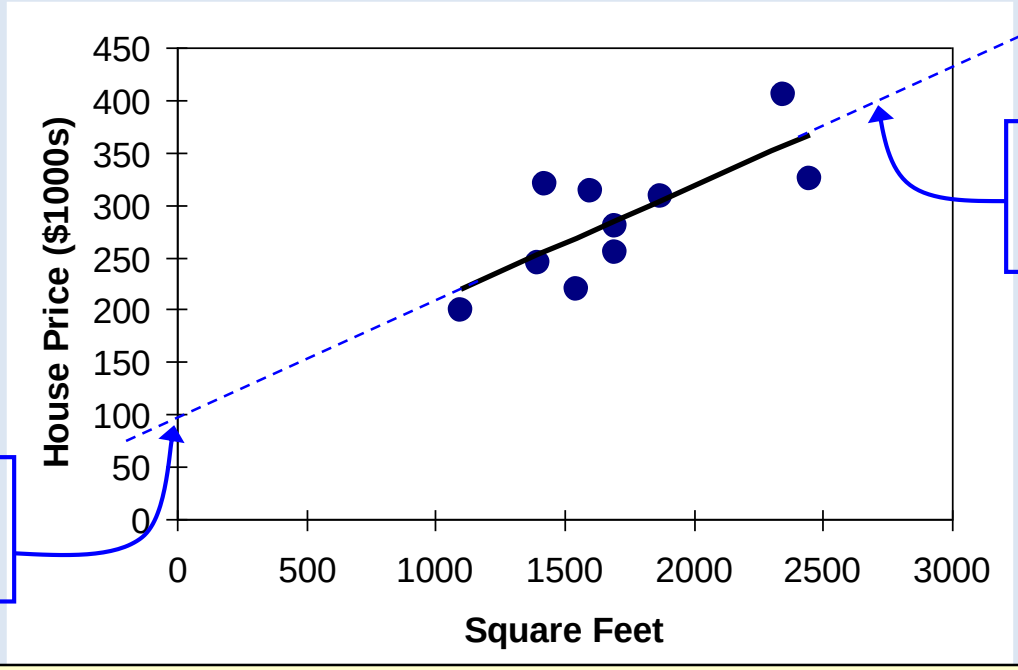
ANOVA	*df*	*SS*	*MS*	*F*	*Significance F*
Regression	1	18934.9348	18934.9348	11.0848	0.01039
Residual	8	13665.5652	1708.1957		
Total	9	32600.5000			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	98.24833	58.03348	1.69296	0.12892	-35.57720	232.07386
Square Feet	0.10977	0.03297	3.32938	0.01039	0.03374	0.18580



Graphical Presentation

- House price model: scatter plot and regression line



$$\widehat{\text{house price}} = 98.24833 + 0.10977\ (\text{square feet})$$

Interpretation of the Intercept, b_0

$$\widehat{\text{house price}} = 98.24833 + 0.10977\ (\text{square feet})$$

- b_0 is the estimated average value of Y when the value of X is zero (if x = 0 is in the range of observed x values)
  - Here, no houses had 0 square feet, so b_0 = 98.24833 just indicates that, for houses within the range of sizes observed, $98,248.33 is t portion of the house price not explained by square feet

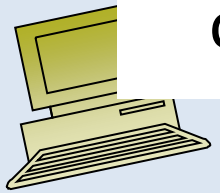
Interpretation of the Slope Coefficient, b_1

$$\widehat{\text{house price}} = 98.24833 + \boxed{0.10977}\,(\text{square feet})$$

- b_1 measures the estimated change in the average value of Y as a result of a one-unit change in X
  - Here, b_1 = .10977 tells us that the average value of a house increases by .10977($1000) = $109.77, on average, for each additional one square foot of size

Least Squares Regression Properties

- The sum of the residuals from the least squares regression line is 0 $\left(\sum (y - \hat{y}) = 0\right)$
- The sum of the squared residuals is a minimum (minimized $\sum (y - \hat{y})^2$)
- The simple regression line always passes through the mean of the y variable and the mean of the x variable
- The least squares coefficients are unbiased estimates of β_0 and β_1



Explained and Unexplained Variation

- Total variation is made up of two parts:

$$SST = SSE + SSR$$

Total sum of Squares	Sum of Squares Error	Sum of Squares Regression
$SST = \sum (y - \bar{y})^2$	$SSE = \sum (y - \hat{y})^2$	$SSR = \sum (\hat{y} - \bar{y})^2$

where:

$\bar{y}$ = Average value of the dependent variable

y = Observed values of the dependent variable

$\hat{y}$ = Estimated value of y for the given x value

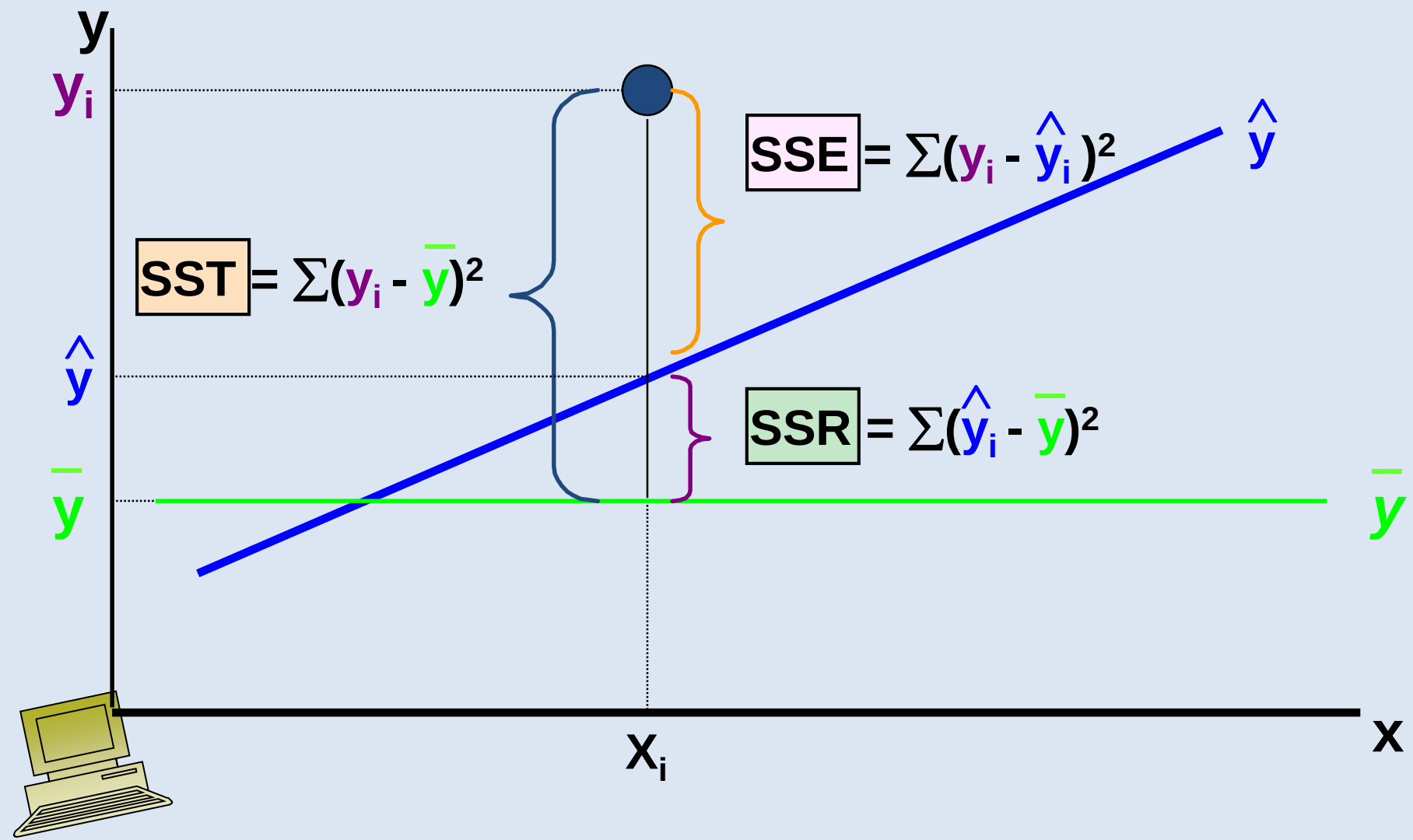
Explained and Unexplained Variation

(continued)

- SST = total sum of squares
  - Measures the variation of the y_i values around their mean y
  - SSE = error sum of squares
  - Variation attributable to factors other than the relationship between x and y
- SSR = regression sum of squares
  - Explained variation attributable to the relationship between x and y

Explained and Unexplained Variation

(continued)



Coefficient of Determination, R^2

- The coefficient of determination is the portion of the total variation in the dependent variable that is explained by variation in the independent variable

- The coefficient of determination is also called R-squared and is denoted as R^2

$$R^2 = \frac{SSR}{SST}$$

where $0 \le R^2 \le 1$

Coefficient of Determination, R^2

(continued)

Coefficient of determination

$$R^2 = \frac{SSR}{SST} = \frac{\text{sum of squares explained by regression}}{\text{total sum of squares}}$$

Note: In the single independent variable case, the coefficient of determination is

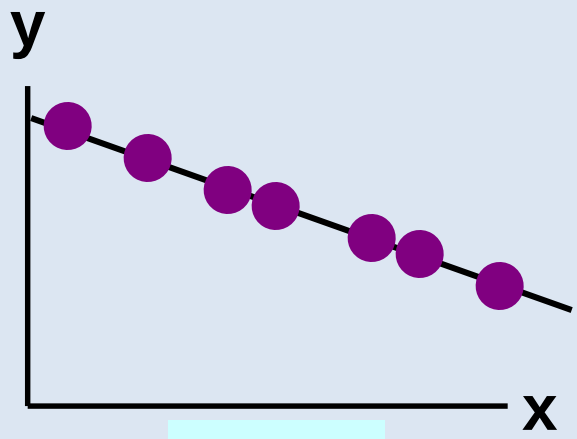
$$R^2 = r^2$$

where:

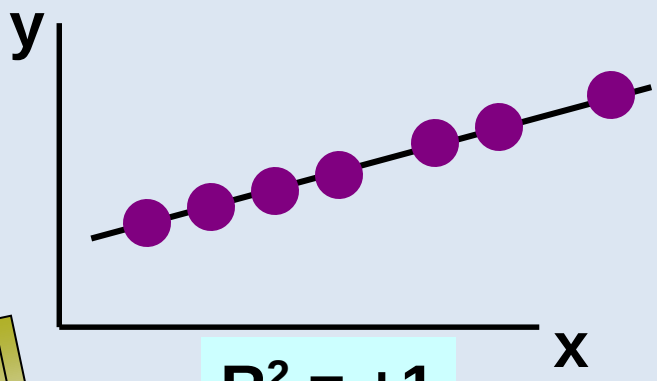
R^2 = Coefficient of determination

r = Simple correlation coefficient

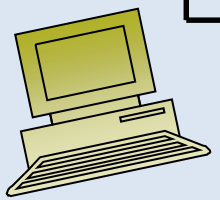
Examples of Approximate R^2 Values



$R^2 = 1$



$R^2 = +1$

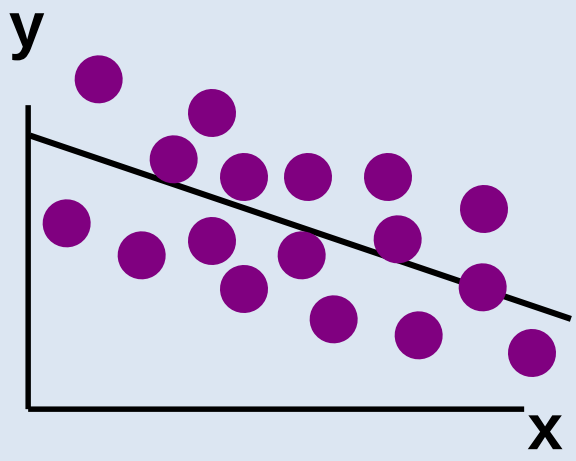


$R^2 = 1$

Perfect linear relationship between x and y:

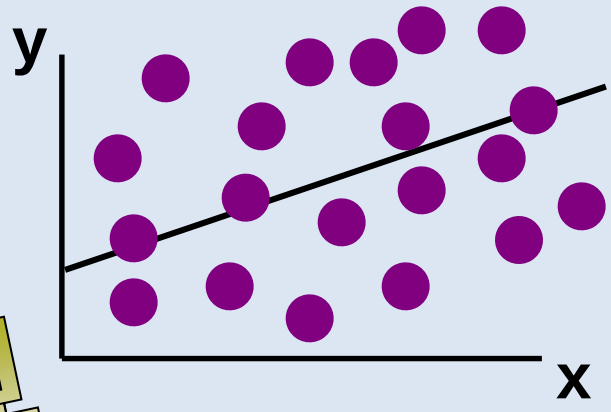
100% of the variation in y is explained by variation in x

Examples of Approximate R^2 Values

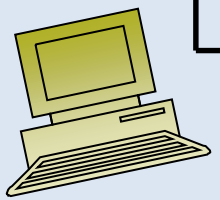


$0 < R^2 < 1$

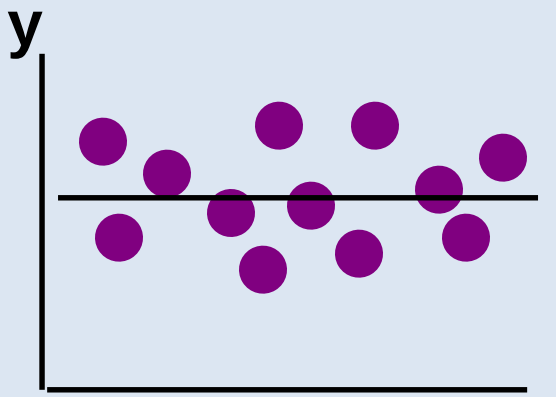
Weaker linear relationship between x and y:



Some but not all of the variation in y is explained by variation in x



Examples of Approximate R^2 Values



$R^2 = 0$

$R^2 = 0$

No linear relationship between x and y:

The value of Y does not depend on x. (None of the variation in y is explained by variation in x)

Standard Error of Estimate

- The standard deviation of the variation of observations around the regression line is estimated by

$$s_{\varepsilon} = \sqrt{\frac{SSE}{n-k-1}}$$

Where

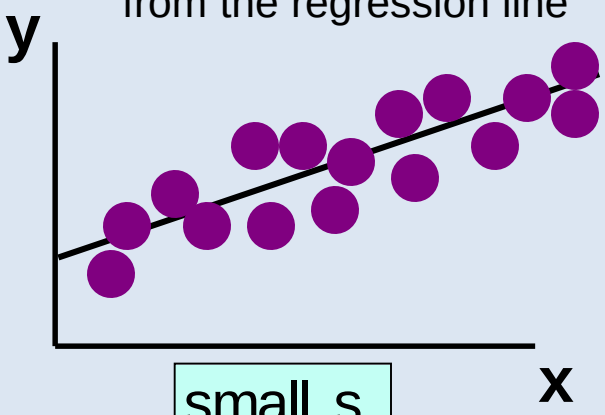
SSE = Sum of squares error

n = Sample size

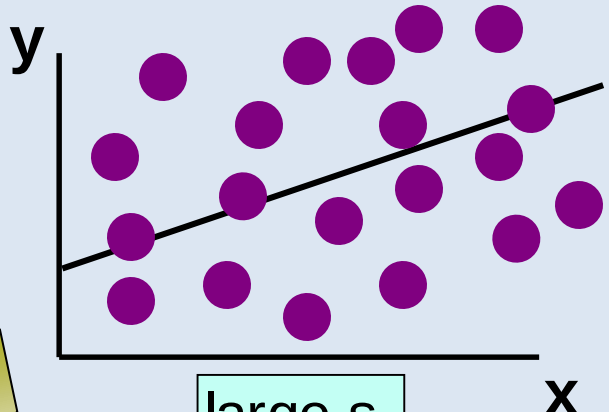
k = number of independent variables in the model

Comparing Standard Errors

Variation of observed y values from the regression line

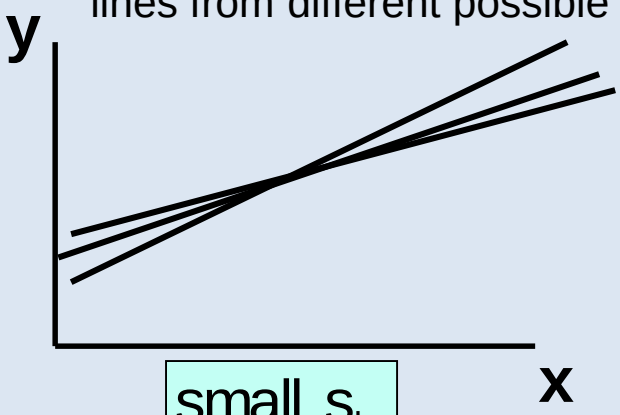


small s_ε

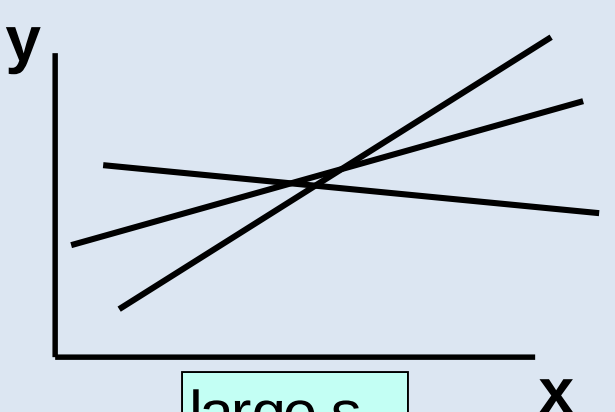


large s_ε

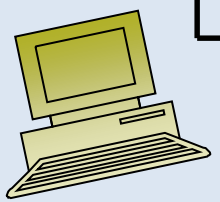
Variation in the slope of regression lines from different possible samples



small s_{b_1}



large s_{b_1}



Inference about the Slope: t Test

- t test for a population slope
  - Is there a linear relationship between x and y?
- Null and alternative hypotheses
  - H_0: $\beta_1 = 0$ (no linear relationship)
  - H_1: $\beta_1 \neq 0$ (linear relationship does exist)
- Test statistic
  -

$$t = \frac{b_1 - \beta_1}{s_{b_1}}$$

where:

b_1 = Sample regression slope coefficient

β_1 = Hypothesized slope

s_{b1} = Estimator of the standard error of the slope

$$d.f. = n - 2$$

Inference about the Slope: t Test

(continued)

House Price in $1000s (y)	Square Feet (x)
245	1400
312	1600
279	1700
308	1875
199	1100
219	1550
405	2350
324	2450
319	1425
255	1700

Estimated Regression Equation:

$$\widehat{\text{house price}} = 98.25 + 0.1098\ (\text{sq.ft.})$$

The slope of this model is 0.1098

Does square footage of the house affect its sales price?

Inferences about the Slope: t Test Example

Test Statistic: **t = 3.329**

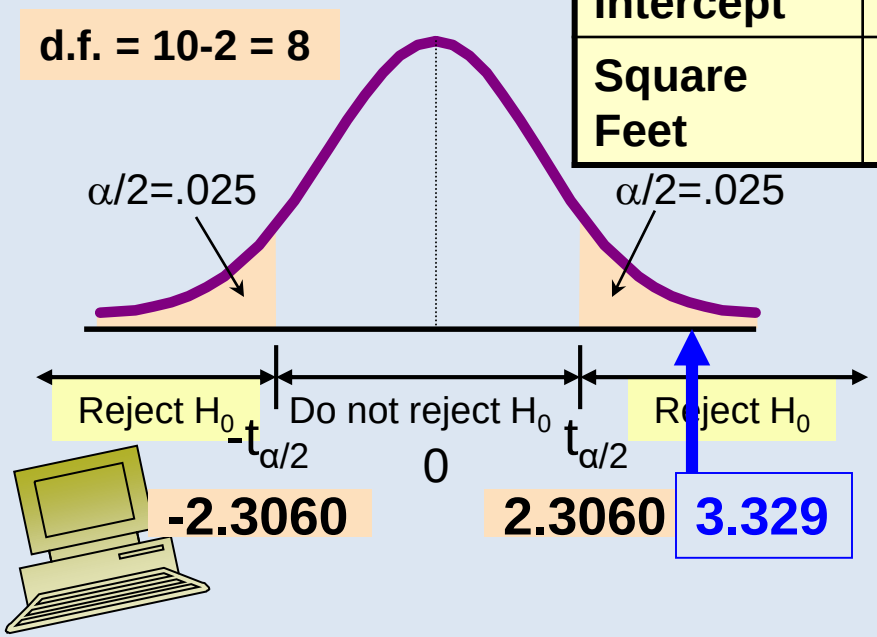
$H_0: \beta_1 = 0$

$H_A: \beta_1 \neq 0$

From Excel output:

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	**98.24833**	**58.03348**	**1.69296**	**0.12892**
Square Feet	**0.10977**	**0.03297**	**3.32938**	**0.01039**

b_1 S_{b_1} t



Reject H_0

There is sufficient evidence that square footage affects house price

Regression Analysis for Description

Confidence Interval Estimate of the Slope:

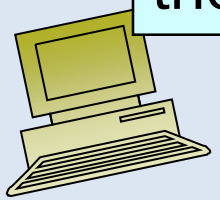
$$b_1 \pm t_{\alpha/2} s_{b_1}$$

d.f. = n - 2

Excel Printout for House Prices:

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	**98.24833**	**58.03348**	**1.69296**	**0.12892**	**-35.57720**	**232.07386**
Square Feet	**0.10977**	**0.03297**	**3.32938**	**0.01039**	**0.03374**	**0.18580**

At 95% level of confidence, the confidence interval for the slope is (0.0337, 0.1858)



Regression Analysis for Description

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	**98.24833**	**58.03348**	**1.69296**	**0.12892**	**-35.57720**	**232.07386**
Square Feet	**0.10977**	**0.03297**	**3.32938**	**0.01039**	**0.03374**	**0.18580**

Since the units of the house price variable is $1000, we are 95% confident that the average impact on sales price is between $33.70 and $185.80 per square foot of house size

This 95% confidence interval does not include 0.

Conclusion: There is a significant relationship between house price and square feet at the .05 level of significance